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MTH501 Assignment 1 Solution

Fall 2026

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MTH501: Linear Algebra

Assignment No. 1 — Fall 2026 | Total Marks: 20

Question 1 [Marks = 5]

Let T : $R^2 \rightarrow R^2$ be defined by

$$T(x_1, x_2) = (3x_1 - 2x_2, 4x_1 + x_2)$$

Verify whether the transformation T satisfies the following two properties:

$$(i)\ \ T(2u + v) = 2T(u) + T(v)$$

$$(ii)\ \ T(3u - v) = 3T(u) - T(v)$$

where u, v R^2.

Solution:

Let $u = (u_1, u_2)$ and $v = (v_1, v_2)$.

(i) First, $2u + v = (2u_1 + v_1, 2u_2 + v_2)$. Then

$$T(2u + v) = (3(2u_1 + v_1) - 2(2u_2 + v_2), 4(2u_1 + v_1) + (2u_2 + v_2))$$

$$= (6u_1 + 3v_1 - 4u_2 - 2v_2, 8u_1 + 4v_1 + 2u_2 + v_2)$$

Also,

$$2T(u) + T(v) = 2(3u_1 - 2u_2, 4u_1 + u_2) + (3v_1 - 2v_2, 4v_1 + v_2)$$

$$= (6u_1 - 4u_2 + 3v_1 - 2v_2, 8u_1 + 2u_2 + 4v_1 + v_2)$$

Hence T(2u + v) = 2T(u) + T(v). Property (i) holds.

(ii) Now, $3u - v = (3u_1 - v_1, 3u_2 - v_2)$. Then

$$T(3u - v) = (3(3u_1 - v_1) - 2(3u_2 - v_2), 4(3u_1 - v_1) + (3u_2 - v_2))$$

$$= (9u_1 - 3v_1 - 6u_2 + 2v_2, 12u_1 - 4v_1 + 3u_2 - v_2)$$

Also,

$$3T(u) - T(v) = 3(3u_1 - 2u_2, 4u_1 + u_2) - (3v_1 - 2v_2, 4v_1 + v_2)$$

$$= (9u_1 - 6u_2 - 3v_1 + 2v_2, 12u_1 + 3u_2 - 4v_1 - v_2)$$

Hence T(3u − v) = 3T(u) − T(v). Property (ii) holds.

Therefore, the transformation T satisfies both properties.

Question 2 [Marks = 5]

Apply elementary row operations to transform the following augmented matrix into reduced row echelon form (RREF).

$$A = \left[\begin{array}{ccc|c} 2 & 1 & -1 & 4 \\ 1 & 1 & 1 & 6 \\ 3 & -1 & 2 & 5 \end{array}\right]$$

Solution:

$R_1 \leftrightarrow R_2$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & 1 & -1 & 4 \\ 3 & -1 & 2 & 5 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - 3R_1$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -1 & -3 & -8 \\ 0 & -4 & -1 & -13 \end{array}\right]$$

$R_2 \rightarrow -R_2$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 3 & 8 \\ 0 & -4 & -1 & -13 \end{array}\right]$$

$R_3 \rightarrow R_3 + 4R_2$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 3 & 8 \\ 0 & 0 & 11 & 19 \end{array}\right]$$

$R_3 \rightarrow R_3/11$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 3 & 8 \\ 0 & 0 & 1 & \frac{19}{11} \end{array}\right]$$

$R_2 \rightarrow R_2 - 3R_3$, $R_1 \rightarrow R_1 - R_3$:

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & \frac{47}{11} \\ 0 & 1 & 0 & \frac{31}{11} \\ 0 & 0 & 1 & \frac{19}{11} \end{array}\right]$$

$R_1 \rightarrow R_1 - R_2$:

$$RREF = \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{16}{11} \\ 0 & 1 & 0 & \frac{31}{11} \\ 0 & 0 & 1 & \frac{19}{11} \end{array}\right]$$

Hence x = 16/11, y = 31/11 and z = 19/11.

Question 3 [Marks = 5]

Let the vectors be

$$v_1 = (1, 2, 1), \quad v_2 = (2, -1, 1), \quad v_3 = (1, 1, 2)$$

and let

$$b = (3, 8, 7)$$

Express b as a linear combination of v_1, v_2 and v_3 by forming a system of linear equations. Determine whether such a linear combination exists. If it exists, find the values of the scalars c_1, c_2 and c_3.

Solution:

Let $b = c_1v_1 + c_2v_2 + c_3v_3$. Then

$$c_1(1, 2, 1) + c_2(2, -1, 1) + c_3(1, 1, 2) = (3, 8, 7)$$

This gives the system of equations:

$$c_1 + 2c_2 + c_3 = 3$$

$$2c_1 - c_2 + c_3 = 8$$

$$c_1 + c_2 + 2c_3 = 7$$

From the first equation, $c_1 = 3 - 2c_2 - c_3$. Substituting in the second and third equations:

$$-5c_2 - c_3 = 2$$

$$-c_2 + c_3 = 4$$

From the second of these, $c_3 = 4 + c_2$. Substituting in the first:

$$-5c_2 - (4 + c_2) = 2,\ -6c_2 = 6,\ c_2 = -1$$

Then $c_3 = 4 + (-1) = 3$ and $c_1 = 3 - 2(-1) - 3 = 2$.

Check: $2(1, 2, 1) - (2, -1, 1) + 3(1, 1, 2) = (2, 4, 2) + (-2, 1, -1) + (3, 3, 6) = (3, 8, 7) = b$.

Hence the linear combination exists, and $b = 2v_1 - v_2 + 3v_3$.

Question 4 [Marks = 5]

Consider the matrices

$$A = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix},\quad B = \begin{bmatrix} x & 4 \\ 2 & y \end{bmatrix},\quad C = \begin{bmatrix} 5 & 3 \\ 1 & -2 \end{bmatrix}$$

(a) If A + B = C, find the values of x and y.

(b) Calculate AB.

Solution:

(a) A + B = C gives

$$\begin{bmatrix} 2 + x & 5 \\ 5 & -1 + y \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 1 & -2 \end{bmatrix}$$

Equating the corresponding entries, $2 + x = 5$ and $-1 + y = -2$. Hence $x = 3$ and $y = -1$.

(b) With $x = 3$ and $y = -1$, B =

$$B = \begin{bmatrix} 3 & 4 \\ 2 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 2(3) + 1(2) & 2(4) + 1(-1) \\ 3(3) + (-1)(2) & 3(4) + (-1)(-1) \end{bmatrix}$$

$$AB = \begin{bmatrix} 8 & 7 \\ 7 & 13 \end{bmatrix}$$