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MTH501 Assignment 1 Solution (Tanzeela Anwar)

Fall 2026

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MTH501: Linear Algebra

Assignment No. 1 — Fall 2026 | Total Marks: 20

Section Incharge: Tanzeela Anwar

Lectures 1 to 10.

Question 1 (5 Marks)

Find the solution vector $x = [x_1, x_2, x_3]^T$ of the following system of linear equations by Gaussian elimination.

$x_1 + 2x_2 - x_3 = 2$

$2x_1 - x_2 + x_3 = 3$

$3x_1 + x_2 + 2x_3 = 11$

Start with the augmented matrix of the system.

1	**2**	**−1**	**2**
2	−1	1	3
3	1	2	11

Apply R2 − 2R1 to the second row and R3 − 3R1 to the third row.

1	**2**	**−1**	**2**
0	−5	3	−1
0	−5	5	5

Apply R3 − R2 to the third row.

1	**2**	**−1**	**2**
0	−5	3	−1
0	0	2	6

Use back substitution.

From the last row, $2x_3 = 6$, so $x_3 = 3$.

From the second row, $-5x_2 + 3(3) = -1$, so $-5x_2 = -10$ and $x_2 = 2$.

From the first row, $x_1 + 2(2) - 3 = 2$, so $x_1 = 1$.

$x = [1, 2, 3]^T$

Question 2 (5 Marks)

Consider the following vectors in $\mathbb{R}^3$:

$v_1 = [1, 2, 0]^T$ $v_2 = [0, 1, 1]^T$ $v_3 = [2, 5, 1]^T$

Decide whether the set $\{v_1, v_2, v_3\}$ is linearly independent or linearly dependent. Justify the answer with elementary row operations.

Put the vectors as the columns of a matrix.

1	**0**	**2**
2	1	5
0	1	1

Apply R2 − 2R1 to the second row.

1	**0**	**2**
0	1	1
0	1	1

Apply R3 − R2 to the third row.

1	**0**	**2**
0	1	1
0	0	0

The last row is all zeros, so the rank of the matrix is 2, which is less than 3.

Hence the vectors are linearly dependent.

From the reduced form, the dependence relation is $-2v_1 - v_2 + v_3 = 0$.

That is, $v_3 = 2v_1 + v_2$.

Question 3 (5 Marks)

Let T: $\mathbb{R}^2 \to \mathbb{R}^2$ be defined by $T(x_1, x_2) = (2x_1 + x_2, x_1 - 3x_2)$.

Check whether T is a linear transformation by verifying the two properties below.

(i) $T(u + v) = T(u) + T(v)$

Take $u = (u_1, u_2)$ and $v = (v_1, v_2)$.

$T(u + v) = (2(u_1 + v_1) + (u_2 + v_2), (u_1 + v_1) - 3(u_2 + v_2))$

$= (2u_1 + u_2 + 2v_1 + v_2, u_1 - 3u_2 + v_1 - 3v_2)$

$= (2u_1 + u_2, u_1 - 3u_2) + (2v_1 + v_2, v_1 - 3v_2)$

$= T(u) + T(v)$, so the first property holds.

(ii) $T(cu) = cT(u)$

Take any scalar c.

$T(cu) = (2cu_1 + cu_2, cu_1 - 3cu_2)$

$= c(2u_1 + u_2, u_1 - 3u_2)$

$= cT(u)$, so the second property holds.

Since both properties hold, T is a linear transformation.

Question 4 (5 Marks)

Let $T: \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that $T(x_1, x_2) = [x_1 + 4x_2, -2x_1, 3x_1 - x_2]^T$.

(a) Find $T(e_1)$ and $T(e_2)$, where e_1 and e_2 are the columns of the 2×2 identity matrix.

$e_1 = [1, 0]^T$ and $e_2 = [0, 1]^T$.

$T(e_1) = [1 + 4(0), -2(1), 3(1) - 0]^T = [1, -2, 3]^T$.

$T(e_2) = [0 + 4(1), -2(0), 3(0) - 1]^T = [4, 0, -1]^T$.

(b) Find the standard matrix A of the linear transformation T.

The standard matrix has $T(e_1)$ and $T(e_2)$ as its columns.

1	**4**
−2	0
3	−1

(c) Find T(x) for $x = [2, 3]^T$.

$T(x) = A\,x = [1(2) + 4(3), -2(2) + 0(3), 3(2) - 1(3)]^T$.

$T(x) = [14, -4, 3]^T$.