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MTH104

MTH104

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MTH104

Sets and Logic

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Topic no 01

Introduction

Introduction

This course, **MTH104: Sets and Logic**, introduces two fundamental pillars of modern mathematics: **Set Theory** and **Mathematical Logic**. These two areas form the foundation upon which all of mathematics is built.

Why do we study it?

Understanding sets and logic is essential for: 

- Developing rigorous mathematical thinking
- Understanding how mathematical proofs are constructed
- Building a solid foundation for advanced mathematics (calculus, algebra, analysis, etc.)
- Learning to reason clearly and avoid logical fallacies

What is the relation between sets and logic?

- Set theory provides the **language** of mathematics – almost every mathematical object can be defined as a set.
- Logic provides the **rules of reasoning** – the framework for constructing valid arguments and proofs.
- Together, they form the foundation of mathematics: set theory gives us objects to work with, and logic gives us rules to reason about them.

Proof and Mathematical Logic

In mathematics, a **proof** is a demonstration that, assuming certain axioms, some statement is necessarily true. Proof is a **logical argument**, not an empirical one.

Key terminology:

- **Theorem:** A proposition that has been proved true in all cases.
- **Conjecture:** An unproven proposition for which there is some sort of empirical evidence.
- **Mathematical Logic:** The framework upon which rigorous proofs are built. It is the study of the principles and criteria of valid inference and demonstrations.

To prove a statement is a theorem, one must demonstrate that it holds in **all cases**, not just in a few examples. Mathematical logic provides the tools to ensure that proofs are valid and conclusions follow correctly from premises.

Set Theory

In set theory, we study **sets**, which are informally collections of objects. Although any type of object can be collected into a set, set theory is applied most often to objects that are relevant to mathematics.

A **rigorous analysis of set theory** belongs to the foundations of mathematics and mathematical logic. Set theorists explore what is **provable**—and what is **not provable**—when one is allowed to use all of the usual tools of mathematical constructions and proofs.

Tools of Set Theory

Set theorists use various mathematical tools to construct and analyze sets, including:

- **Function spaces:** Sets of functions between sets
- **Completions:** Adding limit points to spaces (like completing rational numbers to real numbers)
- **Quotients:** Forming new sets by identifying elements (like equivalence classes)
- **Products:** Cartesian products of sets (ordered tuples)
- **Induction and recursion:** Defining and proving properties of infinite structures

Summary of Topic 01: Introduction

- This course covers the foundations of mathematics: **Sets** and **Logic**.
- A **proof** is a logical argument demonstrating a statement must be true.
- A **theorem** is a proved proposition; a **conjecture** is an unproven proposition.
- **Mathematical logic** provides the rules for valid reasoning and proof construction.
- **Set theory** studies collections of objects and forms the language of modern mathematics.
- Set theorists use tools like function spaces, completions, quotients, products, induction, and recursion.
- Set theory and logic together form the **foundation** upon which all of mathematics is built.

Topic no 02

Basic Concepts of Sets

Sets and Elements

A fundamental concept in all branches of mathematics is that of a **set**. A set is any well-defined list, collection, or class of objects. The objects in sets can be anything: numbers, people, letters, rivers, etc. These objects are called the **elements** or **members** of the set.

Examples of Sets:

1. The numbers 1, 3, 7 and 10.
2. The solution of the equation $x^2 - 3x + 2 = 0$.
3. The vowels of the English alphabet: a, e, i, o, u.
4. The people living on the earth.
5. The even integers 2, 4, 6, ...

Notation

- Sets will usually be denoted by capital letters: **A, B, X, Y, ...**
- Elements in sets will usually be represented by lower case letters: **a, b, x, y, ...**
- The statement "*p belongs to A*" is written as **$p \in A$** .
- The statement "*p is not an element of A*" is written as **$p \notin A$** .

Tabular Form (Roster Form)

If we define a particular set by actually listing its members, the elements are separated by commas and enclosed in curly brackets **{ }**. This is called the **tabular form** of a set.

Example:

Let A consist of the numbers 1, 3, 7 and 10, then we write: **$A = \{1, 3, 7, 10\}$**

Set-Builder Form

If we define a particular set by stating properties which its elements must satisfy, we use a letter (usually **x**) to represent an arbitrary element and write:

$$B = \{ x \mid x \text{ satisfies property } P \}$$

This is called the **set-builder form** of a set.

Example:

Let B be the set of all even numbers. Then we write: **$B = \{ x \mid x \text{ is even} \}$**

This is read as: "B is the set of all x such that x is even."

Summary of Set Representations:

- **Tabular Form:** Listing all elements inside **{ }**. Example: **$\{1, 2, 3\}$**
- **Set-Builder Form:** Describing elements by a property. Example: **$\{x \mid x \text{ is a positive integer}\}$**

Topic no 03

Empty Set, Subsets and Universal Set

Universal Set

All sets under investigation in any application of set theory are assumed to be contained in some large fixed set called the **universal set** or **universe**. We will denote the universal set by **U**.

Examples:

- In plane geometry, the universal set consists of all points in the plane.
- In human population studies, the universal set consists of all people in the world.

Empty Set (Null Set)

It is convenient to introduce the concept of the **empty set**, that is, a set which contains no elements. This set is sometimes called the **null set**. We denote it by the symbol $\emptyset$ (or sometimes $\{ \}$).

Examples of Empty Set:

1. Let A be the set of people in the world who are older than 200 years. Then $A = \emptyset$.
2. Let $B = \{ x \mid x^2 = 4, x \text{ is odd} \}$. Then B is the empty set because no odd number satisfies $x^2 = 4$.
3. The set $S = \{ x : x \text{ is a positive integer, } x^2 = 3 \}$ has no elements since no positive integer has square equal to 3. Therefore $S = \emptyset$.

Subset

Suppose every element in a set A is also an element of a set B ; then **A is called a subset of B** . We also say that A is **contained in B** or **B contains A** . This relationship is written as:

$$A \subseteq B \quad \text{or} \quad B \supseteq A$$

Example:

Let $A = \{1, 2\}$ and $B = \{1, 2, 3\}$. Then every element of A (1 and 2) is also an element of B . Therefore, $A \subseteq B$.

Properties of Subsets

- Every set A is a subset of the universal set U since, by definition, all the elements of A belong to U .
- The empty set $\emptyset$ is a subset of every set A . That is, $\emptyset \subseteq A$ for any set A .
- Every set A is a subset of itself since, trivially, the elements of A belong to A . That is, $A \subseteq A$.
- If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$. (Transitive property)
- If $A \subseteq B$ and $B \subseteq A$, then $A = B$. (Antisymmetric property)

Proper Subset

If $A \subseteq B$ but $A \neq B$, we say that **A is a proper subset of B** . This is denoted by $A \subset B$.

Example:

Suppose $A = \{1, 2\}$, $B = \{1, 2, 3\}$, and $C = \{1, 3, 2\}$.

Then A and B are both subsets of C . But A is a proper subset of C (because $A \neq C$), whereas B is not a proper subset of C (because $B = C$, since order does not matter in sets).

Important Note: Two sets are equal if and only if they have exactly the same elements, regardless of the order in which they are listed. For example, $\{1, 2, 3\} = \{1, 3, 2\}$.

Topic no 04**Set Operations**

In this topic, we will study a number of **set operations**, including the basic operations of **union**, **intersection**, and **difference** of sets, where new sets will be assigned to pairs of sets A and B .

Union of Sets

The **union** of two sets A and B , denoted by $A \cup B$, is the set of all elements which belong to A or B (or both). That is:

$$A \cup B = \{ x : x \in A \text{ or } x \in B \}$$

Example 1:

Let U denote the set of students at a university, and let M and F denote, respectively, the set of male and female students at the university.

Then $M \cup F = U$, since each student in U is either male or female.

Example 2:

Let $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$.

Then $A \cup B = \{1, 2, 3, 4, 5\}$, since we take all distinct elements from both sets.

Intersection of Sets

The **intersection** of two sets A and B , denoted by $A \cap B$, is the set of all elements which belong to **both** A and B . That is:

$$A \cap B = \{ x : x \in A \text{ and } x \in B \}$$

Example 1:

Using the same university example: $M \cap F = \emptyset$, since no student belongs to both the male set and the female set (assuming no student is both male and female).

Example 2:

Let $A = \{1, 2, 3\}$ and $B = \{3, 4, 5\}$.

Then $A \cap B = \{3\}$, since 3 is the only element common in both sets.

Properties of Union and Intersection

- $A \cap B \subseteq A$ and $A \cap B \subseteq B$. (The intersection is a subset of each individual set.)
- $A \subseteq A \cup B$ and $B \subseteq A \cup B$. (Each individual set is a subset of their union.)
- Combining the above: $A \cap B \subseteq A \subseteq A \cup B$ and $A \cap B \subseteq B \subseteq A \cup B$.

Complement of a Set

The **complement** of a set A , denoted by A^c (or sometimes A'), is the set of elements which belong to the universal set U but which do **not** belong to A . That is:

$$A^c = \{ x : x \in U, x \notin A \}$$

Example 1:

Let $U = \{a, b, c, \dots, y, z\}$ be the universal set (all letters of the alphabet). Let $A = \{a, b, c, d, e\}$, $B = \{e, f, g\}$, and $V = \{a, e, i, o, u\}$ (vowels). Then:

- $A^c = \{f, g, h, \dots, z\}$ (all consonants except a, b, c, d, e)
- $B^c = \{a, b, c, d, h, i, \dots, z\}$ (all letters except e, f, g)
- $V^c =$ all consonants (all letters that are not vowels)

Example 2:

Suppose the set R of real numbers is the universal set. Let Q denote the set of rational numbers. Then Q^c denotes the set of **irrational numbers** (real numbers that cannot be expressed as a ratio of integers).

Difference of Sets

Let A and B be sets. The **difference** of A and B , denoted by A / B (or $A - B$), is the set of elements which belong to A but which do **not** belong to B . That is:

$$A / B = \{ x : x \in A, x \notin B \}$$

Example:

Consider the sets $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7\}$, and $C = \{6, 7, 8, 9\}$.

Then:

- $A / B = \{1, 2\}$ (elements in A but not in B)
- $B / A = \{5, 6, 7\}$ (elements in B but not in A)
- $A / C = \{1, 2, 3, 4\}$ (since A and C have no common elements)

Symmetric Difference

The **symmetric difference** of the sets A and B, denoted by $A \oplus B$, can be defined in two equivalent ways:

$$A \oplus B = (A \cup B) / (A \cap B)$$

or

$$A \oplus B = (A / B) \cup (B / A)$$

The symmetric difference consists of all elements that are in A or in B, but not in both. In other words, it is the union of the two sets minus their intersection.

Example:

Consider $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6, 7\}$.

First, $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$ and $A \cap B = \{3, 4\}$.

Then $A \oplus B = (A \cup B) / (A \cap B) = \{1, 2, 5, 6, 7\}$.

Alternatively, $A / B = \{1, 2\}$ and $B / A = \{5, 6, 7\}$, so $(A / B) \cup (B / A) = \{1, 2, 5, 6, 7\}$.

Summary of Set Operations:

- **Union ($\cup$):** Elements in A or B (or both).
- **Intersection ($\cap$):** Elements in both A and B.
- **Complement (c):** Elements in U but not in A.
- **Difference ($/$ or $-$):** Elements in A but not in B.
- **Symmetric Difference ($\oplus$):** Elements in A or B, but not in both.

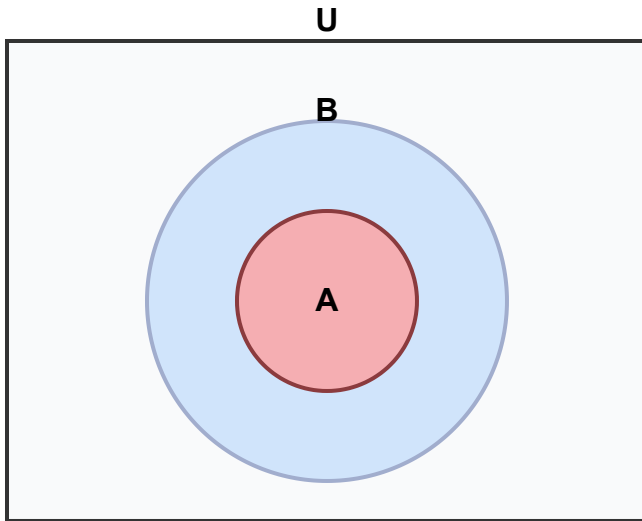
Topic no 05

Venn Diagrams

A **Venn diagram** is a pictorial representation of sets where sets are represented by enclosed areas in the plane. The universal set **U** is represented by the points in a rectangle, and other sets are represented by circles (or disks) lying within the rectangle.

1. Subset Relationship ($A \subseteq B$)

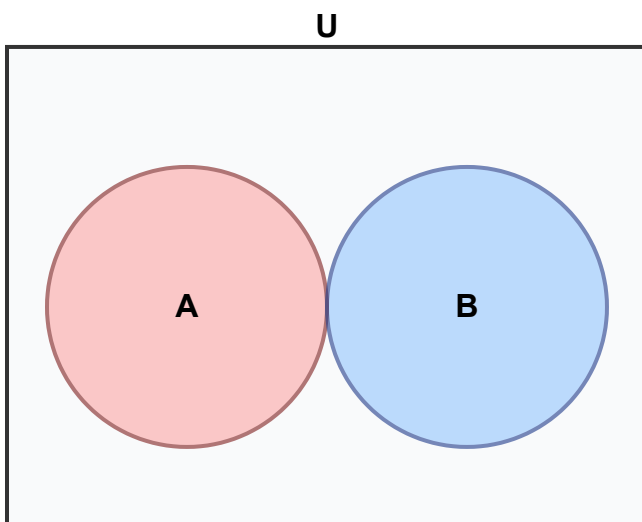
If $A \subseteq B$, then the circle representing A lies entirely within the circle representing B.



Explanation: Since A is completely inside B, every element of A is also an element of B. Therefore, $A \subseteq B$.

2. Disjoint Sets ($A \cap B = \emptyset$)

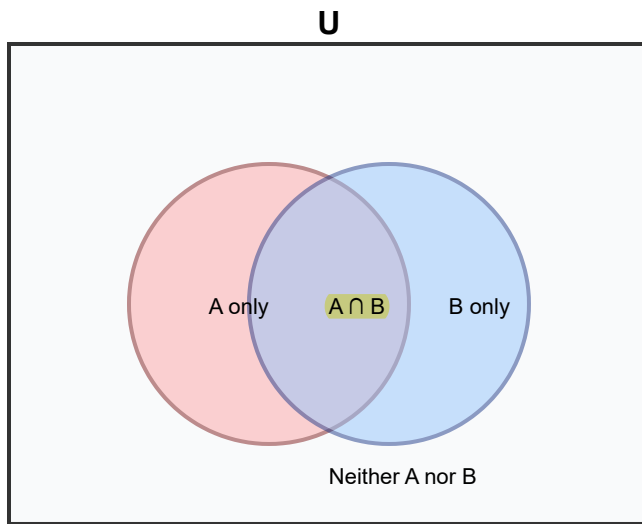
If A and B are disjoint (no common elements), the circles are completely separated with no overlap.



Explanation: The circles do not overlap, meaning A and B have no elements in common. Hence, $A \cap B = \emptyset$.

3. Arbitrary Sets (General Case with Overlap)

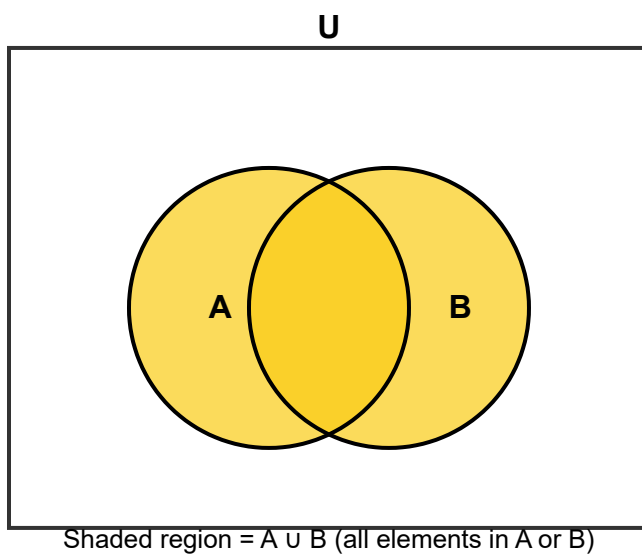
When A and B are arbitrary sets, they may have some elements in common, some unique to A, some unique to B, and some in neither.



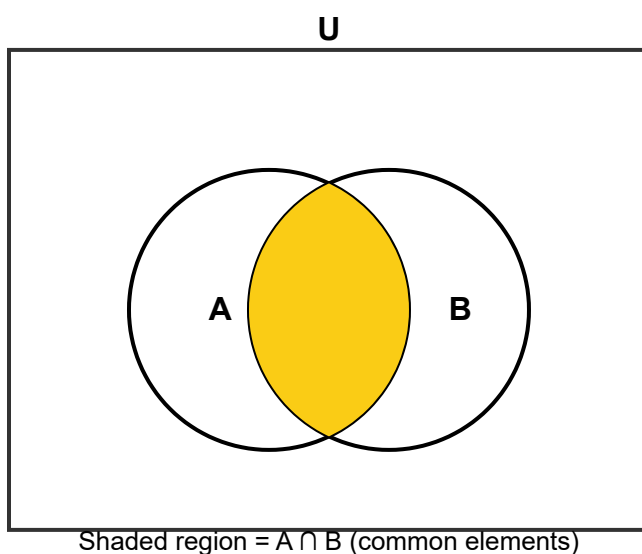
Explanation: The overlapping region represents $A \cap B$. The left-only part is $A \setminus B$, the right-only part is $B \setminus A$.

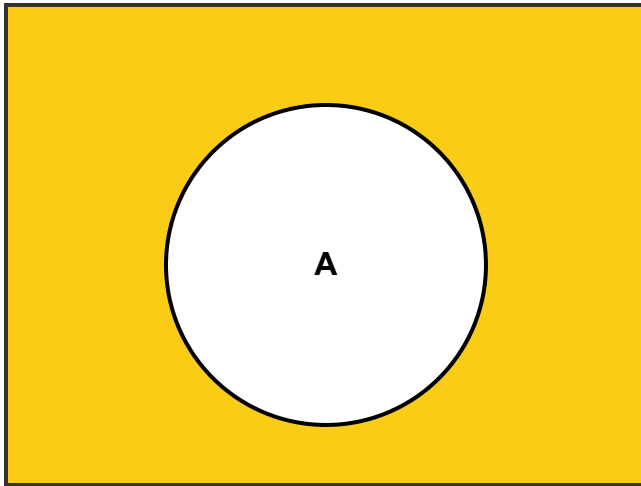
Venn Diagram Representations of Set Operations

1. Union of A and B ($A \cup B$)

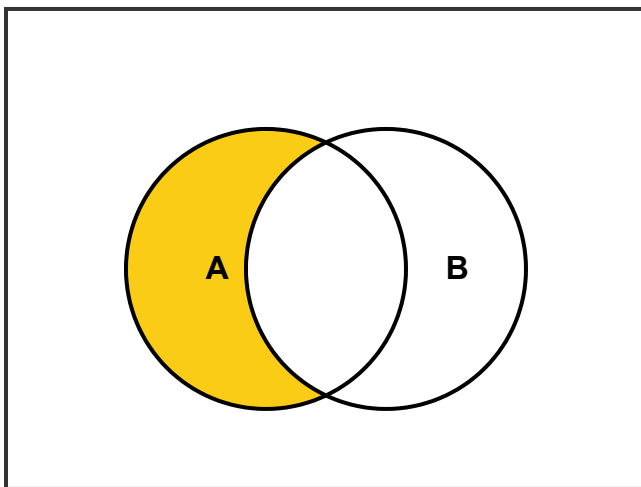


2. Intersection of A and B ($A \cap B$)

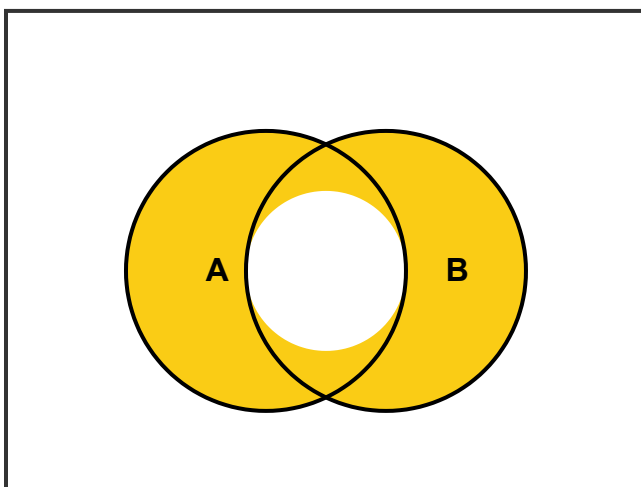


3. Complement of A (A^c)

Shaded region = A^c (everything outside A)

4. Difference of A and B ($A \setminus B$)

Shaded region = $A \setminus B$ (elements in A but not in B)

5. Symmetric Difference ($A \oplus B$)

Shaded region = $A \oplus B$ (in A or B but not both)

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