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HIGHLIGHTED HANDOUT

MTH603

Numerical Analysis

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Dated: / / 20

Modified Euler's method:

$$y_{m+1} = y_m + h \left[\frac{f(t_m, y_m) + f(t_{m+1}, y_{m+1})}{2} \right]$$

Runge-Kutta method:

$$k_1 = hf(t_m, y_m)$$

$$k_2 = hf\left(t + \frac{3}{2}h, y + \frac{3}{2}k_1\right)$$

mostly use k_2 is

$$k_2 = hf\left(t_m + \frac{h}{2}, y_m + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(t_m + \frac{h}{2}, y_m + \frac{k_2}{2}\right)$$

$$k_4 = hf(t_m + h, y_m + k_3)$$

$$k = \frac{1}{6}(k_1 + k_2 + k_3 + k_4)$$

MTWTFES: important formulas: Dated: / / 20

Newton's backward interpolation formula:

$$y_c = y_n + p \nabla y_n + \frac{p(p+1)}{2!} \nabla^2 y_n + \frac{p(p+1)(p+2)}{3!} \nabla^3 y_n$$

here

$$p = \frac{x - x_n}{h}$$

Lagrange interpolation formula:

$$y = f(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} y_1 + \dots + \frac{(x-x_0)(x-x_1)\dots(x-x_{i-1})(x-x_{i+1})\dots(x-x_n)}{(x_i-x_0)(x_i-x_1)\dots(x_i-x_{i-1})(x_i-x_{i+1})\dots(x_i-x_n)} y_i + \dots$$

using method in question

	x_0	x_1	x_2	x_3
x	1	3	4	6
$y=f(x)$	-3	6	30	132
	y_0	y_1	y_2	y_3

value of $x=5$

فرض کریں کہ ہم x میں نفی کر دیں اور نفی کی گئی x کو
dividing term میں x میں نفی کر دیں

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

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Newton divided difference formula

$$y = f(x) = y_0 + (x - x_0)y[x_0, x_1] + (x - x_0)(x - x_1)y[x_0, x_1, x_2]$$

Backward difference operator:

$$\frac{dy}{dx} = \frac{1}{h} \left[\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4} + \dots \right]$$

$$D^2 f(x) = \frac{1}{h^2} [\nabla^2 f(x) - \nabla^3 f(x) + \nabla^4 f(x) - \nabla^5 f(x)]$$

Backward difference formula:

$$y_n = \frac{1}{h} \left[\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4} + \dots \right]$$

$$D^2 f(x) = \frac{1}{h} \left[\Delta^2 f(x) - \Delta^3 f(x) + \frac{11}{12} \Delta^4 f(x) - \frac{5}{6} \Delta^5 f(x) \right]$$

Differentiation using interpolation.

$$\frac{y(x_i + h) - y(x_i)}{h}$$

Richardson's extrapolation method:

$$f(h) = \frac{y(x+h) - y(x-h)}{2h}$$

use the given function at the place of $(x+h)$, and $(x-h)$

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The Trapezoidal rule:

$$\int_a^b f(x) dx = \frac{h}{2} [(y_0 + y_n) + 4(\text{odd terms}) + 2(\text{even terms})]$$

$$= \frac{h}{2} [(y_0 + y_n) + 4(y_1 + y_3 + y_5 + \dots) + 2(y_2 + y_4 + y_6 + \dots)]$$

Simpson's $1/3$ rule:Simpson's $1/3$ rule are use when interval is even

$$\int_a^b f(x) dx = \frac{h}{3} [(y_0 + y_n) + 4(\text{odd terms}) + 2(\text{even term})]$$

$$= \frac{h}{3} [(y_0 + y_n) + 4(y_1 + y_3 + y_5 + \dots) + 2(y_2 + y_4 + y_6 + \dots)]$$

Simpson's $3/8$ ruleSimpson's $3/8$ rule are used in interval in multiple of 3

$$\int_a^b f(x) dx = \frac{3h}{8} [(y_0 + y_n) + 3(\text{remaining terms}) + 2(\text{multiple of 3})]$$

$$= \frac{3h}{8} [(y_0 + y_n) + 3(y_1 + y_2 + y_4 + \dots) + 2(y_3 + y_6 + y_9 + \dots)]$$

Euler method:

$$y_{m+1} = y_m + hf(t_m, y_m)$$

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M T W T F S : important formulas: Date: / / 20

Newton's backward interpolation formula

$$y_{br} = y_n + p \nabla y_n + \frac{p(p+1)}{2!} \nabla^2 y_n + \frac{p(p+1)(p+2)}{3!} \nabla^3 y_n$$

here -

$$p = \frac{x - x_n}{h}$$

Lagrange interpolation formula

$$y = f(x) = \frac{(x-x_1)(x-x_2)\dots(x-x_n)}{(x_0-x_1)(x_0-x_2)\dots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\dots(x-x_n)}{(x_1-x_0)(x_1-x_2)\dots(x_1-x_n)} y_1 +$$

$$+ \frac{(x-x_0)(x-x_1)\dots(x-x_{i-1})(x-x_{i+1})\dots(x-x_n)}{(x_i-x_0)(x_i-x_1)\dots(x_i-x_{i-1})(x_i-x_{i+1})\dots(x_i-x_n)} y_i +$$

using method in question

	x_0	x_1	x_2	x_3
x	1	3	4	6
$y = f(x)$	-3	6	30	132
	y_0	y_1	y_2	y_3

Value of $x = 5$

فرسٹ ٹرم کو سمیٹا کے باقی کو x میں تقسیم کر دیں اور تقسیم کی گئی ٹرم کو

Then continue this method (dividing term x_0 میں تقسیم کر دیں)

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 +$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

