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HIGHLIGHTED HANDOUT

EDUA510

Teaching of Mathematics in Early Childhood

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VU STUDY HANDOUT

EDUA510

Teaching of Mathematics in Early Childhood

This handout prepares early-childhood educators to teach mathematics to young children (ages 3-8): how number sense develops, play-based pedagogy, the learning trajectories for counting, operations, geometry and measurement, and developmentally appropriate assessment.

13 lecture-wise topics | key terms | exam pointers



Table of Contents

UNIT 1: FOUNDATIONS

Lecture 1: Why Early Mathematics Matters

Lecture 2: How Young Children Learn Mathematics

Lecture 3: The Early Childhood Math Curriculum

UNIT 2: NUMBER AND OPERATIONS

Lecture 4: Developing Counting and Cardinality

Lecture 5: Number Sense, Comparing and Place Value Beginnings

Lecture 6: Addition, Subtraction and Problem Solving

UNIT 3: GEOMETRY, MEASUREMENT AND DATA

Lecture 7: Geometry and Spatial Sense

Lecture 8: Measurement

Lecture 9: Patterns, Sorting and Data

UNIT 4: PEDAGOGY AND ASSESSMENT

Lecture 10: Play-Based Pedagogy and the Learning Environment

Lecture 11: Manipulatives, Technology and Resources

Lecture 12: Assessment in Early Childhood Mathematics

Lecture 13: Supporting All Learners

UNIT 1: FOUNDATIONS

Lecture 1: Why Early Mathematics Matters

Learning Objectives

- Argue the importance of early math
- Distinguish math readiness myths
- Describe early math and later achievement

Key Terms & Definitions

Early numeracy

Foundational number knowledge and skills before formal schooling.

Math gap

Socioeconomic disparities in early math knowledge.

Developmentally Appropriate Practice (DAP)

Teaching matched to children's age, experience and individual needs (NAEYC).

Longitudinal research (Duncan et al.) finds **early math knowledge predicts later school achievement** - even more than early reading or attention. Number sense at school entry forecasts Grade-5 mathematics. Early childhood is therefore not "too early" for math; it is the **highest-leverage window**.

Myths to discard: "children aren't ready for math" (they quantify spontaneously from infancy - subitizing, comparing), "math is worksheets" (it is pattern, quantity and space in play), "earlier formal instruction is better" (drill without understanding backfires). **Readiness** is built through rich experience, not waited for.

The **math gap**: by age 4, low-income children can lag a year behind in number knowledge - but targeted play-based interventions close it substantially. Equity demands **intentional** early math, not incidental exposure.

DAP (NAEYC): teaching that is age-appropriate, individually appropriate, and culturally/socially appropriate.

- **Early math predicts later achievement** - the evidence is strong and causal-ish.
- **Play is the pedagogy**, not a break from it.
- **Intentional teaching** closes the math gap; incidental exposure widens it.
- **DAP**: age, individual, and cultural appropriateness together.

VERY IMPORTANT — Exam Pointer

Very important: summarize the evidence that early math predicts later achievement and its implication for teachers.

Section Summary

Early mathematics is the strongest school-readiness predictor; teach it intentionally through play.

Lecture 2: How Young Children Learn Mathematics

Learning Objectives

- Explain constructivist learning
- Describe learning trajectories
- Apply sociocultural theory

Key Terms & Definitions

Constructivism (Piaget)

Children build knowledge by acting on objects and resolving contradictions.

Learning trajectory

Hypothesized developmental path: goal -> developmental progression -> instruction (Clements & Sarama).

Zone of Proximal Development (Vygotsky)

What a child can do with guidance, beyond independent reach.

Piaget: children construct number concepts by acting on objects - **conservation** (quantity stays same despite arrangement) marks operational thinking; pre-operational children are fooled by appearance. Implication: **manipulatives first**, symbols later; let children discover through guided play, not told rules.

Learning trajectories (Clements & Sarama): for each topic, a **goal**, the **developmental progression** of thinking levels (e.g. counting: rote -> one-to-one -> cardinality -> ...), and **instructional tasks** matched to each level. Teaching = diagnosing the child's level and offering the next task. This is the scientific core of early math pedagogy.

Vygotsky: learning is social - **ZPD** is where teaching works, via **scaffolding** (questions, hints, modeling) that fades as competence grows. **Language** mediates math: math talk ("more", "fewer", "how many?") builds concepts. **Private speech** while counting is thinking aloud - don't shush it.

- **Trajectory = goal + progression + tasks:** diagnose level, teach the next step.
- **Scaffold in the ZPD:** support, then fade.
- **Math talk** throughout the day builds vocabulary and concepts.
- **Concrete -> pictorial -> abstract** (Bruner's CPA): the representation sequence.

Example

A child counts "1,2,3,4,5" but recounts when asked "how many?" - she is at one-to-one tagging, not cardinality. The next task: count sets and emphasize the last number as the total.

VERY IMPORTANT — Exam Pointer

Very important: explain learning trajectories and demonstrate diagnosing a child's counting level.

Section Summary

Children construct math actively along predictable trajectories; teachers scaffold the next step.

Lecture 3: The Early Childhood Math Curriculum

Learning Objectives

- List the five content strands
- Explain process standards
- Plan balanced programs

Key Terms & Definitions

Content strands

Number & operations, algebra (patterns), geometry, measurement, data.

Process standards (NCTM)

Problem solving, reasoning, communication, connections, representation.

Integration

Weaving math into play, stories, art, outdoor time.

Content strands for early years: **number and operations** (the core: counting, comparing, composing), **algebra** as patterning and sorting, **geometry** (shapes, spatial relations), **measurement** (length, weight, time, money), **data** (sorting, simple graphs). Number deserves the largest share but not the only share.

Process standards (NCTM): **problem solving**, **reasoning and proof** (age-appropriate justifying: "how do you know?"), **communication** (math talk, drawing), **connections** (math-to-life, math-to-math), **representation** (objects, pictures, symbols). A lesson strong on content but silent on processes is half a lesson.

Curriculum design: balance **planned activities** (focused small-group games), **daily routines** (calendar, attendance counting, snack distribution), and **play-based discovery** (blocks, sand/water). **Integration**: a story about three bears teaches seriation; block play teaches geometry. Avoid fragmented "math time" isolated from life.

- **Five strands, five processes**: plan for both dimensions.
- **Routines are curriculum**: calendar, counting, sharing embed daily math.
- **Integrate, don't isolate**: math lives in stories, art, outdoor play.
- **Depth over coverage**: fewer topics, deeper trajectories.

VERY IMPORTANT — Exam Pointer

Exam pointer: design one week's integrated plan covering all five strands for KG.

Section Summary

A balanced program covers five strands through planned play, routines, and rich processes.

UNIT 2: NUMBER AND OPERATIONS

Lecture 4: Developing Counting and Cardinality

Learning Objectives

- Describe counting principles
- Teach subitizing
- Assess counting levels

Key Terms & Definitions

One-to-one correspondence

Exactly one number word per object counted.

Cardinality

The last number word tells how many (Gelman & Gallistel).

Subitizing

Instantly recognizing small quantities without counting.

Gelman & Gallistel's five principles: one-to-one correspondence, stable order, **cardinality**, abstraction (anything countable), order irrelevance. Children master them in sequence; many 4-year-olds tag objects correctly but **recount** when asked "how many?" - cardinality is the conceptual leap.

Subitizing: perceptual (instantly seeing 3 dots) and **conceptual** (seeing 6 as 3+3). Dot cards, dice, dominoes, and ten-frames build it. Subitizing underpins **composing/decomposing** numbers - the foundation of mental arithmetic.

Teaching: counting songs with objects (not rote chanting alone), "how many?" with emphasis on the last word, counting collections, **counting on** from a number, **backward counting**. **Assess** by watching: does she tag each object once? Does the last word answer "how many?" Can she count out 5 from a pile (harder than counting existing sets)?

- **Cardinality** is the key milestone - teach it explicitly.
- **Ten-frames and dot patterns** build subitizing and part-whole sense.
- **Count out N** is harder than **count N present** - sequence tasks accordingly.
- **Zero** and **counting backward** come later - plan for them.

Example

Dot-card flash: "How many?" (child: "5!") "How did you see it?" ("3 and 2") - conceptual subitizing plus decomposition in 30 seconds.

VERY IMPORTANT — Exam Pointer

Very important: list Gelman & Gallistel's principles and explain how to teach cardinality.

Section Summary

Counting matures from rote chanting to cardinality and subitizing - the gateway to arithmetic.

Lecture 5: Number Sense, Comparing and Place Value Beginnings

Learning Objectives

- Build number sense
- Teach comparison language
- Introduce tens and ones

Key Terms & Definitions

Number sense

Flexible, intuitive grasp of numbers, magnitudes and relationships.

One more/one less

The successor/predecessor relationships building mental number line.

Place value

A digit's value depends on its position (tens, ones).

Number sense = understanding magnitude (7 is bigger than 5 by 2), relationships ($6 = 5+1 = 3+3$), and reasonableness. Build with **number lines** (floor lines children walk), **benchmarks** (5 and 10), estimation games ("more or less than 10?"), and **part-part-whole** mats.

Comparing: language first - **more/less/fewer**, **greater than/less than**, **equal**. Use matching (pairing objects) before symbols; introduce **> < =** only after verbal mastery. **Ordering** 3-4 numerals; **one more/one less** games build the mental number line.

Place value beginnings: bundle straws/sticks into tens ("10 ones = 1 ten"), teen numbers as "10 and some more" ($14 = 10 + 4$ - explicitly, since "fourteen" hides the ten). Avoid formal column algorithms; build the **ten-structure** through bundling games. **Common error**: writing "fourteen" as 41 - shows place value not yet constructed.

- **Magnitude before symbols**: compare, order, estimate first.
- **Bundle to make tens** - the physical root of place value.
- **Teen numbers as 10 + n**: say "ten-four" before "fourteen" if needed.
- **Number talks**: children explain strategies - builds flexibility.

VERY IMPORTANT — Exam Pointer

Exam pointer: a child writes 41 for "fourteen" - diagnose and prescribe a bundling intervention.

Section Summary

Number sense grows through comparing, composing and bundling into tens.

Lecture 6: Addition, Subtraction and Problem Solving

Learning Objectives

- Teach operations conceptually
- Use CGI problem types
- Develop fact fluency

Key Terms & Definitions

Cognitively Guided Instruction (CGI)

Teaching built on children's intuitive problem-solving strategies.

Join/Separate/Compare problems

The three semantic structures of addition/subtraction.

Fact fluency

Quick, accurate, flexible recall of number combinations.

CGI problem types: **join** (5 birds, 3 more join - result unknown/change unknown/start unknown), **separate**, **part-part-whole**, **compare** (how many more?). Children solve first by **direct modeling** (counters), then **counting strategies** (counting on), then **derived facts** ($6+7 = 6+6+1$). Teach all structures - compare problems are hardest.

Strategies, not just answers: ask "how did you solve it?" and let children share; name efficient strategies (making ten, doubles). **Fluency** = accuracy + efficiency + flexibility (not timed-drill speed). Games (concentration, dice sums) build facts; **worksheets of 50 sums** kill interest.

Subtraction as missing addend: "9 - 4" as " $4 + ? = 9$ " connects operations. **Word problems** daily, with children **posing** their own. **Assessment:** watch strategies, not scores - a child counting all vs counting on needs different next tasks.

- **Teach all CGI structures**, especially compare and start-unknown.
- **Strategy progression:** model → count → derive.
- **Fluency through games**, not drill sheets.
- **Children pose problems** - the deepest comprehension check.

Example

Compare problem: "Ali has 8 marbles, Sara has 5. How many more does Ali have?" - children who can join may still struggle; matching pairs makes the difference visible.

VERY IMPORTANT — Exam Pointer

Very important: name the CGI problem structures and the strategy progression.

Section Summary

Operations grow from modeling real problems through counting to flexible derived facts.

UNIT 3: GEOMETRY, MEASUREMENT AND DATA

Lecture 7: Geometry and Spatial Sense

Learning Objectives

- Teach shapes developmentally

- Build spatial reasoning
- Use van Hiele levels

Key Terms & Definitions

van Hiele levels

Visualization -> analysis -> informal deduction (early years: first two).

Spatial reasoning

Mental manipulation of shapes and positions.

Attributes

Defining (sides, corners) vs non-defining (color, size) features.

van Hiele: Level 0 **visualization** ("it looks like a door" - rectangles), Level 1 **analysis** ("it has 4 sides, 4 corners"). Preschool-KG lives at visualization moving to analysis. **Never** define shapes only by typical orientation - show tilted, skinny, large triangles, or children think "upside-down triangle isn't a triangle".

Attributes: sort by **defining** attributes (number of sides) not color/size. Games: "I'm thinking of a shape with 3 straight sides" (guess my rule), shape hunts, **pattern blocks**, tangrams, geoboards. **Compose/decompose:** two triangles make a square - the spatial root of fractions and area.

Spatial language and reasoning: over/under, beside, between; **mental rotation** puzzles; **mapping** (classroom maps, treasure hunts); **symmetry** with mirrors/folding. Block play is geometry laboratory - provide varied blocks, arches, and time.

- **Vary examples AND non-examples** of every shape.
- **Defining attributes only** for sorting rules.
- **Compose/decompose shapes** daily - tangrams, pattern blocks.
- **Spatial language** all day: position words in routines.

VERY IMPORTANT — Exam Pointer

Exam pointer: a child rejects a rotated square as "not a square" - explain via van Hiele and fix it.

Section Summary

Geometry moves from recognizing shapes to analyzing attributes and reasoning spatially.

Lecture 8: Measurement

Learning Objectives

- Teach the measurement progression
- Compare length, weight, capacity
- Introduce time and money

Key Terms & Definitions

Measurement progression

Direct comparison -> nonstandard units -> standard units.

Transitivity

If $A > B$ and $B > C$, then $A > C$ - the logic of indirect comparison.

Unit iteration

Repeating a unit without gaps/overlaps to measure.

Progression: (1) **direct comparison** (hold two ribbons side by side), (2) **indirect** via a third object (**transitivity**), (3) **nonstandard units** (hand spans, cubes - discovering the need for identical units when spans differ!), (4) **standard units** (cm, kg) in later grades. Rushing to rulers skips the concept.

Key ideas: choice of **attribute** (length vs weight), **unit iteration** (no gaps/overlaps - watch children leave spaces between cubes), **unit-size inverse relation** (bigger units -> smaller counts). Measure **length, weight** (balance scales), **capacity** (sand/water pouring), **area** informally (covering with tiles).

Time: sequence vocabulary (before/after, morning/evening), days/weeks, reading o'clock then half-hours.

Money: coin recognition and values through **play shops** - computes addition/subtraction authentically. **Common**

error: starting measurement at 1 on the ruler - from iterating units with gaps.

- **Compare -> nonstandard -> standard:** never skip steps.
- **Identical units** is the big discovery - engineer it (different hand spans!).
- **Unit iteration** without gaps: check physically.
- **Play shops** integrate money, adding, and social skills.

Example

Children measure the table with hand spans: Ali gets 8, Sara 10 - "Why different?" - the identical-unit insight, earned not told.

VERY IMPORTANT — Exam Pointer

Very important: outline the measurement progression and the identical-units discovery.

Section Summary

Measurement is learned by comparing, unitizing, and iterating - standard units come last.

Lecture 9: Patterns, Sorting and Data

Learning Objectives

- Teach patterning
- Classify and sort
- Build early data literacy

Key Terms & Definitions

Pattern

A repeating or growing regularity (AB, AAB, growing).

Classification

Grouping by attributes - the root of logical thinking.

Pictograph

Picture-based graph where each icon represents data.

Patterns are early algebra: **repeat** patterns (red-blue-red-blue: AB; clap-stomp-stomp: ABB), then **growing** patterns. Teach: copy, continue, **extend**, create, and **identify the unit of repeat**. "What comes next? How do you know?" Patterning predicts later math achievement strongly.

Sorting/classifying: by one attribute, then **re-sort by another** (flexibility), Venn-style overlaps informally. attribute blocks, button sorting, nature collections. **Seriation**: ordering by size/length - the root of number order.

Data: pose real questions ("which fruit should we serve Friday?"), **collect** (vote with cubes), **represent** (pictographs, bar-like cube towers), **interpret** ("most/fewest", "how many more?"). One-to-one icon correspondence mirrors counting principles.

- **Unit of repeat** is the concept - always ask "what repeats?"
- **Re-sort the same set** by a new rule - builds flexible thinking.
- **Real questions -> real data -> real graphs.**
- **Interpret**, don't just make, graphs.

VERY IMPORTANT — Exam Pointer

Exam pointer: run a complete data cycle (question to interpretation) for a KG class.

Section Summary

Patterns, sorting and data build algebraic and statistical thinking through real investigations.

UNIT 4: PEDAGOGY AND ASSESSMENT

Lecture 10: Play-Based Pedagogy and the Learning Environment

Learning Objectives

- Structure play-based math
- Set up math-rich environments
- Use questioning well

Key Terms & Definitions

Free play

Child-directed; teacher observes and extends.

Guided play

Teacher-initiated with learning goals, child-directed within.

Math-rich environment

Centers stocked with manipulatives, games, books, real tools.

Play types: **free play** (teacher observes, notes levels, extends with a question), **guided play** (the powerhouse: teacher sets up a bakery shop to target money/counting, children direct the play), **direct instruction** (brief, explicit - for a new game's rules or a stuck child). Research favors guided play over both extremes.

Environment: math center (manipulatives, puzzles, games rotated), block area, sand/water, dramatic play with math props (cash register, measuring cups), **math in books** (counting stories), outdoor math (number hunts, measuring shadows). **Label** quantities in the room; keep materials **accessible and organized**.

Teacher moves: observe first, then **question** ("what do you notice?", "how many more do you need?", "can you prove it?"), **parallel talk** (narrate the child's math: "you gave each bear one plate - that's fair sharing!"), **extend** ("what if two more friends come?"), and know when to step back.

- **Guided play** is the evidence-based sweet spot.
- **Observe -> question -> extend:** the teacher's play cycle.
- **Rotate materials** to sustain novelty and target trajectories.
- **Narrate the math** children do - it makes thinking visible.

Example

In the block area: "Your tower is taller than Ali's. How many blocks taller? Can you build one exactly as tall?"
- measurement and comparison inside free play.

VERY IMPORTANT — Exam Pointer

Very important: contrast free play, guided play and direct instruction with when each fits.

Section Summary

Rich environments plus skillful questioning turn play into mathematics.

Lecture 11: Manipulatives, Technology and Resources

Learning Objectives

- Select manipulatives
- Use technology appropriately
- Make low-cost resources

Key Terms & Definitions

Manipulatives

Physical objects children handle to model math ideas.

Virtual manipulatives

Digital versions (apps, applets) with similar affordances.

Concrete-pictorial-abstract

Bruner's sequence from objects to symbols.

Essential manipulatives: counters, linking cubes, ten-frames, base-ten blocks, pattern blocks, attribute buttons, dice, dominoes, balance scales, measuring cups, play money, geoboards. **Principles:** introduce one at a time with **explicit** modeling of its mathematical use (blocks are not toys first day), connect manipulative to picture to symbol **every time**.

Technology: high-quality apps (e.g. Montessori-style number apps, virtual ten-frames) as **supplements**, never replacements - the hand-brain link of physical objects matters. **Evaluate apps:** math fidelity (no gimmicks teaching wrong ideas), feedback quality, no ads, limited session design. **Interactive whiteboards** for shared problem-solving.

Low-cost/no-cost: bottle caps, sticks, seeds, pebbles, cardboard ten-frames, handmade dot cards, market flyers for numbers, **teacher-made games** (board games with dice = counting + one-to-one). Resourcefulness beats budgets - but **organization** (labeled trays, routines) beats piles of stuff.

- **Teach the manipulative's math use explicitly** before free exploration.
- **Always bridge:** object -> drawing -> symbol.
- **Apps supplement, never replace** hands-on work.
- **Low-cost materials** work when organized and purposeful.

VERY IMPORTANT — Exam Pointer

Exam pointer: list criteria for evaluating a math app for KG.

Section Summary

The right objects, bridged to symbols, make abstract number concrete - at any budget.

Lecture 12: Assessment in Early Childhood Mathematics

Learning Objectives

- Use observation-based assessment
- Conduct diagnostic interviews
- Involve families

Key Terms & Definitions

Authentic assessment

Evaluating through real activities, not isolated tests.

Diagnostic interview

One-on-one task-based conversation revealing thinking.

Portfolio

Collection of work samples showing growth over time.

Principles: assess **during** learning (observation, questioning), use **multiple sources**, assess **processes** (strategies) not just answers, keep it **low-stress** and **playful**. Formal paper tests are invalid for young children - anxiety and reading demands mask math knowledge.

Tools: **observation checklists** aligned to trajectories (can subitize to 5? counts out 10?), **anecdotal records** (dated notes with quotes), **diagnostic interviews** ("count these bears... how many? how did you know?"), **work samples/photos**, **portfolios** with child-selected pieces and reflections. **Rubrics** describe levels, not just scores.

Using data: group children by trajectory level for guided play (flexible, changing groups), plan next tasks, share **growth-focused** reports with parents ("Ayesha now counts to 20 and is working on counting out objects - play counting games at home"). **Family math:** home activities (cooking measures, board games) multiply school effects.

- **Observe strategies, not just answers.**
- **Interviews reveal thinking** that worksheets hide.
- **Portfolios show growth** - include child voice.
- **Report growth + next steps**, never just scores, to families.

Example

Diagnostic interview: "Here are 12 buttons. Give me 7." Watching whether she counts out (mature) or counts all then removes (earlier level) diagnoses precisely.

VERY IMPORTANT — Exam Pointer

Very important: design an observation checklist for counting aligned to the developmental progression.

Section Summary

Assess authentically, diagnose precisely, and partner with families for growth.

Lecture 13: Supporting All Learners

Learning Objectives

- Differentiate instruction
- Support math difficulties
- Enrich advanced learners

Key Terms & Definitions

Differentiation

Adjusting task, support or outcome to learner needs.

Dyscalculia

Persistent difficulty with number sense despite instruction.

Scaffolding

Temporary support calibrated to the learner's level.

Differentiation: same trajectory, different entry points - tiered tasks (count to 10/20/50), **choice boards**, flexible small groups, varied manipulatives. **Universal Design:** multiple representations, clear routines, visual supports help everyone, especially **multilingual** learners (preview vocabulary, pair words with objects).

Struggling learners: more **concrete** time (never rush to symbols), **systematic** small steps, **cumulative** review, explicit math language, **subitizing** interventions (often the root gap), short frequent sessions. **Dyscalculia** signs: cannot subitize 3, no number-line sense by Grade 1 - refer for assessment; adapt, don't just repeat.

Advanced learners: **depth not acceleration** - pose "how many ways?", larger numbers in real contexts, pattern generalization, peer **tutoring** (explaining deepens their own understanding). **Mindset:** praise strategies and effort ("you found a clever way"), never "smart" - growth mindset predicts persistence.

- **Differentiate by trajectory level**, not by worksheets quantity.
- **Subitize first** when a child struggles - it is often the root.
- **Depth > speed** for advanced learners.
- **Growth mindset language** for every child, every day.

VERY IMPORTANT — Exam Pointer 

Exam pointer: a KG child cannot subitize 4 and counts all for 3+2 - plan a two-week intervention.

Section Summary

Meet each child at their trajectory level: scaffold strugglers, deepen advancers, include all.

Glossary of Key Terms

DAP — Developmentally Appropriate Practice (NAEYC).

Learning trajectory — Goal + developmental progression + instructional tasks.

Subitizing — Instant recognition of small quantities.

Cardinality — Last number word = total quantity.

CGI — Cognitively Guided Instruction: teaching from children's strategies.

van Hiele — Levels of geometric thinking (visualization -> analysis...).

ZPD — Zone of Proximal Development: teachable with guidance.

CPA — Concrete-Pictorial-Abstract representation sequence.