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HIGHLIGHTED HANDOUT

MTH403

Calculus and Analytical Geometry - II

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MTH403 - Calculus and Analytical Geometry - II

A structured study handout compiled from the course's lecture topics. Read each topic's objectives first, study the key concepts, then use the summary and exam tips for revision. 

MTH403

Calculus and Analytical Geometry - II

Based on 45 lecture topics

Compiled for VU students as a structured study guide

*Concepts are summarized in the writer's own words;
use alongside your lecture videos for best results.*

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Course Introduction

Calculus II takes the ideas of limits, derivatives, and integrals into deeper and wider territory: advanced integration techniques, infinite sequences and series, parametric curves, polar coordinates, and an introduction to vectors and multivariable thinking.

Where Calculus I taught differentiation and basic integration, this course teaches power: computing integrals that resist elementary methods, determining whether infinite sums converge, describing curves that functions cannot, and extending calculus toward three dimensions.

The emphasis is on technique and understanding together: each method (substitution, parts, partial fractions) is motivated by why it works, series are understood through what convergence means, and geometry (polar, parametric, vectors) keeps the analysis visual and grounded.

How to use this handout: Each topic lists its learning objectives, explains the key concepts in simple words, defines important terms, gives an example where helpful, and closes with a summary plus exam tips. Topics marked **Very important** deserve extra revision.

Unit 1: Integration Techniques

Topic 1: Review: The Definite Integral

Learning Objectives

- Recall Riemann sums
- State the Fundamental Theorem
- Apply basic properties

Key Concepts

The definite integral as a limit of Riemann sums: area under a curve, accumulated quantity. The Fundamental Theorem connects differentiation and integration: if $F' = f$, then the integral of f from a to b equals $F(b) - F(a)$ - differentiation reversed.

Properties: linearity, additivity over intervals, comparison, and symmetry tricks (even/odd functions over symmetric intervals halve or kill integrals). These simplify before technique begins.

Strategy mindset: integration is pattern recognition - identify the integral's form, choose the method that fits, and verify by differentiating. There is no universal algorithm; experience builds the repertoire.

Key Points

- Integral as Riemann limit; FTC links it to antiderivatives.
- Linearity, additivity, symmetry shortcuts.
- Integration is pattern recognition + verification.

Key Terms

Riemann sum: Approximating area with rectangles.

Fundamental Theorem: Linking differentiation and integration.

Antiderivative: A function whose derivative is given.

Example

- Integral of x^2 from 0 to 1 = $1/3$ via $x^3/3$.

Summary: The integral accumulates; the FTC computes.

Exam Tips

- ★ Very important: FTC statement.
- ★ Remember even/odd shortcuts.

Topic 2: Substitution Revisited

Learning Objectives

- Apply u-substitution fluently
- Handle definite integrals
- Recognize substitution patterns

Key Concepts

Substitution reverses the chain rule: spot an inner function $g(x)$ whose derivative (up to constants) also appears, set $u = g(x)$, and transform. For definite integrals, change the limits with u - avoiding back-substitution.

Patterns: integrals of $f(g(x))g'(x)$; rationalizing substitutions ($u = \sqrt{x}$); trigonometric substitutions previewed. The art is choosing u : look for the 'inside' function, especially one whose derivative lurks nearby.

Common errors: forgetting du 's constant factors, not changing limits, and choosing substitutions that complicate. Practice builds the eye.

Key Points

- u = inner function; du absorbs the derivative.
- Change limits for definite integrals.
- Look for the inside function with its derivative nearby.

Key Terms

u-substitution: Reversing the chain rule.

Inner function: The composed function's inside part.

Back-substitution: Returning from u to x .

Example

- Integral of $2x \cos(x^2)$: $u = x^2$.

Summary: Substitution undoes composition.

Exam Tips

- ★ Key point: changing limits.
- ★ Remember the pattern recognition.

Topic 3: Integration by Parts

Learning Objectives

- State the parts formula
- Choose u and dv (LIATE)

- Handle repeated parts

Key Concepts

Integration by parts reverses the product rule: $\int u \, dv = uv - \int v \, du$. Choose u to simplify when differentiated, dv to remain integrable when integrated. LIATE guides: Logarithmic, Inverse trig, Algebraic, Trigonometric, Exponential - earlier letters become u .

Repeated parts: for $e^x \sin x$, apply twice and solve for the original integral algebraically. Tabular (DI) method speeds repeated cases.

Applications: integrals of $\ln x$, $\arctan x$, $x^n e^x$, and reduction formulas (e.g. for $\sin^n x$). Parts is the workhorse for products of unlike functions.

Key Points

- $uv - \int v \, du$; LIATE chooses u .
- Repeated parts may need algebraic solving.
- Products of unlike functions signal parts.

Key Terms

LIATE: Priority order for choosing u .

Reduction formula: Recurrence lowering an integral's power.

Tabular method: Fast repeated integration by parts.

Example

- Integral of $x \cdot e^x$: $u = x$, $dv = e^x \, dx$.

Summary: Parts undoes the product rule.

Exam Tips

- ★ Very important: LIATE.
- ★ Remember the twice-and-solve trick.

Topic 4: Trigonometric Integrals

Learning Objectives

- Integrate powers of $\sin/\cos$
- Handle $\sec/\tan$ powers
- Use power-reduction

Key Concepts

Powers of sine and cosine: for odd powers, peel one factor and convert the rest with $\sin^2 + \cos^2 = 1$, then substitute $u =$ the other function. For even powers, use half-angle identities (power reduction) to linearize.

Secant/tangent: $\sec^2$'s derivative is $\sec^2 \tan$ - peel $\sec^2$ for odd $\sec$ powers; convert $\tan^2 = \sec^2 - 1$. 1. The integral of $\sec x$ (multiply by $(\sec+\tan)/(\sec+\tan)$) is a classic trick.

Products $\sin(mx)\cos(nx)$: product-to-sum identities convert to easy integrals. Strategy: identities first, substitution second.

Key Points

- Odd powers: peel and convert; even: half-angle reduce.
- sec/tan: exploit $\sec^2$'s derivative.
- Product-to-sum for mixed products.

Key Terms

Power reduction: Lowering trig powers via identities.

Half-angle identity: $\sin^2 = (1 - \cos 2x)/2$.

Product-to-sum: Converting products to sums.

Example

- *Integral of $\sin^3 x$: peel $\sin x$. convert $\sin^2$.*

Summary: Identities tame trigonometric integrals.

Exam Tips

- ★ Key point: odd vs. even strategy.
- ★ Remember $\sec x$ trick.

Topic 5: Trigonometric Substitution

Learning Objectives

- Apply trig substitution
- Match forms to substitutions
- Complete the method

Key Concepts

For radicals of quadratics: $\sqrt{a^2 - x^2} \rightarrow x = a \sin(\theta)$; $\sqrt{a^2 + x^2} \rightarrow x = a \tan(\theta)$; $\sqrt{x^2 - a^2} \rightarrow x = a \sec(\theta)$. Each exploits a trig identity to eliminate the radical.

Procedure: substitute, simplify with identities, integrate in θ , then back-substitute using a reference triangle (opposite/adjacent/hypotenuse from the substitution).

Completing squares extends the method to general quadratics. After substitution, the integral often needs parts or trig-integral techniques - methods combine.

Key Points

- Three forms, three substitutions ($\sin$, $\tan$, $\sec$).
- Reference triangles back-substitute.
- Complete the square for general quadratics.

Key Terms

Trigonometric substitution: Removing radicals via trig identities.

Reference triangle: Right triangle encoding the substitution.

Completing the square: Rewriting quadratics as sums of squares.

Example

- *$\sqrt{4 - x^2}$: $x = 2 \sin(\theta)$.*

Summary: Trig substitution dissolves quadratic radicals.

Exam Tips

★ Very important: match form to substitution. 

★ Remember reference triangles.

Topic 6: Partial Fractions

Learning Objectives

- Decompose rational functions
- Handle repeated/irreducible factors
- Integrate the decomposition

Key Concepts

Partial fractions split $P(x)/Q(x)$ (proper: $\deg P < \deg Q$; else divide first) into integrable pieces. Factor Q : linear factors give $A/(x-a)$; repeated linears give chains $A/(x-a) + B/(x-a)^2$; irreducible quadratics give $(Ax+B)/(\text{quadratic})$.

Find constants by equating coefficients or strategic x -values (cover-up for distinct linears). Each piece integrates elementarily: logs for linears, arctan for quadratics (after completing the square).

This is the method that 'always works' for rational functions - the theoretical guarantee that every rational function has an elementary antiderivative.

Key Points

- Proper form first; factor denominator.
- Linear/repeated/quadratic factors get specific forms.
- Every rational function integrates elementarily.

Key Terms

Partial fractions: Decomposing rational functions.

Proper fraction: Numerator degree below denominator's.

Cover-up method: Quick constants for distinct linears.

Example

$$\bullet \quad 1/((x-1)(x+2)) = A/(x-1) + B/(x+2).$$

Summary: Partial fractions guarantee rational integrals.

Exam Tips

- ★ Very important: the three factor forms.
- ★ Remember polynomial division first.

Topic 7: Improper Integrals

Learning Objectives

- Define improper integrals
- Evaluate via limits
- Apply comparison tests

Key Concepts

Improper integrals extend integration to infinite limits or unbounded integrands: replace the problem point with a limit and evaluate. Converges if the limit exists finitely; diverges otherwise.

Examples: integral of $1/x^2$ from 1 to infinity converges ($= 1$); integral of $1/x$ diverges (logarithmically).
The p-test: $1/x^p$ converges at infinity iff $p > 1$; near zero iff $p < 1$.

Comparison: if $0 \leq f \leq g$ and g converges, f converges. Improper integrals model total quantities over infinite horizons (probabilities, work) - convergence is the question that matters.

Key Points

- Infinite limits or unbounded integrands via limits.
- p-test: $1/x^p$ converges at infinity iff $p > 1$.
- Comparison test transfers convergence.

Key Terms

Improper integral: Integral over infinite/unbounded domains.

Convergence: The limit existing finitely.

p-test: Convergence criterion for $1/x^p$.

Example

- *Integral $1/x^2$ from 1 to $\infty = 1$.*

Summary: Limits extend integration to infinity.

Exam Tips

- ★ Very important: the p-test both ways.
- ★ Remember limit definition.

Unit 2: Sequences and Series

Topic 8: Sequences

Learning Objectives

- Define sequences and limits
- Test convergence
- Recognize key sequences

Key Concepts

A sequence is an ordered list $a_1, a_2, \dots$; convergence means terms approach a limit L . Techniques: divide by highest power for rationals, squeeze theorem, and recognizing forms (r^n converges iff $|r| < 1$... careful: $r=1$ gives 1, $r=-1$ oscillates).

Monotone bounded sequences converge (completeness of reals) - useful when limits are hard to compute directly. Recursive sequences (Newton's method, continued fractions) need monotonicity analysis.

Sequences underlie series: a series converges iff its partial-sum sequence converges. Master sequences first; series follow.

Key Points

- Convergence: terms approaching a limit.
- r^n converges iff $-1 < r \leq 1$ (to 0 unless $r=1$).
- Monotone + bounded implies convergence.

Key Terms

Sequence: An ordered list of numbers.

Limit: The value terms approach.

Squeeze theorem: Bounding between converging sequences.

Example

- $(n/(n+1)) \rightarrow 1$; $(1/2)^n \rightarrow 0$.

Summary: Sequences are series' foundation.

Exam Tips

- ★ Key point: r^n cases.
- ★ Remember monotone-bounded theorem.

Topic 9: Infinite Series: Basics

Learning Objectives

- Define series convergence
- State the divergence test
- Sum geometric and telescoping series

Key Concepts

A series sums infinitely: convergence means partial sums approach a finite limit. The divergence (nth-term) test: if terms don't approach 0, the series diverges - but terms approaching 0 guarantees nothing (harmonic series diverges!).

Geometric series: sum of $ar^n = a/(1-r)$ for $|r| < 1$ - the workhorse (finance, probabilities, fractals). Telescoping series collapse via cancellation (partial fractions reveal them).

The harmonic series $(1/n)$ diverges - slowly (logarithmically). This cautionary example motivates the convergence tests ahead.

Key Points

- Convergence via partial sums; nth-term test for divergence.
- Geometric: $a/(1-r)$ for $|r| < 1$.
- Harmonic diverges despite terms $\rightarrow 0$.

Key Terms

Series: Sum of a sequence's terms.

Partial sum: Sum of the first n terms.

Divergence test: Terms not $\rightarrow 0$ implies divergence.

Example

- $1 + 1/2 + 1/4 + \dots = 2$.

Summary: Infinite sums need convergence proof.

Exam Tips

- ★ Very important: geometric sum formula.
- ★ Remember harmonic caution.

Topic 10: Positive-Term Tests

Learning Objectives

- Apply integral, comparison, ratio, root tests
- Choose tests strategically
- Handle p-series

Key Concepts

Integral test: $\sum f(n)$ converges iff integral of f converges (f positive, decreasing) - links series to improper integrals. P-series $(1/n^p)$: converges iff $p > 1$.

Comparison tests: direct (bound by known series) and limit comparison (asymptotic equivalence - compare growth rates). Ratio test: if $|a_{n+1}/a_n| \rightarrow L$, converges if $L < 1$, diverges if $L > 1$ (inconclusive at 1) - ideal for factorials and exponentials. Root test: similar with n th roots - ideal for n th powers.

Strategy: recognize the dominant growth (factorials beat exponentials beat polynomials beat logs), then choose the test that exploits it.

Key Points

- Integral test; p-series converges iff $p > 1$.
- Comparison and limit comparison.
- Ratio (factorials), root (powers); inconclusive at 1.

Key Terms

p-series: Sum of $1/n^p$.

Ratio test: Using successive-term ratios.

Limit comparison: Comparing asymptotic growth.

Example

- $\sum n!/n^n$: ratio test.

Summary: Tests classify series by growth rates.

Exam Tips

- ★ Very important: p-series rule.
- ★ Remember ratio-test use cases.

Topic 11: Alternating Series and Absolute Convergence

Learning Objectives

- State the alternating series test
- Distinguish absolute/conditional
- Estimate remainders

Key Concepts

Alternating series (signs alternate): converge if terms decrease to 0 (Leibniz test) - e.g. the alternating harmonic series converges (to $\ln 2$). The remainder is bounded by the first omitted term - easy error estimates.

Absolute vs. conditional: absolutely convergent ($\sum |a_n|$ converges) series behave perfectly - rearrangement-safe, product-safe. Conditionally convergent series (convergent but not absolutely) are delicate: Riemann showed rearrangement can sum to anything!

Moral: prove absolute convergence when possible; handle conditional cases with the alternating test and care.

Key Points

- Alternating test: decreasing terms $\rightarrow 0$.
- Absolute convergence is robust; conditional is delicate.
- Remainder bounded by first omitted term.

Key Terms

Absolute convergence: Sum of absolute values converges.

Conditional convergence: Convergent but not absolutely.

Leibniz test: The alternating series test.

Example

- *Alternating harmonic converges conditionally to $\ln 2$.*

Summary: Signs matter: absolute safety vs. conditional delicacy.

Exam Tips

- ★ Very important: absolute vs. conditional.
- ★ Remember remainder estimate.

Topic 12: Power Series

Learning Objectives

- Define power series
- Find radius/interval of convergence
- Differentiate and integrate termwise

Key Concepts

Power series $\sum c_n (x-a)^n$: functions as infinite polynomials. The ratio/root test gives a radius R : converges absolutely inside $(a-R, a+R)$, diverges outside; endpoints checked individually - the interval of convergence.

Inside the interval, power series differentiate and integrate termwise - making them calculus-friendly representations. Operations (addition, multiplication, composition) work within radii.

Power series are the bridge to Taylor series: every sufficiently smooth function may equal its Taylor expansion - analysis's most powerful representation.

Key Points

- Radius R via ratio/root; check endpoints separately.
- Termwise differentiation/integration inside interval.
- Functions as infinite polynomials.

Key Terms

Power series: Sum of $c_n(x-a)^n$.

Radius of convergence: Distance of guaranteed convergence.

Interval of convergence: Full set of convergence points.

Example

- $\sum x^n/n!$ converges for all x ($R = \infty$).

Summary: Power series: calculus with infinite polynomials.

Exam Tips

- ★ Very important: endpoint checking.
- ★ Remember termwise operations.

Topic 13: Taylor and Maclaurin Series

Learning Objectives

- Construct Taylor series
- Memorize key Maclaurin series
- Estimate with Taylor's inequality

Key Concepts

Taylor series: $f(x) = \sum f^{(n)}(a)(x-a)^n/n!$ - matching all derivatives at a . Maclaurin is $a = 0$. Key series: e^x , $\sin x$, $\cos x$, $1/(1-x)$, $\ln(1+x)$, $(1+x)^k$ (binomial) - memorize these; derive others by substitution, differentiation, integration.

Taylor's inequality bounds the remainder: $|R_n| \leq M|x-a|^{n+1}/(n+1)!$ - quantifying approximation accuracy, essential for numerical work.

Applications: limits (via series expansion), approximations ($\sin x \approx x - x^3/6$), proving e 's irrationality, and defining functions (e^x for complex x). Series are analysis's microscope.

Key Points

- Taylor: match derivatives; Maclaurin at 0.
- Memorize e^x , $\sin$, $\cos$, $1/(1-x)$, $\ln(1+x)$.
- Taylor's inequality bounds approximation error.

Key Terms

Taylor series: Expansion matching derivatives at a point.

Maclaurin series: Taylor series at 0.

Remainder: The truncation error.

Example

- $e^x = 1 + x + x^2/2! + \dots$

Summary: Taylor series: functions revealed as infinite polynomials.

Exam Tips

- ★ Very important: memorize the key series.
- ★ Remember remainder bound.

Unit 3: Parametric, Polar, and Vectors

Topic 14: Parametric Curves

Learning Objectives

- Parametrize curves
- Compute parametric derivatives
- Find arc lengths

Key Concepts

Parametric equations $(x(t), y(t))$ describe curves functions cannot: circles, cycloids, projectile paths - with t as time, tracing direction matters. Eliminating t recovers Cartesian form when possible.

Derivatives: $dy/dx = (dy/dt)/(dx/dt)$ - tangents via parameter; second derivatives need the quotient rule in t . Arc length: integral of $\sqrt{(dx/dt)^2 + (dy/dt)^2} dt$ - distance traveled.

Applications: motion (velocity/acceleration vectors from position), Bezier curves in design, and plane curves in engineering. Parameters add the dimension of 'when'.

Key Points

- $(x(t), y(t))$: curves with direction and time.
- $dy/dx = y'(t)/x'(t)$; arc length via speed integral.
- Motion and design applications.

Key Terms

Parameter: The variable tracing the curve.

Cycloid: Curve traced by a rolling wheel.

Arc length: Curve's length via integral.

Example

- Circle: $x = \cos t$, $y = \sin t$.

Summary: Parameters describe motion, not just shape.

Exam Tips

- ★ Key point: the dy/dx formula.
- ★ Remember arc length integrand.

Topic 15: Polar Coordinates

Learning Objectives

- Plot in polar coordinates
- Convert between systems
- Graph polar curves

Key Concepts

Polar coordinates (r, θ) : position by distance and angle - natural for circular/spiral symmetry. Conversion: $x = r \cos(\theta)$, $y = r \sin(\theta)$; $r^2 = x^2 + y^2$, $\tan(\theta) = y/x$ (mind quadrants!).

Curves: circles ($r = a \cos(\theta)$), cardioids ($r = a(1 - \cos \theta)$), roses ($r = a \cos(n \theta)$: n petals, or $2n$ if n even), spirals ($r = \theta$), lemniscates. Symmetry tests (about x -axis, y -axis, origin) simplify graphing.

Derivatives and tangents in polar need the parametric view ($x(\theta)$, $y(\theta)$). Areas: $(1/2) \int r^2 d(\theta)$ - sectors, not rectangles.

Key Points

- (r, θ) : distance and angle; conversion formulas.
- Cardioids, roses, spirals, lemniscates.
- Area = $(1/2) \int r^2 d(\theta)$.

Key Terms

Polar coordinates: Position by radius and angle.

Cardioid: Heart-shaped $r = a(1 - \cos \theta)$.

Rose: Petal curve $r = a \cos(n \theta)$.

Example

- $r = 2$: circle radius 2 centered at origin.

Summary: Polar coordinates suit circular symmetry.

Exam Tips

- ★ Very important: area formula.
- ★ Remember rose petal counts.

Topic 16: Vectors in the Plane and Space

Learning Objectives

- Perform vector operations
- Explain dot and cross products
- Describe lines and planes

Key Concepts

Vectors: magnitude and direction; operations: addition (tip-to-tail), scalar multiplication, components. In 3D: with i, j, k basis.

Dot product: $a \cdot b = |a||b|\cos(\theta)$ - measures alignment; zero means perpendicular; projections decompose vectors. Cross product: $a \times b$ (3D only) - perpendicular vector with $|a \times b| = \text{area of parallelogram}$; direction by right-hand rule.

Lines: $r = r_0 + t v$ (point + direction). Planes: $n \cdot (r - r_0) = 0$ (point + normal). Distances: point-to-plane via projection. Vectors geometrize 3D.

Key Points

- Dot: alignment/projection; zero = perpendicular.
- Cross: perpendicular vector; magnitude = parallelogram area.
- Lines: point + $t \cdot \text{direction}$; planes: point + normal.

Key Terms

Dot product: Scalar measure of alignment.

Cross product: Perpendicular vector product.

Normal vector: Perpendicular to a plane.

Example

- *Work = force . displacement.*

Summary: Vectors: algebra for geometry.

Exam Tips

- ★ Very important: dot vs. cross meanings.
- ★ Remember plane equation.

Topic 17: Vector Calculus Preview

Learning Objectives

- Differentiate vector functions
- Explain velocity/acceleration
- Preview multivariable ideas

Key Concepts

Vector functions $\mathbf{r}(t)$: derivatives give velocity $\mathbf{v} = \mathbf{r}'$, speed $= |\mathbf{v}|$, acceleration $\mathbf{a} = \mathbf{v}'$. Unit tangent $\mathbf{T} = \mathbf{v}/|\mathbf{v}|$; curvature measures bending - the geometry of motion.

Projectile motion: $\mathbf{r}(t)$ from constant gravitational acceleration - parametric equations with physics. Kepler's laws emerge from central-force motion (optional enrichment).

Preview: functions of several variables $f(x, y)$ - partial derivatives, gradients pointing uphill, and double integrals - Calculus III's territory, glimpsed here.

Key Points

- $\mathbf{r}'(t)$ = velocity; speed $= |\text{velocity}|$.
- Unit tangent and curvature describe motion geometry.
- Gradient and partial derivatives preview what's next.

Key Terms

Velocity: Derivative of position.

Curvature: Measure of bending.

Partial derivative: Derivative holding others fixed.

Example

- *Projectile: $\mathbf{r}(t) =$.*

Summary: Vector calculus: motion mathematized.

Exam Tips

- ★ Key point: velocity/acceleration relations.
- ★ Remember gradient direction.

Topic 18: Applications of Integration

Learning Objectives

- Compute areas and volumes
- Explain work and average value
- Model with integrals

Key Concepts

Areas between curves: integral of (top - bottom). Volumes: disks/washers (revolve around axis) and shells (cylindrical) - choose by which gives simpler integrals; known cross-sections generalize.

Physics: work = integral of force (variable forces: springs, pumping water); center of mass and centroids; hydrostatic pressure; average value of a function $(1/(b-a)) \int_a^b f(x) dx$.

Modeling: integrals accumulate - from areas to probabilities (continuous distributions) to economics (consumer surplus). The integral is applied mathematics' accumulator.

Key Points

- Volumes: disks, washers, shells - choose wisely.
- Work, center of mass, average value.
- Integrals accumulate: area, work, probability.

Key Terms

Method of shells: Volume via cylindrical shells.

Centroid: Geometric center of mass.

Average value: Mean of a function over an interval.

Example

- *Sphere volume via disks: $(4/3) \pi r^3$.*

Summary: Integration computes the physical world's totals.

Exam Tips

- ★ Key point: disks vs. shells choice.
- ★ Remember work integral.

Topic 19: Differential Equations Preview

Learning Objectives

- Recognize basic ODEs
- Solve separable equations
- Model with ODEs

Key Concepts

Separable equations: $dy/dx = g(x)h(y)$ - separate and integrate: $\int dy/h(y) = \int g(x) dx$. Exponential growth/decay ($dy/dt = ky$), Newton's cooling, logistic growth (with carrying capacity) - the classic models.

First-order linear: $dy/dx + P(x)y = Q(x)$ - integrating factor $\mu = e^{\int P dx}$ makes the left an exact derivative. Mixing problems and circuits apply it.

Direction fields visualize solutions without solving - qualitative ODE theory previewed. Differential equations are calculus's purpose: describing change.

Key Points

- Separable: split and integrate both sides.
- Linear first-order: integrating factor method.
- Growth, cooling, logistic, mixing models.

Key Terms

Separable equation: Splittable into x- and y-parts.

Integrating factor: Multiplier enabling direct integration.

Direction field: Slope visualization of ODE solutions.

Example

- $dv/dx = 2x$ with $v(0)=1$: $v = x^2 + 1$.

Summary: Differential equations: calculus describing change.

Exam Tips

- ★ Key point: separable procedure.
- ★ Remember integrating factor formula.

Topic 20: Problem-Solving Strategies and Review

Learning Objectives

- Choose integration methods
- Plan series problems
- Review the course

Key Concepts

Integration strategy: simplify first (algebra, trig identities), then classify - substitution (composition), parts (products), trig techniques, trig substitution (quadratic radicals), partial fractions (rationals). When stuck: try a different method, differentiate to check, or consider numerical/special functions.

Series strategy: check nth term; identify type (geometric, p-series, alternating); choose test by growth form; for power/Taylor: radius first, endpoints second.

The course's arc: from antidifferentiation techniques through infinite summation to new geometries - each unit generalizing the last. Mastery means method selection, not just execution.

Key Points

- Simplify, classify, choose method, verify.
- Series: nth-term, type, test, endpoints.
- Method selection is the real skill.

Key Terms

Strategy: Planned method selection.

Verification: Checking by differentiation.

Classification: Identifying problem type.

Example

- Seeing $\sqrt{x^2+9}$ and choosing $x = 3 \tan(\theta)$.

Summary: Calculus II: a toolkit wielded with judgment.

Exam Tips

- ★ Key point: the integration decision tree.
- ★ Remember verify-by-differentiation.

Glossary of Key Terms

Antiderivative: A function whose derivative is the given function.

u-substitution: Integration by reversing the chain rule.

Integration by parts: Reversing the product rule: $uv - \int v \, du$.

Partial fractions: Decomposing rational functions for integration.

Improper integral: Integral over infinite or unbounded domains.

p-series: Sum of $1/n^p$; converges iff $p > 1$.

Ratio test: Convergence via successive-term ratios.

Absolute convergence: Convergence of absolute values.

Power series: Infinite polynomial sum $\sum c_n(x-a)^n$.

Taylor series: Function expansion matching derivatives.

Polar coordinates: Position by radius r and angle θ .

Dot product: Scalar product measuring alignment.

Cross product: Vector product perpendicular to both factors.

Separable equation: ODE splittable into single-variable parts.

Parametric equations: Curves described via a parameter t .