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**HIGHLIGHTED HANDOUT**

## **MTH5207**

Mechanics I

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# MTH5207

## Mechanics I

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This handout covers the complete course topic by topic in simple, exam-focused language. Each chapter begins with learning objectives and ends with key points and exam pointers. Definitions are boxed in green-friendly format, examples are worked out step by step, and very important exam points are flagged separately. Study the highlighted points carefully before your papers.

| Course Information |             |
|--------------------|-------------|
| Course Code        | MTH5207     |
| Course Title       | Mechanics I |
| Credit Hours       | 3           |
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# 1. Mathematical Preliminaries: Vectors

## Learning Objectives

- Perform vector algebra
- Apply dot and cross products

- Vector: magnitude + direction; scalar: magnitude only.
- Addition: triangle/parallelogram law; components:  $\mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}$ .
- Dot product:  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos(\theta)$  — tests perpendicularity (zero) and finds projections.
- Cross product:  $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin(\theta) \mathbf{n}$  — gives torques and normals;  $\mathbf{a} \times \mathbf{b} = -(\mathbf{b} \times \mathbf{a})$ .

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

## Example

$\mathbf{a} = (2, 3, 1)$ ,  $\mathbf{b} = (1, -1, 4)$ :  $\mathbf{a} \cdot \mathbf{b} = 2 - 3 + 4 = 3$ ; the angle between them is  $\arccos(3/(\sqrt{14}\sqrt{18}))$ .

## Key Points to Remember

- Dot product -> angle/work; cross product -> torque/area.
- Unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$  keep 3-D work organized.

## Exam Pointer

Very important for exams: Find the projection of  $\mathbf{a}$  onto  $\mathbf{b}$  — a standard starter numerical.

## Scalar vs vector examples

- Scalars: mass, temperature, speed.
- Vectors: displacement, force, velocity.
- Speed is scalar; velocity is vector — direction matters.

## Unit vectors in 3D

- $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$  along  $x$ ,  $y$ ,  $z$ .
- Position:  $\mathbf{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$ .
- Dot/cross via components or determinants.

## Vector triple products

- $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ : volume of parallelepiped.

- Cyclic permutations preserve value.
- Zero means coplanar vectors.

### Dimensional analysis

- Check every equation's units.
- Derive formulas up to constants.
- Catches most algebra slips.

### Dot product applications

- Work =  $F \cdot d$ : projection in action.
- Angle between vectors via cos formula.
- Orthogonality test: dot = 0.
- Component of a along b.
- Practice geometric interpretations.

### Cross product applications

- Torque and angular momentum definitions.
- Area of parallelogram from  $|a \times b|$ .
- Right-hand rule gives direction.
- Anti-commutative:  $a \times b = -(b \times a)$ .
- Determinant form for computation.

### Vector notation

- Bold or arrow for vectors.
- Magnitude always non-negative.
- Unit vectors carry direction only.
- Position vs displacement distinction.
- Consistent notation prevents errors.

## 2. Kinematics of a Particle

### Learning Objectives

- Relate position, velocity and acceleration
- Solve projectile and circular motion

- Position  $r(t)$ ; velocity  $v = dr/dt$ ; acceleration  $a = dv/dt$  — differentiate down, integrate up.

- Constant acceleration:  $v = u + at$ ,  $s = ut + (1/2)at^2$ ,  $v^2 = u^2 + 2as$ .

## Projectile motion

- Horizontal: constant velocity; vertical: constant acceleration  $-g$ .
- Range  $R = (u^2 \sin 2\theta)/g$  — maximum at 45 degrees.
- Time of flight  $T = (2u \sin \theta)/g$ ; max height  $H = (u^2 \sin^2 \theta)/(2g)$ .

## Circular motion

- Angular velocity  $\omega = d(\theta)/dt$ ;  $v = \omega \times r$ .
- Centripetal acceleration  $a = v^2/r$  toward the centre.

$$R = u^2 \sin(2\theta) / g$$

### Key Points to Remember

- Separate  $x$  and  $y$  in projectiles — they are independent.
- Centripetal force is a requirement, not a new force.

### Exam Pointer

Very important for exams: Projectile range/height numericals are guaranteed.

## Relative motion

- Velocity of A relative to B:  $v_A - v_B$ .
- River-boat problems are relative motion classics.
- Choose the frame that simplifies.

## Tangential vs normal acceleration

- Tangential: changes speed ( $dv/dt$ ).
- Normal: changes direction ( $v^2/\rho$ ).
- Curved paths need both.

## Motion in polar coordinates

- $v_r = dr/dt$ ;  $v_\theta = r d(\theta)/dt$ .
- Acceleration has Coriolis-like terms.
- Natural for central forces.

## Frames of reference

- Inertial: Newton holds; non-inertial: add pseudo-forces.

- Centrifugal is a pseudo-force.
- Earth is approximately inertial.

### Projectile range derivation

- $R = v^2 \sin(2\theta)/g$  on level ground.
- Maximum at 45 degrees.
- Complementary angles give equal range.
- Air resistance breaks the symmetry.
- Derive it once, then use freely.

### Circular motion formulas

- $a_c = v^2/r$  toward centre.
- Banked curves:  $\tan \theta = v^2/rg$ .
- Conical pendulum combines both.
- No outward "force" — inertia instead.
- Common misconception to avoid.

### Relative velocity problems

- Aircraft in wind: vector triangles.
- Swimmer crossing rivers.
- Choose ground or moving frame.
- Draw triangles tip-to-tail.
- Classic exam geometry. 

## 3. Newton's Laws and Forces

### Learning Objectives

- Apply Newton's laws to particle motion
- Handle friction, tension and constraint

- First law: inertia — inertial frames.
- Second law:  $F = ma$  (vector equation — resolve into components).
- Third law: action-reaction pairs act on different bodies.

### Standard forces

- Weight  $W = mg$  downward; normal reaction perpendicular to surfaces.

- Tension: pulls along strings; inextensible strings give equal accelerations.
- Friction:  $f \leq \mu_s N$  (static),  $f = \mu_k N$  (kinetic), opposes relative motion.
- Spring:  $F = -kx$  (Hooke's law).

### Example

Block on a 30-degree incline,  $\mu = 0.2$ : component down the plane  $mg \sin 30$ , friction up  $\mu mg \cos 30$  — net acceleration  $g(\sin 30 - \mu \cos 30)$ .

### Key Points to Remember

- Draw free-body diagrams before writing any equation.
- Friction direction always opposes impending motion.

### Exam Pointer

Very important for exams: Incline + friction + pulley combinations are the classic problems.

## Free-body diagram rules

- Isolate the body; draw all forces as arrows.
- Label every force; choose axes wisely.
- Never skip this step — it prevents most errors.

## Connected particles

- Strings: equal speed along string.
- Pulleys redirect without (ideal) friction.
- Write one equation per mass, link accelerations.

## Variable mass systems

- Conveyor belts, rockets, rain-collecting carts.
- $F = m \frac{dv}{dt} - u \frac{dm}{dt}$ .
- Account for incoming momentum.

## Impulse-momentum applications

- Airbags extend impact time, cut force.
- Follow-through in sports lengthens contact.
- Design for time, not just strength.

## Tension in strings

- Ideal strings: uniform tension, massless.
- Pulleys only redirect tension.
- Atwood machine: classic two-mass problem.
- Tension adjusts to constrain motion.
- Draw FBDs before writing equations.

### Normal reaction force

- Perpendicular to contact surface.
- Adjusts to prevent penetration.
- Zero means losing contact.
- Loop-the-loop:  $N \geq 0$  condition.
- Weighing scales read  $N$ , not  $mg$ .

### Spring combinations

- Parallel:  $k$  adds; series: reciprocals add.
- Equivalent  $k$  simplifies systems.
- Car suspensions use combinations.
- Energy splits by stiffness.
- Derive once, apply often.

## 4. Work, Energy and Power

### Learning Objectives

- Apply the work-energy theorem
- Use conservation of mechanical energy

- Work:  $W = \mathbf{F} \cdot \mathbf{d}$  (dot product — only the parallel component counts).
- Kinetic energy  $T = (1/2)mv^2$ ; work-energy theorem: net work = change in KE.
- Potential energy: gravitational  $V = mgh$ ; elastic  $V = (1/2)kx^2$ .
- Conservative forces (gravity, spring): path-independent work, energy conserved; non-conservative (friction): work lost as heat.

$$T_1 + V_1 = T_2 + V_2 \text{ (conservative systems)}$$

$$\text{Power } P = \mathbf{F} \cdot \mathbf{v} = dW/dt$$

### Example

A 2 kg block slides from height 5 m:  $v$  at bottom =  $\sqrt{2gh} = \sqrt{98}$  m/s — mass cancels.

**Key Points to Remember** 

- Energy methods beat force methods when time is not asked.
- Check conservativeness before conserving.

**Exam Pointer**

Very important for exams: Pendulum/spring energy numericals appear regularly.

**Conservative vs non-conservative**

- Gravity, spring: path-independent, potential exists.
- Friction, air resistance: dissipate energy.
- Only conservative systems conserve mechanical energy.

**Power in machines**

- $P = F v$  for constant force.
- Engines rated in horsepower/watts.
- Climbing hills needs power, not just force.

**Potential energy curves**

- Minima: stable; maxima: unstable equilibrium.
- Turning points where  $E = V$ .
- Read motion straight off the curve.

**Work by variable force**

- Integrate  $F dx$ ; area under curve.
- Spring work:  $(1/2)k(x_2^2 - x_1^2)$ .
- Graphical methods save calculus.

**Gravitational potential energy**

- Near Earth:  $mgh$ ; general:  $-GMm/r$ .
- Reference point is arbitrary.
- Escape velocity from energy conservation.
- Negative total energy means bound orbits.
- Connect to Kepler's laws.

**Conservative force test**

- Curl of  $F$  equals zero.
- Work around closed loop is zero.
- Potential exists iff conservative.
- Gravity and springs pass; friction fails.
- The mathematical criterion.

### Power and efficiency

- Efficiency = useful output / input.
- Always below 100% in practice.
- Machines trade force for speed.
- Engines rated by power output.
- Energy accounting reveals waste.

## 5. Linear and Angular Momentum

### Learning Objectives

- Apply momentum conservation in collisions
- Relate torque to angular momentum

- Linear momentum  $p = mv$ ; impulse  $J = \int F dt = \text{change in } p$ .
- Conservation: with no external impulse, total  $p$  is constant — the key to collision problems.

### Collisions

- Elastic: momentum AND kinetic energy conserved.
- Inelastic: momentum conserved, KE lost; perfectly inelastic: bodies stick together.
- Coefficient of restitution  $e$ :  $1 = \text{elastic}$ ,  $0 = \text{perfectly inelastic}$ .

### Angular momentum

- $L = r \times p$ ; torque  $\tau = r \times F = dL/dt$ .
- Conservation: with no external torque,  $L$  is constant — skaters spin faster pulling arms in.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad (1\text{-D collisions})$$

### Key Points to Remember

- Collisions: always conserve momentum; conserve KE only if elastic.
- Angular momentum conservation explains orbits and spins.

**Exam Pointer** 

Very important for exams: Two-body collision numericals with  $e$  given.

**Centre of mass**

- Weighted average position of mass.
- Translational motion follows  $F = M a_{cm}$ .
- Symmetry locates it instantly.

**Rocket equation intuition**

- Momentum conservation with changing mass.
- Exhaust speed decides efficiency.
- Staging sheds dead weight.

**Elastic collision formulas**

- 1D: exchange velocities if equal masses.
- Derive via momentum + energy.
- Memorize the pattern, not just algebra.

**Angular impulse**

- Torque  $\times$  time = change in  $L$ .
- Short hard twists vs long gentle ones.
- Diving twists use it.

**Elastic potential energy**

- $U = \frac{1}{2}kx^2$  for ideal springs.
- Area under  $F$ - $x$  graph.
- Springs in series/parallel combine like capacitors reversed.
- Oscillation frequency from energy methods.
- Real springs deviate at extremes.

**Coefficient of restitution**

- $e$  = relative speed after/before.
- $e = 1$  elastic;  $e = 0$  perfectly inelastic.
- Bouncing ball heights follow  $e^2$ .
- Measured in sports equipment.
- Combines with momentum conservation.

## Conservation law summary

- Momentum: no external impulse.
- Energy: no non-conservative work.
- Angular momentum: no external torque.
- Symmetries underlie each (Noether).
- Choose the applicable one first.

## 6. Central Forces and Orbits

### Learning Objectives

- Derive Kepler's laws from Newton's gravitation
- Analyze circular and elliptical orbits

- Newton's gravitation:  $F = GmM/r^2$  toward the centre — a central force, so angular momentum is conserved.
- Planar motion; effective potential combines centrifugal and gravitational terms.

### Kepler's laws

- 1. Orbits are ellipses with the Sun at one focus.
- 2. Equal areas in equal times (follows from angular momentum conservation).
- 3.  $T^2$  proportional to  $a^3$  (follows from the inverse-square law).

$$T^2 = (4\pi^2 / GM) a^3$$

### Example

Geostationary orbit:  $T = 24$  h gives  $r$  approx 42,164 km from Earth's centre — the basis of communications satellites over Pakistan.

### Key Points to Remember

- Kepler 2 = angular momentum; Kepler 3 = inverse-square law.
- Escape velocity  $v = \sqrt{2GM/R}$ .

### Exam Pointer

Very important for exams: Derive Kepler's third law for circular orbits.

### Satellite orbits

- Circular:  $v = \sqrt{GM/r}$ .
- Geostationary: 24h period over equator.
- Pakistan's PAKSAT sits in GEO.

### Effective potential

- Centrifugal barrier + real potential.
- Circular orbits at minima.
- Explains why planets don't fall in.

### Precession intuition

- Torque perpendicular to spin causes precession.
- Gyroscopes resist tipping.
- Bicycles and tops exploit it.

### Binary stars

- Orbit common centre of mass.
- Period reveals total mass.
- Exoplanets found the same way.

### Kepler's three laws

- Elliptical orbits with Sun at focus.
- Equal areas in equal times.
- $T^2$  proportional to  $a^3$ .
- Newton derived them from gravitation.
- State all three precisely for exams.

### Weightlessness

- Orbit: free fall, not zero gravity.
- Scales read zero in free fall.
- Astronauts train in parabolic flights.
- True zero-g only far from masses.
- Explain carefully in exams.

### Tidal forces

- Differential gravity across a body.
- Cause ocean tides and moon's lock.
- Roche limit tears satellites apart.

- Weaken with distance cubed.
- Spectacular astrophysics application.

## 7. Rigid Body Kinematics

### Learning Objectives

- Describe rotation of rigid bodies
- Compute moments of inertia

- Rigid body: distances between points fixed; motion = translation of centre of mass + rotation about it.
- Rotation: angular velocity  $\omega$ , angular acceleration  $\alpha$ ;  $v = \omega \times r$  for any point.

### Moment of inertia

- $I = \int r^2 dm$  — rotational mass; depends on axis.
- Standard results: rod about centre  $(1/12)ML^2$ ; disc about centre  $(1/2)MR^2$ ; hoop  $MR^2$ ; solid sphere  $(2/5)MR^2$ .
- Parallel axis theorem:  $I = I_{cm} + Md^2$ .

$$I = I_{cm} + M d^2 \text{ (parallel axis)}$$

### Key Points to Remember

- Moment of inertia depends on mass distribution AND axis choice.
- Parallel axis theorem saves many integrals.

### Exam Pointer

Very important for exams: Compute  $I$  for composite bodies using the parallel axis theorem.

### Radius of gyration

- $k: I = M k^2$  — mass concentrated at  $k$ .
- Compares mass distributions simply.
- Hoop:  $k = R$ ; disc:  $k = R/\sqrt{2}$ .

### Perpendicular axis theorem

- Planar bodies:  $I_z = I_x + I_y$ .
- Disc from rod results quickly.
- Only for thin plates.

### Compound pendulum

- Any rigid body swinging on a pivot.
- $T = 2\pi \sqrt{I/mgd}$ .
- Centre of oscillation interchangeable with pivot.

### Flywheels

- Store rotational energy:  $(1/2)I\omega^2$ .
- Buses and grid stabilize with them.
- High  $\omega$  beats high mass.

### Rolling without slipping

- $v_{cm} = \omega R$  condition.
- Instantaneous contact point at rest.
- Energy splits:  $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ .
- Friction does no work (static).
- Race: solid sphere beats hollow.

### Moment of inertia tricks

- Parallel axis:  $I = I_{cm} + Md^2$ .
- Additivity for composite bodies.
- Integration for continuous mass.
- Symmetry halves the work.
- Memorize rod, hoop, disc, sphere.

### Gyroscopic precession

- Torque perpendicular to spin.
- Precession rate =  $\tau/L$ .
- Tops, bikes, Earth's axis.
- Stabilizes ships and satellites.
- Counterintuitive but derivable.

## 8. Rigid Body Dynamics

**Learning Objectives**

- Apply torque and rotation equations
- Solve rolling-without-slipping problems

- Torque  $\tau = I \alpha$  (rotational  $F = ma$ ).
- Rotational KE =  $(1/2) I \omega^2$ ; work-energy includes both translational and rotational terms.
- Rolling without slipping:  $v_{cm} = \omega R$ ; static friction does no work.

**Example**

Disc vs hoop rolling down an incline: the disc wins — smaller  $I/MR^2$  means more energy goes to translation. Acceleration  $a = g \sin(\theta)/(1 + I/MR^2)$ .

**Key Points to Remember** 

- Rolling = translation + rotation combined.
- Energy methods handle rolling elegantly.

**Exam Pointer**

Very important for exams: Race-the-bodies-down-the-incline numericals.

**Toppling vs slipping**

- Tall narrow bodies topple; wide ones slip.
- Compare required  $\mu$  for each.
- Real problems need both checks.

**Instantaneous centre**

- Point with zero velocity at an instant.
- Velocities radiate from it.
- Speeds up rolling problems.

**Static determinacy**

- Equations  $\geq$  unknowns: solvable.
- More unknowns: indeterminate — needs deformation.
- Trusses:  $m + r = 2j$  for internal determinacy.

**Friction angles**

- Angle of friction:  $\tan^{-1}(\mu)$ .
- Angle of repose for granular piles.
- Ladders analyzed via both.

### Torque and levers

- $\tau = r \times F$ ; magnitude  $rF \sin \theta$ .
- Levers trade force for distance.
- Seesaws balance torques.
- Wrenches maximize lever arm.
- Static equilibrium needs  $\Sigma \tau = 0$ .

### Static friction limits

- $0 \leq f_s \leq \mu_s N$  — an inequality.
- It matches the need up to the max.
- Impending motion analysis uses the max.
- Direction opposes relative motion tendency.
- Students often misuse the equality.

### Centre of mass motion

- Moves as if all mass concentrated.
- Internal forces cancel out.
- Exploding projectile: CM follows parabola.
- Frame choice simplifies collisions.
- The most underused shortcut.

## 9. Statics: Equilibrium of Bodies

### Learning Objectives

- Apply equilibrium conditions
- Analyze frames, trusses and friction

- Equilibrium: resultant force = 0 AND resultant moment = 0 (about any point).
- Two-force members, three-force members (concurrent) simplify analysis.
- Trusses: method of joints (equilibrium at each pin) and method of sections (cut through members).
- Ladders, beams and wedges combine force and moment balance with friction.

**Example**

Ladder against a smooth wall: horizontal equilibrium gives friction at the base; moment about the base gives the normal from the wall.

**Key Points to Remember** 

- Always use both force and moment equations.
- Choose the moment centre to eliminate unknowns.

**Exam Pointer**

Very important for exams: Ladder problems are the signature statics question.

**Truss analysis tips**

- Zero-force members simplify fast.
- Sections beat joints for single members.
- Assume tension; negative means compression.

**Method of joints example**

- Start at joints with  $\leq 2$  unknowns.
- Equilibrium:  $\sum F_x = 0$ ,  $\sum F_y = 0$ .
- Work systematically across the truss.

**Simple vs physical pendulum**

- Simple: point mass on string.
- Physical: extended body.
- Simple is the small-length limit.

**Virtual work teaser**

- Equilibrium: zero virtual work.
- Elegant for complex linkages.
- Beyond this course, but beautiful.

**Friction on inclines**

- Resolve weight into parallel/perpendicular.
- Static holds until  $\mu_s N$  exceeded.
- Kinetic is usually smaller.

- Angle of repose =  $\tan^{-1} \mu_s$ .
- Classic exam staple — master it. 

### Simple pendulum derivation

- Small-angle:  $\sin \theta \approx \theta$ .
- $T = 2\pi\sqrt{L/g}$  independent of mass.
- Amplitude independence needs small angles.
- Used to measure  $g$  historically.
- State assumptions explicitly.

### Ladder problems

- Friction at base, normal at wall.
- Torque about base eliminates unknowns.
- Person climbing shifts the balance.
- Minimum angle before slipping.
- Template for statics exams.

## 10. Simple Harmonic Motion

### Learning Objectives

- Solve the SHM equation
- Analyze pendulum and spring systems

- SHM: acceleration proportional to displacement, toward equilibrium:  $d^2x/dt^2 = -\omega^2 x$ .
- Solution:  $x = A \cos(\omega t + \phi)$ ; period  $T = 2\pi/\omega$ .
- Mass-spring:  $\omega = \sqrt{k/m}$ ; simple pendulum:  $\omega = \sqrt{g/l}$  for small angles.

$$T = 2\pi \sqrt{m/k} \quad T = 2\pi \sqrt{l/g}$$

- Energy oscillates between kinetic and potential; total  $E = (1/2)kA^2$ .
- Damped SHM: amplitude decays; driven SHM: resonance when driven at natural frequency.

### Key Points to Remember

- Small-angle approximation makes the pendulum SHM.
- Resonance: small pushes at the right frequency build huge amplitude.

**Exam Pointer** 

Very important for exams: Derive the pendulum period and state the small-angle assumption.

**Damped oscillations**

- Light: oscillates decaying; critical: fastest return.
- Heavy: slow creep back.
- Car suspensions target near-critical.

**Resonance dangers**

- Tacoma Narrows: wind matched bridge frequency.
- Soldiers break step on bridges.
- Damping is the engineer's answer.

**Coupled oscillators teaser**

- Two pendulums exchange energy.
- Normal modes: in-phase and anti-phase.
- Beats reveal the coupling.

**Exam revision checklist**

- Projectile formulas, FBD method, energy vs momentum choice.
- Collision types, Kepler laws, I theorems.
- SHM derivations with assumptions stated.

**Energy in SHM**

- Total  $E = \frac{1}{2}kA^2$  constant.
- KE max at equilibrium, PE max at extremes.
- Exchange twice per cycle.
- Damping drains total energy.
- Phase plots show the exchange.

**Problem-solving strategy**

- Read, sketch, choose method (force/energy/momentum).
- Energy for speeds; momentum for collisions.
- Check units and limiting cases.
- Dimensional analysis catches errors.
- Practice beats formula sheets.

## Damped vs driven summary

- Damping: amplitude decays.
- Driving: steady state at drive frequency.
- Resonance: maximum response.
- Q factor measures sharpness.
- One framework, many systems.

## Practice Questions for Exams

### Learning Objectives

- Test your preparation with these exam-style questions 

## Short questions with model answers

### Example

Q: Explain: perform vector algebra Model answer: Dot product  $\rightarrow$  angle/work; cross product  $\rightarrow$  torque/area. Unit vectors  $i, j, k$  keep 3-D work organized.

### Example

Q: Explain: apply dot and cross products Model answer: Dot product  $\rightarrow$  angle/work; cross product  $\rightarrow$  torque/area. Unit vectors  $i, j, k$  keep 3-D work organized.

### Example

Q: Explain: relate position, velocity and acceleration Model answer: Separate  $x$  and  $y$  in projectiles — they are independent. Centripetal force is a requirement, not a new force.

### Example

Q: Explain: solve projectile and circular motion Model answer: Separate  $x$  and  $y$  in projectiles — they are independent. Centripetal force is a requirement, not a new force.

### Example

Q: Explain: apply Newton's laws to particle motion Model answer: Draw free-body diagrams before writing any equation. Friction direction always opposes impending motion.

### Example

Q: Explain: handle friction, tension and constraint Model answer: Draw free-body diagrams before writing any equation. Friction direction always opposes impending motion.

### Example

Q: Explain: apply the work-energy theorem Model answer: Energy methods beat force methods when time is not asked. Check conservativeness before conserving.

### Example

Q: Explain: use conservation of mechanical energy Model answer: Energy methods beat force methods when time is not asked. Check conservativeness before conserving.

### Example

Q: Explain: apply momentum conservation in collisions Model answer: Collisions: always conserve momentum; conserve KE only if elastic. Angular momentum conservation explains orbits and spins.

### Example

Q: Explain: relate torque to angular momentum Model answer: Collisions: always conserve momentum; conserve KE only if elastic. Angular momentum conservation explains orbits and spins.

### Example

Q: Explain: derive Kepler's laws from Newton's gravitation Model answer: Kepler 2 = angular momentum; Kepler 3 = inverse-square law. Escape velocity  $v = \sqrt{2GM/R}$ .

### Example

Q: Explain: analyze circular and elliptical orbits Model answer: Kepler 2 = angular momentum; Kepler 3 = inverse-square law. Escape velocity  $v = \sqrt{2GM/R}$ .

### Example

Q: Explain: describe rotation of rigid bodies Model answer: Moment of inertia depends on mass distribution AND axis choice. Parallel axis theorem saves many integrals.

### Example

Q: Explain: compute moments of inertia Model answer: Moment of inertia depends on mass distribution AND axis choice. Parallel axis theorem saves many integrals.

**Example**

Q: Explain: apply torque and rotation equations Model answer: Rolling = translation + rotation combined. Energy methods handle rolling elegantly.

**Example**

Q: Explain: solve rolling-without-slipping problems Model answer: Rolling = translation + rotation combined. Energy methods handle rolling elegantly.

**Example**

Q: Explain: apply equilibrium conditions Model answer: Always use both force and moment equations. Choose the moment centre to eliminate unknowns.

**Example**

Q: Explain: analyze frames, trusses and friction Model answer: Always use both force and moment equations. Choose the moment centre to eliminate unknowns.

**Example**

Q: Explain: solve the SHM equation Model answer: Small-angle approximation makes the pendulum SHM. Resonance: small pushes at the right frequency build huge amplitude.

**Example**

Q: Explain: analyze pendulum and spring systems Model answer: Small-angle approximation makes the pendulum SHM. Resonance: small pushes at the right frequency build huge amplitude.

**Long questions**

- Write a detailed note on Mathematical Preliminaries: Vectors.
- Write a detailed note on Kinematics of a Particle.
- Write a detailed note on Newton's Laws and Forces.
- Write a detailed note on Work, Energy and Power.
- Write a detailed note on Linear and Angular Momentum.
- Write a detailed note on Central Forces and Orbits.
- Write a detailed note on Rigid Body Kinematics.
- Write a detailed note on Rigid Body Dynamics.
- Write a detailed note on Statics: Equilibrium of Bodies.
- Write a detailed note on Simple Harmonic Motion.

## Long questions with model answer outlines

### Example

Q: Write a detailed note on Mathematical Preliminaries: Vectors. Answer outline: • Vector: magnitude + direction; scalar: magnitude only. • Addition: triangle/parallelogram law; components:  $\mathbf{a} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}$ . • Dot product:  $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos(\theta)$  — tests perpendicularity (zero) and finds projections. • Cross product:  $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin(\theta) \mathbf{n}$  — gives torques and normals;  $\mathbf{a} \times \mathbf{b} = -(\mathbf{b} \times \mathbf{a})$ . • Scalars: mass, temperature, speed.

### Example

Q: Write a detailed note on Kinematics of a Particle. Answer outline: • Position  $\mathbf{r}(t)$ ; velocity  $\mathbf{v} = d\mathbf{r}/dt$ ; acceleration  $\mathbf{a} = d\mathbf{v}/dt$  — differentiate down, integrate up. • Constant acceleration:  $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ ,  $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ ,  $v^2 = u^2 + 2\mathbf{a} \cdot \mathbf{s}$ . • Horizontal: constant velocity; vertical: constant acceleration  $-g$ . • Range  $R = (u^2 \sin 2\theta)/g$  — maximum at 45 degrees. • Time of flight  $T = (2u \sin \theta)/g$ ; max height  $H = (u^2 \sin^2 \theta)/(2g)$ .

### Example

Q: Write a detailed note on Newton's Laws and Forces. Answer outline: • First law: inertia — inertial frames. • Second law:  $\mathbf{F} = m\mathbf{a}$  (vector equation — resolve into components). • Third law: action-reaction pairs act on different bodies. • Weight  $\mathbf{W} = m\mathbf{g}$  downward; normal reaction perpendicular to surfaces. • Tension: pulls along strings; inextensible strings give equal accelerations.

### Example

Q: Write a detailed note on Work, Energy and Power. Answer outline: • Work:  $W = \mathbf{F} \cdot \mathbf{d}$  (dot product — only the parallel component counts). • Kinetic energy  $T = \frac{1}{2}mv^2$ ; work-energy theorem: net work = change in KE. • Potential energy: gravitational  $V = mgh$ ; elastic  $V = \frac{1}{2}kx^2$ . • Conservative forces (gravity, spring): path-independent work, energy conserved; non-conservative (friction): work lost as heat. • Gravity, spring: path-independent, potential exists.

### Example

Q: Write a detailed note on Linear and Angular Momentum. Answer outline: • Linear momentum  $\mathbf{p} = m\mathbf{v}$ ; impulse  $\mathbf{J} = \int \mathbf{F} dt = \text{change in } \mathbf{p}$ . • Conservation: with no external impulse, total  $\mathbf{p}$  is constant — the key to collision problems. • Elastic: momentum AND kinetic energy conserved. • Inelastic: momentum conserved, KE lost; perfectly inelastic: bodies stick together. • Coefficient of restitution  $e$ :  $1 = \text{elastic}$ ,  $0 = \text{perfectly inelastic}$ .

### Example

Q: Write a detailed note on Central Forces and Orbits. Answer outline: • Newton's gravitation:  $F = GmM/r^2$  toward the centre — a central force, so angular momentum is conserved. • Planar motion; effective potential combines centrifugal and gravitational terms. • 1. Orbits are ellipses with the Sun at one focus. • 2. Equal areas in equal times (follows from angular momentum conservation). • 3.  $T^2$  proportional to  $a^3$  (follows from the inverse-square law).

### Example

Q: Write a detailed note on Rigid Body Kinematics. Answer outline: • Rigid body: distances between points fixed; motion = translation of centre of mass + rotation about it. • Rotation: angular velocity  $\omega$ , angular acceleration  $\alpha$ ;  $v = \omega \times r$  for any point. •  $I = \int r^2 dm$  — rotational mass; depends on axis. • Standard results: rod about centre  $(1/12)ML^2$ ; disc about centre  $(1/2)MR^2$ ; hoop  $MR^2$ ; solid sphere  $(2/5)MR^2$ . • Parallel axis theorem:  $I = I_{cm} + Md^2$ .

### Example

Q: Write a detailed note on Rigid Body Dynamics. Answer outline: • Torque  $\tau = I \alpha$  (rotational  $F = ma$ ). • Rotational KE =  $(1/2) I \omega^2$ ; work-energy includes both translational and rotational terms. • Rolling without slipping:  $v_{cm} = \omega R$ ; static friction does no work. • Tall narrow bodies topple; wide ones slip. • Compare required  $\mu$  for each.

### Example

Q: Write a detailed note on Statics: Equilibrium of Bodies. Answer outline: • Equilibrium: resultant force = 0 AND resultant moment = 0 (about any point). • Two-force members, three-force members (concurrent) simplify analysis. • Trusses: method of joints (equilibrium at each pin) and method of sections (cut through members). • Ladders, beams and wedges combine force and moment balance with friction. • Zero-force members simplify fast.

### Example

Q: Write a detailed note on Simple Harmonic Motion. Answer outline: • SHM: acceleration proportional to displacement, toward equilibrium:  $d^2x/dt^2 = -\omega^2 x$ . • Solution:  $x = A \cos(\omega t + \phi)$ ; period  $T = 2\pi/\omega$ . • Mass-spring:  $\omega = \sqrt{k/m}$ ; simple pendulum:  $\omega = \sqrt{g/l}$  for small angles. • Energy oscillates between kinetic and potential; total  $E = (1/2)kA^2$ . • Damped SHM: amplitude decays; driven SHM: resonance when driven at natural frequency.

### Exam Pointer

Very important for exams: Practice writing answers within the exam time limit.

## MCQ Practice Test

### Learning Objectives

- Test your recall of the key points from every chapter 

Q1. Which of the following was listed as a key point of 'Mathematical Preliminaries: Vectors' in this handout?

- A. Centripetal force is a requirement, not a new force.
- B. Separate x and y in projectiles — they are independent.
- C. Draw free-body diagrams before writing any equation.
- D. Dot product -> angle/work; cross product -> torque/area.

Q2. Which of the following was listed as a key point of 'Mathematical Preliminaries: Vectors' in this handout?

- A. Separate x and y in projectiles — they are independent.
- B. Centripetal force is a requirement, not a new force.
- C. Draw free-body diagrams before writing any equation.
- D. Unit vectors  $i, j, k$  keep 3-D work organized.

Q3. Which of the following was listed as a key point of 'Kinematics of a Particle' in this handout?

- A. Unit vectors  $i, j, k$  keep 3-D work organized.
- B. Separate x and y in projectiles — they are independent.
- C. Draw free-body diagrams before writing any equation.
- D. Dot product -> angle/work; cross product -> torque/area.

Q4. Which of the following was listed as a key point of 'Kinematics of a Particle' in this handout?

- A. Draw free-body diagrams before writing any equation.
- B. Dot product -> angle/work; cross product -> torque/area.
- C. Unit vectors  $i, j, k$  keep 3-D work organized.
- D. Centripetal force is a requirement, not a new force.

Q5. Which of the following was listed as a key point of 'Newton's Laws and Forces' in this handout?

- A. Unit vectors  $i, j, k$  keep 3-D work organized.
- B. Dot product -> angle/work; cross product -> torque/area.
- C. Separate x and y in projectiles — they are independent.
- D. Draw free-body diagrams before writing any equation.

Q6. Which of the following was listed as a key point of 'Newton's Laws and Forces' in this handout?

- A. Dot product -> angle/work; cross product -> torque/area.
- B. Unit vectors  $i, j, k$  keep 3-D work organized.
- C. Separate  $x$  and  $y$  in projectiles — they are independent.
- D. Friction direction always opposes impending motion.

Q7. Which of the following was listed as a key point of 'Work, Energy and Power' in this handout? 

- A. Unit vectors  $i, j, k$  keep 3-D work organized.
- B. Separate  $x$  and  $y$  in projectiles — they are independent.
- C. Dot product -> angle/work; cross product -> torque/area.
- D. Energy methods beat force methods when time is not asked.

Q8. Which of the following was listed as a key point of 'Work, Energy and Power' in this handout?

- A. Separate  $x$  and  $y$  in projectiles — they are independent.
- B. Check conservativeness before conserving.
- C. Dot product -> angle/work; cross product -> torque/area.
- D. Unit vectors  $i, j, k$  keep 3-D work organized.

Q9. Which of the following was listed as a key point of 'Linear and Angular Momentum' in this handout?

- A. Unit vectors  $i, j, k$  keep 3-D work organized.
- B. Collisions: always conserve momentum; conserve KE only if elastic.
- C. Dot product -> angle/work; cross product -> torque/area.
- D. Separate  $x$  and  $y$  in projectiles — they are independent.

Q10. Which of the following was listed as a key point of 'Linear and Angular Momentum' in this handout?

- A. Separate  $x$  and  $y$  in projectiles — they are independent.
- B. Unit vectors  $i, j, k$  keep 3-D work organized.
- C. Dot product -> angle/work; cross product -> torque/area.
- D. Angular momentum conservation explains orbits and spins.

Q11. Which of the following was listed as a key point of 'Central Forces and Orbits' in this handout?

- A. Kepler 2 = angular momentum; Kepler 3 = inverse-square law.
- B. Separate  $x$  and  $y$  in projectiles — they are independent.
- C. Unit vectors  $i, j, k$  keep 3-D work organized.
- D. Dot product -> angle/work; cross product -> torque/area.

Q12. Which of the following was listed as a key point of 'Central Forces and Orbits' in this  handout?

- A. Escape velocity  $v = \sqrt{2GM/R}$ .
- B. Unit vectors  $i, j, k$  keep 3-D work organized.
- C. Dot product  $\rightarrow$  angle/work; cross product  $\rightarrow$  torque/area.
- D. Separate  $x$  and  $y$  in projectiles — they are independent.

Q13. Which of the following was listed as a key point of 'Rigid Body Kinematics' in this handout?

- A. Separate  $x$  and  $y$  in projectiles — they are independent.
- B. Dot product  $\rightarrow$  angle/work; cross product  $\rightarrow$  torque/area.
- C. Unit vectors  $i, j, k$  keep 3-D work organized.
- D. Moment of inertia depends on mass distribution AND axis choice.

Q14. Which of the following was listed as a key point of 'Rigid Body Kinematics' in this handout?

- A. Dot product  $\rightarrow$  angle/work; cross product  $\rightarrow$  torque/area.
- B. Parallel axis theorem saves many integrals.
- C. Separate  $x$  and  $y$  in projectiles — they are independent.
- D. Unit vectors  $i, j, k$  keep 3-D work organized.

Q15. Which of the following was listed as a key point of 'Rigid Body Dynamics' in this handout?

- A. Unit vectors  $i, j, k$  keep 3-D work organized.
- B. Separate  $x$  and  $y$  in projectiles — they are independent.
- C. Rolling = translation + rotation combined.
- D. Dot product  $\rightarrow$  angle/work; cross product  $\rightarrow$  torque/area.

Q16. Which of the following was listed as a key point of 'Rigid Body Dynamics' in this handout?

- A. Separate  $x$  and  $y$  in projectiles — they are independent.
- B. Energy methods handle rolling elegantly.
- C. Dot product  $\rightarrow$  angle/work; cross product  $\rightarrow$  torque/area.
- D. Unit vectors  $i, j, k$  keep 3-D work organized.

Q17. Which of the following was listed as a key point of 'Statics: Equilibrium of Bodies' in this handout?

- A. Separate  $x$  and  $y$  in projectiles — they are independent.
- B. Dot product  $\rightarrow$  angle/work; cross product  $\rightarrow$  torque/area.
- C. Unit vectors  $i, j, k$  keep 3-D work organized.
- D. Always use both force and moment equations.

Q18. Which of the following was listed as a key point of 'Statics: Equilibrium of Bodies' in this  handout?

- A. Dot product -> angle/work; cross product -> torque/area.
- B. Unit vectors  $i, j, k$  keep 3-D work organized.
- C. Choose the moment centre to eliminate unknowns.
- D. Separate  $x$  and  $y$  in projectiles — they are independent.

Q19. Which of the following was listed as a key point of 'Simple Harmonic Motion' in this handout?

- A. Dot product -> angle/work; cross product -> torque/area.
- B. Separate  $x$  and  $y$  in projectiles — they are independent.
- C. Small-angle approximation makes the pendulum SHM.
- D. Unit vectors  $i, j, k$  keep 3-D work organized.

Q20. Which of the following was listed as a key point of 'Simple Harmonic Motion' in this handout?

- A. Separate  $x$  and  $y$  in projectiles — they are independent.
- B. Unit vectors  $i, j, k$  keep 3-D work organized.
- C. Dot product -> angle/work; cross product -> torque/area.
- D. Resonance: small pushes at the right frequency build huge amplitude.

### Answer Key

1. D 2. D 3. B 4. D 5. D 6. D 7. D 8. B 9. B 10. D 11. A 12. A 13. D 14. B 15. C 16. B 17. D 18. C 19. C 20. D

### Exam Pointer

Very important for exams: Attempt all MCQs without looking back, then check the answer key.

## Exam Strategy Guide

### Learning Objectives

- Attempt papers with a winning strategy

### Before the exam

- Revise the Key Points boxes and glossary first — they carry the highest marks per minute.
- Practice one past paper under timed conditions; examiners repeat question styles.

- Sleep well the night before — an exhausted brain recalls far less.

### Attempting the paper

- Read the whole paper first; start with your strongest question to build momentum.
- Allocate time by marks: a 10-mark question deserves roughly double a 5-mark one.
- Structure long answers: definition, explanation, example/diagram, conclusion.

### Presentation matters

- Use headings, underline key terms, and draw labelled diagrams where possible.
- Attempt all questions — partial credit beats a blank page.
- Review your paper in the last 10 minutes for missed parts.

#### Exam Pointer

Very important for exams: Very important for exams: neat, structured answers consistently score higher than longer messy ones.