

MTH501

Linear Algebra

Formula Sheet

32 Chapters, 810 Formulas

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5

$$5 = 5 - 0 = 5$$

$$= - (5 - 0) = -5$$

x_1

$$x_1 = -6x_2 - 3x_4$$

x_2 is free

$$x_3 = 5 + 4x_4$$

$$15. x_1 + 6x_2$$

$$15. x_1 + 6x_2 = 1$$

$$16. x_1 - 3x_2$$

$$16. x_1 - 3x_2 = 1$$

$$2x_1 + 3x_2$$

$$2x_1 + 3x_2 = k$$

$$2x_1 + 6x_2$$

$$2x_1 + 6x_2 = k$$

$$u + v$$

$$u + v = v + u \text{ (Commutative)}$$

(ii)

$$(u + v) + w = u + (v + w) \text{ (Associative)}$$

(iii)

$$u + 0 = 0 + u = u \text{ (Additive Identity)}$$

(iv)

$$u + (-u) = (-u) + u = 0 \text{ (Additive Inverse)}$$

$$(3, 5, 2)$$

$$(3, 5, 2) = 3 \cdot 1v + 5$$

$$x_1 a_1 + x_2 a_2$$

$$x_1 a_1 + x_2 a_2 = b$$

because cv_1

because $cv_1 = cv_1 + 0v_2 + \dots + 0v_p$ i.e every cv_i can be written as a linear

$$(x, y, z)$$

$$(x, y, z) = (1, 2, -3) + t(4, -5, 1)$$

the sum

$$x - x_0 = t_1 v_1 + t_2 v_2$$

or, equivalently,

$$x = x_0 + t_1 v_1 + t_2 v_2 \text{ ----- (1)}$$

x

$$x = t_1 v_1 + t_2 v_2$$

component form of eq. 1 will be

$$(x, y, z) = (x_0, y_0, z_0) + t_1(a_1, b_1, c_1) + t_2(a_2, b_2, c_2)$$

x

$$x = x_0 + a_1 t_1 + a_2 t_2$$

y

$$y = y_0 + b_1 t_1 + b_2 t_2$$

z

$$z = z_0 + c_1 t_1 + c_2 t_2$$

$$(x, y, z)$$

$$(x, y, z) = t_1(1, -2, 3) + t_2(4, 0, 5)$$

plane is

$$x = x_0 + t_1(x_1 - x_0) + t_2(x_2 - x_0)$$

x

$$x = 5 + y - 2z$$

$$x_1 a_1 + x_2 a_2 + \dots + x_n a_n$$

$$x_1 a_1 + x_2 a_2 + \dots + x_n a_n = b$$

$$= aA + bB + cC$$

$$4a + b$$

$$4a + b = 6$$

$$-b + 2c$$

$$-b + 2c = -8$$

$$-2a + 2b + c$$

$$-2a + 2b + c = -1$$

$$-2a + 3b + 4c$$

$$-2a + 3b + 4c = -8$$

$$-b - 3c$$

$$-b - 3c = 7$$

$$5c$$

$$5c = -15 \quad c = -3$$

From (2),

$$-b + 2(-3) = -8 \quad b = 2$$

$$4(1) + 2$$

$$4(1) + 2 = 6$$

$$= 1A + 2B - 3C$$

$$4a + b$$

$$4a + b = 0$$

$$-b + 2c$$

$$-b + 2c = 0$$

$$-2a + 2b + c$$

$$-2a + 2b + c = 0$$

$$-2a + 3b + 4c$$

$$-2a + 3b + 4c = 0$$

Subtracting equation (3) from equation (4) we get

$$b + 3c = 0 \quad (5)$$

$$= 0A + 0B + 0C$$

$$-2a + 2b + c$$

$$-2a + 2b + c = 3$$

$$-2a + 3b + 4c$$

$$-2a + 3b + 4c = 8$$

$$-b - 3c$$

$$-b - 3c = -5$$

From (2),

$$-b + 2(1) = 0 \quad b = 2$$

$$= 1A + 2B + 1C$$

. Verify $A(u + v)$

$$. \text{Verify } A(u + v) = Au + Av.$$

The equation

$$x = su + tv$$

x

$$x = p + tv \quad (t \text{ in } R)$$

$$1. x_1 - 5x_2 + 9x_3$$

$$1. x_1 - 5x_2 + 9x_3 = 0$$

$$2. 3x_1 + 6x_2 - 4x_3 - x_4$$

$$2. 3x_1 + 6x_2 - 4x_3 - x_4 = 0$$

$$-x_1 + 4x_2 - 3x_3$$

$$-x_1 + 4x_2 - 3x_3 = 0$$

$$-5x_1 + 8x_3 + 3x_4$$

$$-5x_1 + 8x_3 + 3x_4 = 0$$

$$2x_1 - 8x_2 + 9x_3$$

$$2x_1 - 8x_2 + 9x_3 = 0$$

$$8x_1 - x_2 + 7x_4$$

$$8x_1 - x_2 + 7x_4 = 0$$

$$3. 5x_1 - x_2 + 3x_3$$

$$3. 5x_1 - x_2 + 3x_3 = 0$$

$$4x_1 - 3x_2 + 7x_3$$

$$4x_1 - 3x_2 + 7x_3 = 0$$

$$5. x_1 + 2x_2 - 7x_3$$

$$5. x_1 + 2x_2 - 7x_3 = 0$$

$$-2x_1 - 3x_2 + 9x_3$$

$$-2x_1 - 3x_2 + 9x_3 = 0$$

$$-2x_1 + 3x_2 + 7x_3$$

$$-2x_1 + 3x_2 + 7x_3 = 0$$

$$-2x_2 + 10x_3$$

$$-2x_2 + 10x_3 = 0$$

$$\text{of } x_1 - 4x_2 + 3x_3$$

$$\text{of } x_1 - 4x_2 + 3x_3 = 7.$$

$$10v_1 - 5v_2 + 5v_3$$

$$10v_1 - 5v_2 + 5v_3 = 0$$

$$c_1v_1 + c_2v_2 + \dots + c_pv_p$$

$$c_1v_1 + c_2v_2 + \dots + c_pv_p = 0$$

$$2. T(cu + dv)$$

$$2. T(cu + dv) = cT(u) + dT(v)$$

$$3. T(c_1v_1 + \dots + c_pv_p)$$

$$3. T(c_1v_1 + \dots + c_pv_p) = c_1T(v_1) + \dots + c_pT(v_p)$$

$$2. \text{ Just apply the definition of linear transformation. i. e}$$

$$T(cu + dv) = T(cu) + T(dv) = cT(u) + dT(v)$$

$$T(c_1v_1 + \dots + c_pv_p)$$

$$T(c_1v_1 + \dots + c_pv_p) = c_1T(v_1) + \dots + c_pT(v_p)$$

$$T(x)$$

$$T(x) = x+1.$$

$$T(cu + dv)$$

$$T(cu + dv) = 3(cu + dv)$$

$$= 3cu + 3dv$$

$$= c(3u) + d(3v)$$

$$= cT(u) + dT(v)$$

$$L(0, 1, 0) = (3, 1), \text{ and}$$

$$L(0, 0, 1) = (-1, 2).$$

Find x such that $T(x)$

$$\text{Find } x \text{ such that } T(x) = (-2, -5).$$

. Find x such that $T(x)$

$$\text{. Find } x \text{ such that } T(x) = (-4, 7, 0).$$

$$A + B$$

$$A + B = B + A$$

$$r(A + B)$$

$$r(A + B) = rA + rB$$

$$(A + B) + C$$

$$(A + B) + C = A + (B + C)$$

$$(r + s)A$$

$$(r + s)A = rA + sA$$

$$= -1 = (1)$$

$$A(Bx)$$

$$A(Bx) = (AB)$$

$$x \text{-----} (1)$$

$$(2 \times 4) + (6 \times 3) + (0 \times 5)$$

$$(2 \times 4) + (6 \times 3) + (0 \times 5) = 26$$

$$(1 \times 3) + (2 \times 1) + (4 \times 2)$$

$$(1 \times 3) + (2 \times 1) + (4 \times 2) = 13$$

$$3) + (6 \times 1) + (0 \times 2)$$

$$3) + (6 \times 1) + (0 \times 2) = 12$$

$$A(B + C)$$

$$A(B + C) = AB + AC$$

$$(B + C)A$$

$$(B + C)A = BA + CA$$

Thus C

$$\text{Thus } C = A^{-1}.$$

(a) A^n is invertible and $(A^n)^{-1}$

$$(a) A^n \text{ is invertible and } (A^n)^{-1} = A^{-n} = (A^{-1})^n$$

B

$$B = A^{-1} \text{ and } A = B^{-1}$$

$\det(A)$

$$\det(A) = 1(20 + 2) - 6(10 - 1) + 4(4 + 4)$$

$$= 22 - 54 + 32$$

$T(x, y)$

$$T(x, y) = xyt + (x + y)t^2$$

be inverse of A .i.e. A^{-1}

be inverse of A .i.e. $A^{-1} = B$, then

A

$$A = (E_p \dots E_1)^{-1}U = LU$$

L

$$L = (E_p \dots E_1)^{-1}$$

because $E_p \dots E_1 L$

because $E_p \dots E_1 L = (E_p \dots E_1)(E_p \dots E_1)^{-1} = I$. This observation is the key to

(4^*)

And rewrite (3^*) as $Ly = b$

$$10x_1 + x_2 - x_3$$

$$10x_1 + x_2 - x_3 = 18$$

$$x_1 + 15x_2 + x_3$$

$$x_1 + 15x_2 + x_3 = -12$$

$$-x_1 + x_2 + 20x_3$$

$$-x_1 + x_2 + 20x_3 = 17$$

Comparing the corresponding entries on both sides, we have

$$10y_1 = -x_2 + x_3 + 18$$

$$15y_2$$

$$15y_2 = -x_1 - x_3 - 12$$

$$20y_3$$

$$20y_3 = x_1 - x_2 + 17$$

$$y_1$$

$$y_1 = (-x_2 + x_3 + 18)/10$$

$$y_2$$

$$y_2 = (-x_1 - x_3 - 12)/15$$

$$y_3$$

$$y_3 = (x_1 - x_2 + 17)/20$$

Thus, $x(2)$

$$\text{Thus, } x(2) = (1.965, -0.9767, 0.98).$$

$$x_3 = (17+0-0)/20 = 0.85$$

Which gives $x(1) = (1.80, -0.80, 0.85)$ ----put this again on RHS of the equivalent system

$$10x + x$$

$$10x + x = 13$$

$$x + x + 10x$$

$$x + x + 10x = 18$$

$x(0) = (0, 0, 0)$ and six iterations.

$$10x_1 + x_2 - x_3 = 18$$

$$x_1 + 15x_2 + x_3$$

$$x_1 + 15x_2 + x_3 = -12$$

$$-x_1 + x_2 + 20x_3$$

$$-x_1 + x_2 + 20x_3 = 17$$

$$10y_1$$

$$10y_1 = -x_2 + x_3 + 18$$

$$y_1 + 15y_2$$

$$y_1 + 15y_2 = -x_3 - 12$$

$$-y_1 + y_2 + 20y_3$$

$$-y_1 + y_2 + 20y_3 = 17$$

regard the highlighted x 's as the values:

$$x_1 = (-x_2 + x_3 + 18)/10$$

$$x_2$$

$$x_2 = (-x_1 - x_3 - 12)/15$$

$$x_3$$

$$x_3 = (x_1 - x_2 + 17)/20$$

$$x_1 - 3x_2 + x_3$$

$$x_1 - 3x_2 + x_3 = -2$$

$$-6x_1 + 4x_2 + 11x_3$$

$$-6x_1 + 4x_2 + 11x_3 = 1$$

$$5x_1 - 2x_2 - 2x_3 = 9$$

$$x_1 - 3x_2 + x_3 = -2$$

$$\det(A)$$

$$\det(A) = ad-bc.$$

$$= 45 + 84 + 96 - 105 + 48 + 72$$

$$= 1 [4(0) - (-1)(-2)] - 5 [2(0) - 0(-1)] + 0 [2(-2) - 4(0)]$$

$$= ab. (1)^2 = ab$$

$$x = -8$$

$$x + 2x = -3$$

$$2x + x + x$$

$$2x + x + x = 4$$

$$2x = 2$$

$$3x + x + 3x = -2$$

$$6sx + 4x$$

$$6sx + 4x = 5$$

$$9x + 2sx$$

$$9x + 2sx = -2$$

$$3sx - 5x$$

$$3sx - 5x = 3$$

$$9x + 5sx$$

$$9x + 5sx = 2$$

$$sx - 2sx$$

$$sx - 2sx = -1$$

$$3x + 6sx$$

$$3x + 6sx = 4$$

$$x = 1$$

$$3sx + 6sx = 2$$

and b_2

$$\text{and } b_2 = -2$$

$$0 + u$$

$$0 + u = u + 0 = u$$

$$-u + u$$

$$-u + u = 0 = u + (-u)$$

$$= (u_1 + v_1, u_2 + v_2, \dots, u_n + v_n) \mathbb{R}^n$$

$$= (u_1 + v_1, u_2 + v_2, \dots, u_n + v_n)$$

$$= (v_1 + u_1, v_2 + u_2, \dots, v_n + u_n) \text{ (By commutative law of real numbers)}$$

$$= (v_1, v_2, \dots, v_n) + (u_1, u_2, \dots, u_n)$$

$$= (u_1 + v_1, u_2 + v_2, \dots, u_n + v_n) + (w_1, w_2, \dots, w_n)$$

$$= ((u_1 + v_1) + w_1, (u_2 + v_2) + w_2, \dots, (u_n + v_n) + w_n)$$

$$\begin{aligned}
 &= (u_1, u_2, \dots, u_n) + (v_1 + w_1, v_2 + w_2, \dots, v_n + w_n) \\
 &= (u_1, u_2, \dots, u_n) + [(v_1, v_2, \dots, v_n) + (w_1, w_2, \dots, w_n)] \\
 &= u + (v + w)
 \end{aligned}$$

$0 + u$

$$\begin{aligned}
 0 + u &= (0, 0, \dots, 0) + (u_1, u_2, \dots, u_n) \\
 &= (0 + u_1, 0 + u_2, \dots, 0 + u_n)
 \end{aligned}$$

Similarly, $u + 0$

$$\text{Similarly, } u + 0 = u$$

Such that $u + (-u)$

$$\begin{aligned}
 \text{Such that } u + (-u) &= (u_1, u_2, \dots, u_n) + (-u_1, -u_2, \dots, -u_n) \\
 &= (u_1 + (-u_1), u_2 + (-u_2), \dots, u_n + (-u_n))
 \end{aligned}$$

Similarly, $(-u) + u$

$$\text{Similarly, } (-u) + u = 0$$

$$\begin{aligned}
 &= k(u_1 + v_1, u_2 + v_2, \dots, u_n + v_n) \\
 &= (k(u_1 + v_1), k(u_2 + v_2), \dots, k(u_n + v_n)) \\
 &= (k u_1 + k v_1, k u_2 + k v_2, \dots, k u_n + k v_n) \\
 &= (k u_1, k u_2, \dots, k u_n) + (k v_1, k v_2, \dots, k v_n) \\
 &= k(u_1, u_2, \dots, u_n) + k(v_1, v_2, \dots, v_n) \\
 &= k u + k v \\
 &= ((k + 1)u_1, (k + 1)u_2, \dots, (k + 1)u_n) \\
 &= (k u_1 + 1 u_1, k u_2 + 1 u_2, \dots, k u_n + 1 u_n) \\
 &= (k u_1, k u_2, \dots, k u_n) + (1 u_1, 1 u_2, \dots, 1 u_n) \\
 &= k(u_1, u_2, \dots, u_n) + 1(u_1, u_2, \dots, u_n) \\
 &= k u + 1 u
 \end{aligned}$$

$u + (v + w)$

$$= u + (v + w)$$

and similarly $u + 0$

and similarly $u + 0 = u$.

$(k+1)u$

$$(k+1) = (k+1)$$

$(f+g)(x)$

$$(f+g)(x) = f(x) + g(x), \text{ for all } x \in R.$$

$(f+g)(x)$

$$(f+g)(x) = f(x) + g(x)$$

$$= g(x) + f(x)$$

$$= (g+f)(x)$$

So that

$$f+g = g+f$$

$((f+g)+h)(x)$

$$((f+g)+h)(x) = (f+g)(x) + h(x)$$

$$= (f(x) + g(x)) + h(x)$$

$$= f(x) + (g(x) + h(x))$$

$$= f(x) + (g+h)(x)$$

$$= (f+(g+h))(x)$$

And so

$$(f+g)+h = f+(g+h)$$

$0(x)$

$$0(x) = 0, \text{ for all } x \in R \text{ because } (0+f)(x) = 0(x) + f(x)$$

$$= 0 + f(x) = f(x)$$

i.e. $0+f$

i.e. $0+f = f$. Similarly, $f+0 = f$.

$(f+(-f))(x)$

$$(f+(-f))(x) = f(x) + (-f)(x)$$

i.e. $f + (-f)$

i.e. $f + (-f) = 0$. Similarly, $(-f) + f = 0$.

Thus $(a + b) f$

Thus $(a + b) f = a f + b f$

$) + ($

$) = (a + a$

$= (a + b) + (a + b) + (a + b)$

$)$

$) = k(a + a$

$a x + b y + c z$

$a x + b y + c z = 0$

Adding these equations gives $a (u_1 + v_1) + b (u_2 +$

Adding these equations gives $a (u_1 + v_1) + b (u_2 + v_2) + c (u_3 + v_3) = 0$

$u + v$

$u + v = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$

$a (ku_1) + b (ku_2) + c (ku_3)$

$a (ku_1) + b (ku_2) + c (ku_3) = 0$

gives $a (-1u_1) + b (-1u_2) + c (-1u_3)$

gives $a (-1u_1) + b (-1u_2) + c (-1u_3) = 0$

$+$

$+ = (u + v, u + v)$

(iii) $(-1) u$

(iii) $(-1) u = -u$

$(-1) u$

$(-1) u = -u$ is in W .

$+$

$+ = (a, b, 0) + (a, b, 0) = (a + a, b + b, 0)$

+

$$+ = (a, b, 1) + (a, b, 1) = (a+a, b+b, 1+1)$$

$$\begin{aligned} \text{Then } u + v &= (u_1, u_1 + u_3, u_3) + (v_1, v_1 + v_3, v_3) \\ &= (u_1 + v_1, u_1 + u_3 + v_1 + v_3, u_3 + v_3) \end{aligned}$$

If k is any scalar and u

If k is any scalar and $u = (u_1, u_1 + u_3, u_3)$ is any vector in W , then

(By definition)

$$= (k u_1, k u_1 + k u_3, k u_3)$$

$$\begin{aligned} \text{Then } u + v &= (u_1, u_1 + u_3 + 1, u_3) + (v_1, v_1 + v_3 + 1, v_3) \\ &= (u_1 + v_1, u_1 + u_3 + 1 + v_1 + v_3 + 1, u_3 + v_3) \\ &= (u_1 + v_1, (u_1 + v_1) + (u_3 + v_3) + 2, u_3 + v_3) \end{aligned}$$

(i) All polynomials $a_0 + a_1 x + a_2 x^2 + a_3 x^3$ for

(i) All polynomials $a_0 + a_1 x + a_2 x^2 + a_3 x^3$ for which $a_0 = 0$

$$a_0 + a_1 + a_2 + a_3$$

$$a_0 + a_1 + a_2 + a_3 = 0.$$

$$= (c_0 + b_0) + (c_1 + b_1)x + (c_2 + b_2)x^2 + (c_3 + b_3)x^3$$

+ 0

$$+ 0 = 0. \text{ Therefore, } u + v \in W$$

where $a + b + c + d$

$$\text{where } a + b + c + d = 0$$

where $a + b + c + d$

$$\text{where } a + b + c + d = 0.$$

h

$$h = 0 + 0 = 0$$

$u + w$

$$u + w = (s_1 v_1 + s_2 v_2) + (t_1 v_1 + t_2 v_2)$$

$$= (s_1 + t_1) v_1 + (s_2 + t_2) v_2$$

$$= (s_1v_1 + s_2v_2 + \dots + s_nv_n) + (t_1v_1 + t_2v_2 + \dots + t_nv_n)$$

$$(x, y, z) + (x, y, z)$$

$$(x, y, z) + (x, y, z) = (x + x, y + y, z + z)$$

$$(x, y) + (x, y)$$

$$(x, y) + (x, y) = (x + x, y + y)$$

$$(x, y) + (x, y)$$

$$(x, y) + (x, y) = (x + x + 1, y + y + 1)$$

matrix multiplication, we find that

$$A(u + v) = Au + Av = 0 + 0 = 0$$

equations

$$a - 2b + 5c = d$$

$$a - 2b + 5c - d$$

$$a - 2b + 5c - d = 0$$

$$-a - b + c$$

$$-a - b + c = 0$$

$$x - 2x$$

$$-x + 3x = 0$$

$$x + 2x - 2x$$

$$x + 2x - 2x = 0$$

$$x$$

$$x = 2x + x - 3x$$

x = free variable

$$x = -2x + 2x$$

$$x - 3x + 2x$$

$$+2x + x = 0$$

$$0x + 3x + 6x + 0x - 3x$$

$$0x + 3x + 6x + 0x - 3x = 0$$

$$2x - 3x - 2x$$

$$+4x + 4x = 0$$

$$3x - 6x + 0x + 6x + 5x$$

$$3x - 6x + 0x + 6x + 5x = 0$$

$$-2x + 9x + 2x - 4x - 5x$$

$$-2x + 9x + 2x - 4x - 5x = 0$$

$$1x +$$

$$2x + (4/3)x = 0$$

$$1x +$$

$$(-1/6)x = 0$$

$$1x +$$

$$(-5/12)x = 0$$

$$x$$

$$x = -2x + (-4/3)x$$

$$x$$

$$x = +(1/6)x$$

$$x$$

$$x = +(5/12)x$$

The solution can be written in the vector form:

$$= (-2, 0, 0, 1, 0)$$

$$= (-4/3, 1/6, 5/12, 0, 1)$$

$$2 \ 1 - 2 + 3 - 3$$

$$2 \ 1 - 2 + 3 - 3 = -9$$

$$2x + 6x - 5x - 2x + 4x - 3x$$

$$2x + 6x - 5x - 2x + 4x - 3x = -1$$

$$2x + 6x$$

$$+ 8x + 4x + 18x = 6$$

$$1x$$

$$1x = (1/3)$$

$$x$$

$$x = -3x - 4x - 2x$$

x

$$x = -3r - 4s - 2t$$

$$(i) T(u + v)$$

$$(i) T(u + v) = T(u) + T(v)$$

$$b : a - 3b - c$$

$$b : a - 3b - c = 0$$

$$= [1 \ -3 \ -1].$$

$$a - 2b$$

$$a - 2b = 4c$$

. For instance, if $p(t)$

. For instance, if $p(t) = 3 + 5t + 7t^2$, then

$$T(A)$$

$$T(A) = A + AT, \text{ where}$$

$$u + v$$

$$u + v = v + u$$

$$u + (v + w)$$

$$u + (v + w) = (u + v) + w$$

$$0 + u$$

$$0 + u = u + 0 = u$$

There exist a vector $-u$ in V such that

$$-u + u = 0 = u + (-u)$$

$$l(u + v)$$

$$l(u + v) = lu + lv$$

$$(l + m)u$$

$$(l + m)u = lu + mu$$

Example 3 Let A be an invertible $n \times n$

matrix - say, $A = [a_1 \dots a_n]$. Then the columns

$c_0, \dots, c_n$ satisfy

$$c_0 \cdot 1 + c_1 t + c_2 t^2 + \dots + c_n t^n = 0(t)$$

$$c_1 v_1 + c_2 v_2 + c_3 v_3$$

$$c_1 v_1 + c_2 v_2 + c_3 v_3 = d_1 w_1 + d_2 w_2 + d_3 w_3$$

$$c_1(2, -3, 1) + c_2(4, 1, 1) + c_3(0, -7, 1)$$

$$c_1(2, -3, 1) + c_2(4, 1, 1) + c_3(0, -7, 1) = d_1(1, 0, 0) + d_2(0, 1, 0) + d_3(0, 0, 1)$$

$$2c + 4c + 0c$$

$$2c + 4c + 0c = 1d + 0d + 0d$$

$$-3c + 1c - 7c$$

$$-3c + 1c - 7c = 0d + 1d + 0d$$

$$1c + 1c + 1c$$

$$1c + 1c + 1c = 0d + 0d + 1d$$

$$-4c + 6c + 8c$$

$$-4c + 6c + 8c = 1d + 0d + 0d$$

$$1c + 5c + 4c$$

$$1c + 5c + 4c = 0d + 1d + 0d$$

$$3c + 2c + 1c$$

$$3c + 2c + 1c = 0d + 0d + 1d$$

solution of

$$c_1 p_1(t) + c_2 p_2(t) + c_3 p_3(t) = 0$$

$$-4c + 6c + 8c$$

$$-4c + 6c + 8c = 0$$

$$1c + 5c + 4c$$

$$1c + 5c + 4c = 0$$

$$3c + 2c + 1c$$

$$3c + 2c + 1c = 0$$

$$3c + 0c + 0c + 1c$$

$$3c + 0c + 0c + 1c = 1d + 0d + 0d + 0d$$

$$6c -$$

$$1c - 8c + 0c = 0d + 1d + 0d + 0d$$

$$3c -$$

$$1c - 12c - 1c = 0d + 0d + 1d + 0d$$

$$-6c + 0c - 4c + 2c$$

$$-6c + 0c - 4c + 2c = 0d + 0d + 0d + 1d$$

$$\det(A)$$

$$\det(A) = 48 \neq 0$$

Every vector in $\text{Span}\{v_1, v_2\}$ belongs to H because

$$c_1 v_1 + c_2 v_2 = c_1 v_1 + c_2 v_2 + 0 v_3$$

substitute

$$x = c_1 v_1 + c_2 v_2 + c_3 (5v_1 + 3v_2)$$

$$= (c_1 + 5c_3) v_1 + (c_2 + 3c_3) v_2$$

$$c_1 v_1 + c_2 v_2 + \dots + c_n v_n$$

$$c_1 v_1 + c_2 v_2 + \dots + c_n v_n = 0$$

Solution Step 1 Form Equation (3),

$$c_1 (1, 2, -2, 1) + c_2 (-3, 0, -4, 3) + c_3 (2, 1, 1, -1) + c_4 (-3, 3, -9, 6) + c_5 (9, 3, 7, -6) = (0, 0, 0, 0).$$

$$c - 3c + 2c - 3c + 9c$$

$$c - 3c + 2c - 3c + 9c = 0$$

$$+ c + 3c + 3c$$

$$+ c + 3c + 3c = 0$$

$$-2c - 4c + c - 9c + 7c$$

$$-2c - 4c + c - 9c + 7c = 0$$

$$c + 3c -$$

$$c + 6c - 6c = 0$$

$$x = 0$$

$$-2x - x + 2x = 0$$

$$x + y + z$$

$$x + y + z = 0$$

$$3x+2y - 2z$$

$$3x+2y - 2z = 0$$

$$4x+3y - z$$

$$4x+3y - z = 0$$

$$6x+5y+ z$$

$$6x+5y+ z = 0$$

14. Determine bases for the following subspace of R^3

(a) the plane $3x - 2y + 5z = 0$

(b) the plane $x - y = 0$

(c) the line $x = 2t, y = -t, z = 4t$

$[v]_s$

$$[v]_s = (-1, 3, 2)$$

$$c + 2c + 3c$$

$$c + 2c + 3c = 5$$

$$2c + 9c + 3c$$

$$2c + 9c + 3c = -1$$

$$4c + 2c + 3c$$

$$4c + 2c + 3c = 5$$

$$+ 9c + 3c$$

$$+ 9c + 3c = -1$$

to get c

$$c + 4(2) = 9$$

Therefore,

$$[v]_s = (1, -1, 2)$$

A

$$A = c_1 A_1 + c_2 A_2 + c_3 A_3 + c_4 A_4$$

Therefore,

$$[v]_s = (-1, 1, -1, 3)$$

yourself)

Thus $x = 3b_1 + 2b_2$ and

$$3 + 2t$$

$$3 + 2t = 2(4 + t + 5t^2) - 5(1 + 2t^2)$$

vector of $p(t)$

vector of $p(t) = 6 + 3t - t^2$ relative to B .

Put value of c in equation

$$1 + c = -1$$

we have

$$c + c = 6$$

$p(t)$

$p(t) = 1 + 4t + 7t^2$ relative to B .

vectors as

$$x = x_1e_1 + x_2e_2 + x_3e_3 + \dots + x_n e_n.$$

$$: 2a - b + 3c + d$$

$$: 2a - b + 3c + d = 0\}.$$

vectors as

$$x = x_1e_1 + x_2e_2 + x_3e_3 + \dots + x_n e_n.$$

$$: 2a - b + 3c + d$$

$$: 2a - b + 3c + d = 0\}.$$

$$= aA_1 + bA_2 + cA_3$$

$$aA + bA + cA$$

$$aA + bA + cA = 0$$

$$- x -$$

$$x + 2x - 3x + x = 0$$

$$- x = 0$$

$$x + x + x = 0$$

$$x$$

$$x = -1x - 1x$$

$$\begin{aligned}
 &6 \{(a, b, c, d): a - 3b + c \\
 &6 \{(a, b, c, d): a - 3b + c = 0\} \\
 &= (-2, -5, 8, 0, -17) \\
 &= (1, 3, -5, 1, 5) \\
 &= (3, 11, -19, 7, 1) \\
 &= (1, 7, -13, 5, -3) \\
 &= (2, 1, 0) \\
 &= (3, -1, 4) \\
 &= (1, -2, 0, 0, 3) \\
 &= (2, -5, -3, -2, 6)
 \end{aligned}$$

$$\begin{aligned}
 &x + x + x \\
 &x + x + x = 0
 \end{aligned}$$

$$\begin{aligned}
 &x - 2x + 3x \\
 &x - 2x + 3x = 0
 \end{aligned}$$

$$\begin{aligned}
 &x - 5x \\
 &x - 5x = 0
 \end{aligned}$$

$$\begin{aligned}
 &\text{rank } A + \dim \text{Nul } A \\
 &\text{rank } A + \dim \text{Nul } A = n
 \end{aligned}$$

$$\begin{aligned}
 &x - 4x - 28x - 37x + 13x \\
 &x - 4x - 28x - 37x + 13x = 0
 \end{aligned}$$

$$\begin{aligned}
 &x - 2x - 12x - 16x + 5x \\
 &x - 2x - 12x - 16x + 5x = 0
 \end{aligned}$$

$$\begin{aligned}
 &x \\
 &x = 4x - 28x + 37x - 13x
 \end{aligned}$$

$$\begin{aligned}
 &x \\
 &x = 2x + 12x + 16x - 5x
 \end{aligned}$$

$$\begin{aligned}
 &x \\
 &x = 4r + 28s + 37t - 13u
 \end{aligned}$$

x

$$x = 2r + 12s + 16t - 5u$$

has 6 columns,

$$\text{so rank}(A) + \text{nullity}(A) = 2 + 4 = 6 = n$$

The system has infinitely many solutions:

$$x_1 = -2x_4 + (-4/3)x_5$$

x2

$$x_2 = (1/6)x_5$$

x3

$$x_3 = (5/12)x_5$$

The solution can be written in the vector form:

$$c_4 = (-2, 0, 0, 1, 0)$$

c5

$$c_5 = (-4/3, 1/6, 5/12, 0, 1)$$

$$\{(-2, 0, 0, 1, 0), (-4/3, 1/6, 5/12, 0, 1)\}$$

The nullity of the matrix is 2. Now $\text{Rank}(A) + \text{nullity}(A) = 3 + 2 = 5 = n$

Solution

$$\text{nullity}(A) = n - \text{rank}(A) = 7 - 3 = 4$$

Solution

$$\text{nullity}(A) = n - \text{rank}(A) = 4 - 0 = 4$$

Suppose that x

$$\text{Suppose that } x = 3b_1 + b_2$$

(a) Find the change-of-coordinates matrix from F to D.

(b) Find $[x]_D$ for $x = f_1 - 2f_2 + 2f_3$.

(a) Find the change-of-coordinates matrix from B to C.

(b) Find $[x]_C$ for $x = 5b_1 + 3b_2$.

feed two different signals into the filter

$$0.35y_k + 2 + 0.5y_k + 1 + 0.35y_k = z_k$$

± 2 so that

$$\{y_k\} = \{\dots, 1, 0.7, 0, -0.7, -1, -0.7, 0, 0.7, 1, 0.7, 0, \dots\}$$

y_{k+2}

$$0.35y_k + 0.5y_{k+1} + 0.35y_{k+2} = z_k$$

$$0.35(1) + 0.5(0.7) + 0.35(0)$$

$$0.35(1) + 0.5(0.7) + 0.35(0) = 0.7$$

$\{w_k\}$

$$\{w_k\} = \{\dots, 1, -0.7, 0, 0.7, -1, 0.7, 0, -0.7, 1, -0.7, 0, \dots\}$$

$y_k = rk$ for some r . Find some solutions of the equation

$$y_{k+3} - 2y_{k+2} - 5y_{k+1} + 6y_k = 0 \text{ for all } k$$

$$r_{k+3} - 2r_{k+2} - 5r_{k+1} + 6r_k$$

$$r_{k+3} - 2r_{k+2} - 5r_{k+1} + 6r_k = 0$$

By synthetic division

$$rk(r^3 - 2r^2 - 5r + 6) = 0$$

So we get

$$rk[(x-1)(x^2 - x - 6)] = 0$$

$$rk[(x-1)(x^2 - 3x + 2x - 6)]$$

$$rk[(x-1)(x^2 - 3x + 2x - 6)] = 0$$

$$rk[(x-1)\{x(x-3) + 2(x-3)\}]$$

$$rk[(x-1)\{x(x-3) + 2(x-3)\}] = 0$$

$$rk(r-1)(r+2)(r-3)$$

$$rk(r-1)(r+2)(r-3) = 0$$

$$y_{k+3} - 2y_{k+2} - 5y_{k+1} + 6y_k$$

$$y_{k+3} - 2y_{k+2} - 5y_{k+1} + 6y_k = 0 \text{ for all } k$$

$$y_{k+2} - 4y_{k+1} + 3y_k$$

$$y_{k+2} - 4y_{k+1} + 3y_k = -4k \text{ for all } k$$

$$y_{k+2} - 4y_{k+1} + 3y_k$$

$$y_{k+2} - 4y_{k+1} + 3y_k = 0$$

$$y_{k+3} - 2y_{k+2} - 5y_{k+1} + 6y_k$$

$$y_{k+3} - 2y_{k+2} - 5y_{k+1} + 6y_k = 0 \text{ for all } k$$

In general, the equation $y_{k+n} + a_1 y_{k+n-1} + \dots + a_n$

In general, the equation $y_{k+n} + a_1 y_{k+n-1} + \dots + a_n - 1y_{k+1} + any_k$
 $= 0$ for all k

1. $2k, (-4)k; y_{k+2} + 2y_{k+1} - 8y_k$

1. $2k, (-4)k; y_{k+2} + 2y_{k+1} - 8y_k = 0$

2. $3k, (-3)k; y_{k+2} - 9y_k$

2. $3k, (-3)k; y_{k+2} - 9y_k = 0$

3. $2k, (-4)k; y_{k+2} + 2y_{k+1} - 8y_k$

3. $2k, (-4)k; y_{k+2} + 2y_{k+1} - 8y_k = 0$

4. $3k, (-3)k; y_{k+2} - 9y_k$

4. $3k, (-3)k; y_{k+2} - 9y_k = 0$

5. $(-3)k, k(-3)k; y_{k+2} + 6y_{k+1} + 9y_k$

5. $(-3)k, k(-3)k; y_{k+2} + 6y_{k+1} + 9y_k = 0$

$k; y_{k+2} + 25y_k$

$k; y_{k+2} + 25y_k = 0$

$k; y_{k+3} - y_{k+2} + 9y_{k+1} - 9y_k$

$k; y_{k+3} - y_{k+2} + 9y_{k+1} - 9y_k = 0$

8. $(-1)k, k(-1)k, 5k; y_{k+3} - 3y_{k+2} - 9y_{k+1} - 5y_k$

8. $(-1)k, k(-1)k, 5k; y_{k+3} - 3y_{k+2} - 9y_{k+1} - 5y_k = 0$

9. $(-1)k, 3k; y_{k+3} + y_{k+2} - 9y_{k+1} - 9y_k$

9. $(-1)k, 3k; y_{k+3} + y_{k+2} - 9y_{k+1} - 9y_k = 0$

10. $1k, (-1)k; y_{k+4} - 2y_{k+2} + y_k$

10. $1k, (-1)k; y_{k+4} - 2y_{k+2} + y_k = 0$

11. $y_{k+2} - 7y_{k+1} + 12y_k$

11. $y_{k+2} - 7y_{k+1} + 12y_k = 0$

12. $16y_{k+2} + 8y_{k+1} - 3y_k$

12. $16y_{k+2} + 8y_{k+1} - 3y_k = 0$

$$15. y_{k+4} - 6y_{k+3} + 8y_{k+2} + 6y_{k+1} - 9y_k$$

$$15. y_{k+4} - 6y_{k+3} + 8y_{k+2} + 6y_{k+1} - 9y_k = 0$$

$$16. y_{k+3} - (3/4)y_{k+2} + (1/16)y_k$$

$$16. y_{k+3} - (3/4)y_{k+2} + (1/16)y_k = 0$$

Thus λ is an eigenvalue of A if and only if the equation
 $(A - \lambda I)x = 0$

$\lambda = -2$, and

(i) Case

(i) Case $\lambda = -2$

(ii) Case

(ii) Case $\lambda = 5$

or equivalently,

$$A = PBP^{-1}.$$

Replacing Q by P^{-1} , we have

$$Q^{-1}BQ = A.$$

Proof If B

Proof If $B = P^{-1}AP$, then

Since

$$\det(P^{-1}) \cdot \det(P) = \det(P^{-1}P)$$

Analyze the long term behavior of the dynamical
 system defined by $x_{k+1} = Ax_k$ ($k = 0, 1, 2, \dots$), with

$$x_k = c_1 v_1 + c_2 \left(\frac{1}{2}\right)^k v_2$$

Continuing in the same way, we get the general equation as

$$x_k = c_1 v_1 + c_2 \left(\frac{1}{2}\right)^k v_2$$

to construct D . By Eqs. (1) – (3). AP

to construct D . By Eqs. (1) – (3). $AP = PD$. This

A

$$A = PDP^{-1}, \text{ where}$$

Let $p(t)$

Let $p(t) = a_0 + a_1 t + a_2 t^2$ and compute

So $T(p)$

So $T(p) = (3a_0 + 4a_1)1 + (5a_1 - a_2)t + (a_0 - 2a_1 + 7a_2)t^2$.

(a) A

(a) $A = (I) - 1AI$, so A is similar to A .

$\det(A - I)$

$\det(A - I) = 0$

corresponding to

corresponding to $= 0.8 - 0.6i$ is

Example 2

Plot several trajectories of the dynamical system $x_{k+1} = Ax_k$,
when

If x_0

If $x_0 = c_1 v_1 + c_2 v_2$, then

A

$A = PDP^{-1}$, we find that

$P y_{k+1}$

$P y_{k+1} = A P y_k = (P D P^{-1}) P y_k = P D y_k$

<, the origin is a saddle point of the

dynamical system. If $x_0 = c_1 v_1 + c_2 v_2$, then

eigen vector for first value of

will be $v = (10, 3)$

out to be

$1 = 6$ and

z

$z = y - y y$

Compute

, p, q where $p(t) = 4 + t$ $q(t) = 5 - 4t^2$

orthogonal to y

orthogonal to $y = (5, -1)$

inequality holds for x

inequality holds for $x = (3, -2)$ and $y = (-2, 1)$

5

$$5 = 5 - 0 = 5$$

$$= - (5 - 0) = -5$$

x_1

$$x_1 = -6x_2 - 3x_4$$

x_2 is free

$$x_3 = 5 + 4x_4$$

$$15. \quad x_1 + hx_2$$

$$15. \quad x_1 + hx_2 = 1$$

$$16. \quad x_1 - 3x_2$$

$$16. \quad x_1 - 3x_2 = 1$$

$$2x_1 + 3x_2$$

$$2x_1 + 3x_2 = k$$

$$2x_1 + hx_2$$

$$2x_1 + hx_2 = k$$

$$u + v$$

$$u + v = v + u \text{ (Commutative)}$$

(ii)

$$(u + v) + w = u + (v + w) \text{ (Associative)}$$

(iii)

$$u + 0 = 0 + u = u \text{ (Additive Identity)}$$

(iv)

$$u + (-u) = (-u) + u = 0 \text{ (Additive Inverse)}$$

$$(3, 5, 2)$$

$$(3, 5, 2) = 3 \cdot 1v + 5$$

$$x_1 a_1 + x_2 a_2$$

$$x_1 a_1 + x_2 a_2 = b$$

because cv_1

because $cv_1 = cv_1 + 0v_2 + \dots + 0v_p$ i.e every cv_i can be written as a linear

(x, y, z)

$$(x, y, z) = (1, 2, -3) + t(4, -5, 1)$$

the sum

$$x - x_0 = t_1 v_1 + t_2 v_2$$

or, equivalently,

$$x = x_0 + t_1 v_1 + t_2 v_2 \text{ ----- (1)}$$

x

$$x = t_1 v_1 + t_2 v_2$$

component form of eq. 1 will be

$$(x, y, z) = (x_0, y_0, z_0) + t_1(a_1, b_1, c_1) + t_2(a_2, b_2, c_2)$$

x

$$x = x_0 + a_1 t_1 + a_2 t_2$$

y

$$y = y_0 + b_1 t_1 + b_2 t_2$$

z

$$z = z_0 + c_1 t_1 + c_2 t_2$$

(x, y, z)

$$(x, y, z) = t_1(1, -2, 3) + t_2(4, 0, 5)$$

plane is

$$x = x_0 + t_1(x_1 - x_0) + t_2(x_2 - x_0)$$

x

$$x = 5 + y - 2z$$

$$x_1 a_1 + x_2 a_2 + \dots + x_n a_n$$

$$x_1 a_1 + x_2 a_2 + \dots + x_n a_n = b$$

$$= aA + bB + cC$$

$$4a + b$$

$$4a + b = 6$$

$$-b + 2c$$

$$-b + 2c = -8$$

$$-2a + 2b + c$$

$$-2a + 2b + c = -1$$

$$-2a + 3b + 4c$$

$$-2a + 3b + 4c = -8$$

$$-b - 3c$$

$$-b - 3c = 7$$

$$5c$$

$$5c = -15 \quad c = -3$$

From (2),

$$-b + 2(-3) = -8 \quad b = 2$$

$$4(1) + 2$$

$$4(1) + 2 = 6$$

$$= 1A + 2B - 3C$$

$$4a + b$$

$$4a + b = 0$$

$$-b + 2c$$

$$-b + 2c = 0$$

$$-2a + 2b + c$$

$$-2a + 2b + c = 0$$

$$-2a + 3b + 4c$$

$$-2a + 3b + 4c = 0$$

Subtracting equation (3) from equation (4) we get

$$b + 3c = 0 \quad (5)$$

$$= 0A + 0B + 0C$$

$$-2a + 2b + c$$

$$-2a + 2b + c = 3$$

$$-2a + 3b + 4c$$

$$-2a + 3b + 4c = 8$$

$$-b - 3c$$

$$-b - 3c = -5$$

From (2),

$$-b + 2(1) = 0 \quad b = 2$$

$$= 1A + 2B + 1C$$

. Verify $A(u + v)$

$$. \text{Verify } A(u + v) = Au + Av.$$

The equation

$$x = su + tv$$

x

$$x = p + tv \quad (t \text{ in } R)$$

$$1. x_1 - 5x_2 + 9x_3$$

$$1. x_1 - 5x_2 + 9x_3 = 0$$

$$2. 3x_1 + 6x_2 - 4x_3 - x_4$$

$$2. 3x_1 + 6x_2 - 4x_3 - x_4 = 0$$

$$-x_1 + 4x_2 - 3x_3$$

$$-x_1 + 4x_2 - 3x_3 = 0$$

$$-5x_1 + 8x_3 + 3x_4$$

$$-5x_1 + 8x_3 + 3x_4 = 0$$

$$2x_1 - 8x_2 + 9x_3$$

$$2x_1 - 8x_2 + 9x_3 = 0$$

$$8x_1 - x_2 + 7x_4$$

$$8x_1 - x_2 + 7x_4 = 0$$

$$3. 5x_1 - x_2 + 3x_3$$

$$3. 5x_1 - x_2 + 3x_3 = 0$$

$$4x_1 - 3x_2 + 7x_3$$

$$4x_1 - 3x_2 + 7x_3 = 0$$

$$5. x_1 + 2x_2 - 7x_3$$

$$5. x_1 + 2x_2 - 7x_3 = 0$$

$$-2x_1 - 3x_2 + 9x_3$$

$$-2x_1 - 3x_2 + 9x_3 = 0$$

$$-2x_1 + 3x_2 + 7x_3$$

$$-2x_1 + 3x_2 + 7x_3 = 0$$

$$-2x_2 + 10x_3$$

$$-2x_2 + 10x_3 = 0$$

$$\text{of } x_1 - 4x_2 + 3x_3$$

$$\text{of } x_1 - 4x_2 + 3x_3 = 7.$$

$$10v_1 - 5v_2 + 5v_3$$

$$10v_1 - 5v_2 + 5v_3 = 0$$

$$c_1v_1 + c_2v_2 + \dots + c_pv_p$$

$$c_1v_1 + c_2v_2 + \dots + c_pv_p = 0$$

$$2. T(cu + dv)$$

$$2. T(cu + dv) = cT(u) + dT(v)$$

$$3. T(c_1v_1 + \dots + c_pv_p)$$

$$3. T(c_1v_1 + \dots + c_pv_p) = c_1T(v_1) + \dots + c_pT(v_p)$$

2. Just apply the definition of linear transformation. i. e

$$T(cu + dv) = T(cu) + T(dv) = cT(u) + dT(v)$$

$$T(c_1v_1 + \dots + c_pv_p)$$

$$T(c_1v_1 + \dots + c_pv_p) = c_1T(v_1) + \dots + c_pT(v_p)$$

$$T(x)$$

$$T(x) = x+1.$$

$$T(cu + dv)$$

$$T(cu + dv) = 3(cu + dv)$$

$$= 3cu + 3dv$$

$$= c(3u) + d(3v)$$

$$= cT(u) + dT(v)$$

$$L(0, 1, 0) = (3, 1), \text{ and}$$

$$L(0, 0, 1) = (-1, 2).$$

Find x such that $T(x)$

$$\text{Find } x \text{ such that } T(x) = (-2, -5).$$

. Find x such that $T(x)$

$$\text{. Find } x \text{ such that } T(x) = (-4, 7, 0).$$

$$A + B$$

$$A + B = B + A$$

$$r(A + B)$$

$$r(A + B) = rA + rB$$

$$(A + B) + C$$

$$(A + B) + C = A + (B + C)$$

$$(r + s)A$$

$$(r + s)A = rA + sA$$

$$= -1 = (1)$$

$$A(Bx)$$

$$A(Bx) = (AB)$$

$$x \text{-----} (1)$$

$$(2 \times 4) + (6 \times 3) + (0 \times 5)$$

$$(2 \times 4) + (6 \times 3) + (0 \times 5) = 26$$

$$(1 \times 3) + (2 \times 1) + (4 \times 2)$$

$$(1 \times 3) + (2 \times 1) + (4 \times 2) = 13$$

$$3) + (6 \times 1) + (0 \times 2)$$

$$3) + (6 \times 1) + (0 \times 2) = 12$$

$$A(B + C)$$

$$A(B + C) = AB + AC$$

$$(B + C)A$$

$$(B + C)A = BA + CA$$

Thus C

$$\text{Thus } C = A^{-1}.$$

(a) A^n is invertible and $(A^n)^{-1}$

$$(a) A^n \text{ is invertible and } (A^n)^{-1} = A^{-n} = (A^{-1})^n$$

B

$$B = A^{-1} \text{ and } A = B^{-1}$$

$\det(A)$

$$\det(A) = 1(20 + 2) - 6(10 - 1) + 4(4 + 4)$$

$$= 22 - 54 + 32$$

$T(x, y)$

$$T(x, y) = xyt + (x + y)t^2$$

be inverse of A .i.e. A^{-1}

be inverse of A .i.e. $A^{-1} = B$, then

A

$$A = (E_p \dots E_1)^{-1}U = LU$$

L

$$L = (E_p \dots E_1)^{-1}$$

because $E_p \dots E_1 L$

because $E_p \dots E_1 L = (E_p \dots E_1)(E_p \dots E_1)^{-1} = I$. This observation is the key to

(4^*)

And rewrite (3^*) as $Ly = b$

$$10x_1 + x_2 - x_3$$

$$10x_1 + x_2 - x_3 = 18$$

$$x_1 + 15x_2 + x_3$$

$$x_1 + 15x_2 + x_3 = -12$$

$$-x_1 + x_2 + 20x_3$$

$$-x_1 + x_2 + 20x_3 = 17$$

Comparing the corresponding entries on both sides, we have

$$10y_1 = -x_2 + x_3 + 18$$

$$15y_2$$

$$15y_2 = -x_1 - x_3 - 12$$

$$20y_3$$

$$20y_3 = x_1 - x_2 + 17$$

$$y_1$$

$$y_1 = (-x_2 + x_3 + 18)/10$$

$$y_2$$

$$y_2 = (-x_1 - x_3 - 12)/15$$

$$y_3$$

$$y_3 = (x_1 - x_2 + 17)/20$$

Thus, $x(2)$

$$\text{Thus, } x(2) = (1.965, -.9767, .98).$$

$$x_3 = (17+0-0)/20 = 0.85$$

Which gives $x(1) = (1.80, -0.80, 0.85)$ ----put this again on RHS of the equivalent system

$$10x + x$$

$$10x + x = 13$$

$$x + x + 10x$$

$$x + x + 10x = 18$$

$x(0) = (0, 0, 0)$ and six iterations.

$$10x_1 + x_2 - x_3 = 18$$

$$x_1 + 15x_2 + x_3$$

$$x_1 + 15x_2 + x_3 = -12$$

$$-x_1 + x_2 + 20x_3$$

$$-x_1 + x_2 + 20x_3 = 17$$

$$10y_1$$

$$10y_1 = -x_2 + x_3 + 18$$

$$y_1 + 15y_2$$

$$y_1 + 15y_2 = -x_3 - 12$$

$$-y_1 + y_2 + 20y_3$$

$$-y_1 + y_2 + 20y_3 = 17$$

regard the highlighted x 's as the values:

$$x_1 = (-x_2 + x_3 + 18)/10$$

$$x_2$$

$$x_2 = (-x_1 - x_3 - 12)/15$$

$$x_3$$

$$x_3 = (x_1 - x_2 + 17)/20$$

$$x_1 - 3x_2 + x_3$$

$$x_1 - 3x_2 + x_3 = -2$$

$$-6x_1 + 4x_2 + 11x_3$$

$$-6x_1 + 4x_2 + 11x_3 = 1$$

$$5x_1 - 2x_2 - 2x_3 = 9$$

$$x_1 - 3x_2 + x_3 = -2$$

$$\det(A)$$

$$\det(A) = ad-bc.$$

$$= 45 + 84 + 96 - 105 + 48 + 72$$

$$= 1 [4(0) - (-1)(-2)] - 5 [2(0) - 0(-1)] + 0 [2(-2) - 4(0)]$$

$$= ab. (1)^2 = ab$$

$$x = -8$$

$$x + 2x = -3$$

$$2x + x + x$$

$$2x + x + x = 4$$

$$2x = 2$$

$$3x + x + 3x = -2$$

$$6sx + 4x$$

$$6sx + 4x = 5$$

$$9x + 2sx$$

$$9x + 2sx = -2$$

$$3sx - 5x$$

$$3sx - 5x = 3$$

$$9x + 5sx$$

$$9x + 5sx = 2$$

$$sx - 2sx$$

$$sx - 2sx = -1$$

$$3x + 6sx$$

$$3x + 6sx = 4$$

$$x = 1$$

$$3sx + 6sx = 2$$

and b_2

$$\text{and } b_2 = -2$$

$$0 + u$$

$$0 + u = u + 0 = u$$

$$-u + u$$

$$-u + u = 0 = u + (-u)$$

$$= (u_1 + v_1, u_2 + v_2, \dots, u_n + v_n) \in R^n$$

$$= (u_1 + v_1, u_2 + v_2, \dots, u_n + v_n)$$

$$= (v_1 + u_1, v_2 + u_2, \dots, v_n + u_n) \text{ (By commutative law of real numbers)}$$

$$= (v_1, v_2, \dots, v_n) + (u_1, u_2, \dots, u_n)$$

$$= (u_1 + v_1, u_2 + v_2, \dots, u_n + v_n) + (w_1, w_2, \dots, w_n)$$

$$= ((u_1 + v_1) + w_1, (u_2 + v_2) + w_2, \dots, (u_n + v_n) + w_n)$$

$$= (u_1, u_2, \dots, u_n) + (v_1 + w_1, v_2 + w_2, \dots, v_n + w_n)$$

$$= (u_1, u_2, \dots, u_n) + [(v_1, v_2, \dots, v_n) + (w_1, w_2, \dots, w_n)]$$

$$= u + (v + w)$$

$$0 + u$$

$$0 + u = (0, 0, \dots, 0) + (u_1, u_2, \dots, u_n)$$

$$= (0 + u_1, 0 + u_2, \dots, 0 + u_n)$$

$$\text{Similarly, } u + 0$$

$$\text{Similarly, } u + 0 = u$$

$$\text{Such that } u + (-u)$$

$$\text{Such that } u + (-u) = (u_1, u_2, \dots, u_n) + (-u_1, -u_2, \dots, -u_n)$$

$$= (u_1 + (-u_1), u_2 + (-u_2), \dots, u_n + (-u_n))$$

$$\text{Similarly, } (-u) + u$$

$$\text{Similarly, } (-u) + u = 0$$

$$= k(u_1 + v_1, u_2 + v_2, \dots, u_n + v_n)$$

$$= (k(u_1 + v_1), k(u_2 + v_2), \dots, k(u_n + v_n))$$

$$= (k u_1 + k v_1, k u_2 + k v_2, \dots, k u_n + k v_n)$$

$$= (k u_1, k u_2, \dots, k u_n) + (k v_1, k v_2, \dots, k v_n)$$

$$= k(u_1, u_2, \dots, u_n) + k(v_1, v_2, \dots, v_n)$$

$$= k u + k v$$

$$= ((k + 1)u_1, (k + 1)u_2, \dots, (k + 1)u_n)$$

$$\begin{aligned}
 &= (k u_1 + l u_1, k u_2 + l u_2, \dots, k u_n + l u_n) \\
 &= (k u_1, k u_2, \dots, k u_n) + (l u_1, l u_2, \dots, l u_n) \\
 &= k (u_1, u_2, \dots, u_n) + l (u_1, u_2, \dots, u_n) \\
 &= k u + l u
 \end{aligned}$$

$$\begin{aligned}
 u + (v + w) \\
 &= u + (v + w)
 \end{aligned}$$

and similarly $u + 0$
and similarly $u + 0 = u$.

$$\begin{aligned}
 (k + l)u \\
 (k + l) = (k + l)
 \end{aligned}$$

$$\begin{aligned}
 (f + g)(x) \\
 (f + g)(x) = f(x) + g(x), \text{ for all } x \in R.
 \end{aligned}$$

$$\begin{aligned}
 (f + g)(x) \\
 (f + g)(x) &= f(x) + g(x) \\
 &= g(x) + f(x) \\
 &= (g + f)(x)
 \end{aligned}$$

So that
 $f + g = g + f$

$$\begin{aligned}
 ((f + g) + h)(x) \\
 ((f + g) + h)(x) &= (f + g)(x) + h(x) \\
 &= (f(x) + g(x)) + h(x) \\
 &= f(x) + (g(x) + h(x)) \\
 &= f(x) + (g + h)(x) \\
 &= (f + (g + h))(x)
 \end{aligned}$$

And so
 $(f + g) + h = f + (g + h)$

$$0(x)$$

$$0(x) = 0, \text{ for all } x \in R \text{ because } (0 + f)(x) = 0(x) + f(x) \\ = 0 + f(x) = f(x)$$

$$\text{i.e. } 0 + f$$

$$\text{i.e. } 0 + f = f. \text{ Similarly, } f + 0 = f.$$

$$(f + (-f))(x)$$

$$(f + (-f))(x) = f(x) + (-f)(x)$$

$$\text{i.e. } f + (-f)$$

$$\text{i.e. } f + (-f) = 0. \text{ Similarly, } (-f) + f = 0.$$

$$\text{Thus } (a + b)f$$

$$\text{Thus } (a + b)f = af + bf$$

$$)+ ($$

$$) = (a + a$$

$$= (a + b) + (a + b) + (a + b)$$

$$)$$

$$) = k(a + a$$

$$ax + by + cz$$

$$ax + by + cz = 0$$

$$\text{Adding these equations gives } a(u_1 + v_1) + b(u_2 +$$

$$\text{Adding these equations gives } a(u_1 + v_1) + b(u_2 + v_2) + c(u_3 + v_3) = 0$$

$$u + v$$

$$u + v = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$$

$$a(ku_1) + b(ku_2) + c(ku_3)$$

$$a(ku_1) + b(ku_2) + c(ku_3) = 0$$

$$\text{gives } a(-1u_1) + b(-1u_2) + c(-1u_3)$$

$$\text{gives } a(-1u_1) + b(-1u_2) + c(-1u_3) = 0$$

$$+$$

$$+ = (u + v, u + v)$$

$$(iii) (-1) u$$

$$(iii) (-1) u = -u$$

$$(-1) u$$

$$(-1) u = -u \text{ is in } W.$$

+

$$+ = (a, b, 0) + (a, b, 0) = (a + a, b + b, 0)$$

+

$$+ = (a, b, 1) + (a, b, 1) = (a + a, b + b, 1 + 1)$$

$$\text{Then } u + v = (u_1, u_1 + u_3, u_3) + (v_1, v_1 + v_3, v_3)$$

$$= (u_1 + v_1, u_1 + u_3 + v_1 + v_3, u_3 + v_3)$$

If k is any scalar and u

If k is any scalar and $u = (u_1, u_1 + u_3, u_3)$ is any vector in W ,
then

(By definition)

$$= (k u_1, k u_1 + k u_3, k u_3)$$

$$\text{Then } u + v = (u_1, u_1 + u_3 + 1, u_3) + (v_1, v_1 + v_3 + 1, v_3)$$

$$= (u_1 + v_1, u_1 + u_3 + 1 + v_1 + v_3 + 1, u_3 + v_3)$$

$$= (u_1 + v_1, (u_1 + v_1) + (u_3 + v_3) + 2, u_3 + v_3)$$

(i) All polynomials $a_0 + a_1 x + a_2 x^2 + a_3 x^3$ for

(i) All polynomials $a_0 + a_1 x + a_2 x^2 + a_3 x^3$ for which $a_0 = 0$

$$a_0 + a_1 + a_2 + a_3$$

$$a_0 + a_1 + a_2 + a_3 = 0.$$

$$= (c_0 + b_0) + (c_1 + b_1) x + (c_2 + b_2) x^2 + (c_3 + b_3) x^3$$

+ 0

$$+ 0 = 0. \text{ Therefore, } u + v \in W$$

where $a + b + c + d$

$$\text{where } a + b + c + d = 0$$

where $a + b + c + d$

$$\text{where } a + b + c + d = 0.$$

h

$$h = 0 + 0 = 0$$

$u + w$

$$u + w = (s_1v_1 + s_2v_2) + (t_1v_1 + t_2v_2)$$

$$= (s_1 + t_1)v_1 + (s_2 + t_2)v_2$$

$$= (s_1v_1 + s_2v_2 + \dots + s_nv_n) + (t_1v_1 + t_2v_2 + \dots + t_nv_n)$$

$(x, y, z) + (x, y, z)$

$$(x, y, z) + (x, y, z) = (x + x, y + y, z + z)$$

$(x, y) + (x, y)$

$$(x, y) + (x, y) = (x + x, y + y)$$

$(x, y) + (x, y)$

$$(x, y) + (x, y) = (x + x + 1, y + y + 1)$$

matrix multiplication, we find that

$$A(u + v) = Au + Av = 0 + 0 = 0$$

equations

$$a - 2b + 5c = d$$

$$a - 2b + 5c - d$$

$$a - 2b + 5c - d = 0$$

$$-a - b + c$$

$$-a - b + c = 0$$

$$x - 2x$$

$$-x + 3x = 0$$

$$x + 2x - 2x$$

$$x + 2x - 2x = 0$$

x

$$x = 2x + x - 3x$$

$x = \text{free variable}$

$$x = -2x + 2x$$

$$x - 3x + 2x$$

$$+2x + x = 0$$

$$0x + 3x + 6x + 0x - 3x$$

$$0x + 3x + 6x + 0x - 3x = 0$$

$$2x - 3x - 2x$$

$$+4x + 4x = 0$$

$$3x - 6x + 0x + 6x + 5x$$

$$3x - 6x + 0x + 6x + 5x = 0$$

$$-2x + 9x + 2x - 4x - 5x$$

$$-2x + 9x + 2x - 4x - 5x = 0$$

$$1x +$$

$$2x + (4/3)x = 0$$

$$1x +$$

$$(-1/6)x = 0$$

$$1x +$$

$$(-5/12)x = 0$$

$$x$$

$$x = -2x + (-4/3)x$$

$$x$$

$$x = +(1/6)x$$

$$x$$

$$x = +(5/12)x$$

The solution can be written in the vector form:

$$= (-2, 0, 0, 1, 0)$$

$$= (-4/3, 1/6, 5/12, 0, 1)$$

$$2 \ 1 \ -2 \ +3 \ -3$$

$$2 \ 1 \ -2 \ +3 \ -3 = -9$$

$$2x + 6x - 5x - 2x + 4x - 3x$$

$$2x + 6x - 5x - 2x + 4x - 3x = -1$$

$$2x + 6x$$

$$+ 8x + 4x + 18x = 6$$

$$1x$$

$$1x = (1/3)$$

$$x$$

$$x = -3x - 4x - 2x$$

$$x$$

$$x = -3r - 4s - 2t$$

$$(i) T(u + v)$$

$$(i) T(u + v) = T(u) + T(v)$$

$$b : a - 3b - c$$

$$b : a - 3b - c = 0$$

$$= [1 \ -3 \ -1].$$

$$a - 2b$$

$$a - 2b = 4c$$

. For instance, if $p(t)$

. For instance, if $p(t) = 3 + 5t + 7t^2$, then

$$T(A)$$

$$T(A) = A + AT, \text{ where}$$

$$u + v$$

$$u + v = v + u$$

$$u + (v + w)$$

$$u + (v + w) = (u + v) + w$$

$$I(u + v)$$

$$I(u + v) = Iu + Iv$$

$$(I + m)u$$

$$(I + m)u = Iu + mu$$

Example 3 Let A be an invertible $n \times n$

$\times$ matrix – say, $A = [a_1 \dots a_n]$. Then the columns

$c_0, \dots, c_n$ satisfy

$$c_0 \cdot 1 + c_1 t + c_2 t^2 + \dots + c_n t^n = 0 \quad (t)$$

$$c_1 v_1 + c_2 v_2 + c_3 v_3$$

$$c_1 v_1 + c_2 v_2 + c_3 v_3 = d_1 w_1 + d_2 w_2 + d_3 w_3$$

$$c_1(2, -3, 1) + c_2(4, 1, 1) + c_3(0, -7, 1)$$

$$c_1(2, -3, 1) + c_2(4, 1, 1) + c_3(0, -7, 1) = d_1(1, 0, 0) + d_2(0, 1, 0) + d_3(0, 0, 1)$$

$$2c + 4c + 0c$$

$$2c + 4c + 0c = 1d + 0d + 0d$$

$$-3c + 1c - 7c$$

$$-3c + 1c - 7c = 0d + 1d + 0d$$

$$1c + 1c + 1c$$

$$1c + 1c + 1c = 0d + 0d + 1d$$

$$-4c + 6c + 8c$$

$$-4c + 6c + 8c = 1d + 0d + 0d$$

$$1c + 5c + 4c$$

$$1c + 5c + 4c = 0d + 1d + 0d$$

$$3c + 2c + 1c$$

$$3c + 2c + 1c = 0d + 0d + 1d$$

solution of

$$c_1 p_1(t) + c_2 p_2(t) + c_3 p_3(t) = 0$$

$$-4c + 6c + 8c$$

$$-4c + 6c + 8c = 0$$

$$1c + 5c + 4c$$

$$1c + 5c + 4c = 0$$

$$3c + 2c + 1c$$

$$3c + 2c + 1c = 0$$

$$3c + 0c + 0c + 1c$$

$$3c + 0c + 0c + 1c = 1d + 0d + 0d + 0d$$

$$6c -$$

$$1c - 8c + 0c = 0d + 1d + 0d + 0d$$

$$3c -$$

$$1c - 12c - 1c = 0d + 0d + 1d + 0d$$

$$-6c + 0c - 4c + 2c$$

$$-6c + 0c - 4c + 2c = 0d + 0d + 0d + 1d$$

$$\det(A)$$

$$\det(A) = 48 \neq 0$$

Every vector in $\text{Span}\{v_1, v_2\}$ belongs to H because

$$c_1 v_1 + c_2 v_2 = c_1 v_1 + c_2 v_2 + 0 v_3$$

substitute

$$x = c_1 v_1 + c_2 v_2 + c_3 (5v_1 + 3v_2)$$

$$= (c_1 + 5c_3) v_1 + (c_2 + 3c_3) v_2$$

$$c_1 v_1 + c_2 v_2 + \dots + c_n v_n$$

$$c_1 v_1 + c_2 v_2 + \dots + c_n v_n = 0$$

Solution Step 1 Form Equation (3),

$$c_1 (1, 2, -2, 1) + c_2 (-3, 0, -4, 3) + c_3 (2, 1, 1, -1) + c_4 (-3, 3, -9, 6) + c_5 (9, 3, 7, -6) = (0, 0, 0, 0).$$

$$c - 3c + 2c - 3c + 9c$$

$$c - 3c + 2c - 3c + 9c = 0$$

$$+ c + 3c + 3c$$

$$+ c + 3c + 3c = 0$$

$$-2c - 4c + c - 9c + 7c$$

$$-2c - 4c + c - 9c + 7c = 0$$

$$c + 3c -$$

$$c + 6c - 6c = 0$$

$$x = 0$$

$$-2x - x + 2x = 0$$

$$x + y + z$$

$$x + y + z = 0$$

$$3x + 2y - 2z$$

$$3x + 2y - 2z = 0$$

$$4x + 3y - z$$

$$4x + 3y - z = 0$$

$$6x + 5y + z$$

$$6x + 5y + z = 0$$

14. Determine bases for the following subspace of R^3

(a) the plane $3x - 2y + 5z = 0$

(b) the plane $x - y = 0$

(c) the line $x = 2t, y = -t, z = 4t$

$$[v]_S$$

$$[v]_S = (-1, 3, 2)$$

$$c + 2c + 3c$$

$$c + 2c + 3c = 5$$

$$2c + 9c + 3c$$

$$2c + 9c + 3c = -1$$

$$4c + 2c + 3c$$

$$4c + 2c + 3c = 5$$

$$+ 9c + 3c$$

$$+ 9c + 3c = -1$$

to get c

$$c + 4(2) = 9$$

Therefore,

$$[v]_S = (1, -1, 2)$$

A

$$A = c_1 A_1 + c_2 A_2 + c_3 A_3 + c_4 A_4$$

Therefore,

$$[v]s = (-1, 1, -1, 3)$$

yourself)

Thus $x = 3b_1 + 2b_2$ and

$$3 + 2t$$

$$3 + 2t = 2(4 + t + 5t^2) - 5(1 + 2t^2)$$

vector of $p(t)$

vector of $p(t) = 6 + 3t - t^2$ relative to B .

Put value of c in equation

$$1 + c = -1$$

we have

$$c + c = 6$$

$p(t)$

$p(t) = 1 + 4t + 7t^2$ relative to B .

vectors as

$$x = x_1e_1 + x_2e_2 + x_3e_3 + \dots + x_n e_n.$$

$$: 2a - b + 3c + d$$

$$: 2a - b + 3c + d = 0\}.$$

$$= aA_1 + bA_2 + cA_3$$

$$aA + bA + cA$$

$$aA + bA + cA = 0$$

$$-x -$$

$$x + 2x - 3x + x = 0$$

$$-x = 0$$

$$x + x + x = 0$$

$$x$$

$$x = -1x - 1x$$

$$6 \{(a, b, c, d): a - 3b + c$$

$$6 \{(a, b, c, d): a - 3b + c = 0\}$$

$$= (-2, -5, 8, 0, -17)$$

$$= (1, 3, -5, 1, 5)$$

$$= (3, 11, -19, 7, 1)$$

$$= (1, 7, -13, 5, -3)$$

$$= (2, 1, 0)$$

$$= (3, -1, 4)$$

$$= (1, -2, 0, 0, 3)$$

$$= (2, -5, -3, -2, 6)$$

$$x + x + x$$

$$x + x + x = 0$$

$$x - 2x + 3x$$

$$x - 2x + 3x = 0$$

$$x - 5x$$

$$x - 5x = 0$$

$$\text{rank } A + \dim \text{Nul } A$$

$$\text{rank } A + \dim \text{Nul } A = n$$

$$x - 4x - 28x - 37x + 13x$$

$$x - 4x - 28x - 37x + 13x = 0$$

$$x - 2x - 12x - 16x + 5x$$

$$x - 2x - 12x - 16x + 5x = 0$$

$$x$$

$$x = 4x - 28x + 37x - 13x$$

$$x$$

$$x = 2x + 12x + 16x - 5x$$

$$x$$

$$x = 4r + 28s + 37t - 13u$$

$$x$$

$$x = 2r + 12s + 16t - 5u$$

has 6 columns,

$$\text{so rank}(A) + \text{nullity}(A) = 2 + 4 = 6 = n$$

The system has infinitely many solutions:

$$x_1 = -2x_4 + (-4/3)x_5$$

x_2

$$x_2 = (1/6)x_5$$

x_3

$$x_3 = (5/12)x_5$$

The solution can be written in the vector form:

$$c_4 = (-2, 0, 0, 1, 0)$$

c_5

$$c_5 = (-4/3, 1/6, 5/12, 0, 1)$$

$$\{(-2, 0, 0, 1, 0), (-4/3, 1/6, 5/12, 0, 1)\}$$

The nullity of the matrix is 2. Now $\text{Rank}(A) + \text{nullity}(A) = 3 + 2 = 5 = n$

Solution

$$\text{nullity}(A) = n - \text{rank}(A) = 7 - 3 = 4$$

Solution

$$\text{nullity}(A) = n - \text{rank}(A) = 4 - 0 = 4$$

Suppose that x

$$\text{Suppose that } x = 3b_1 + b_2$$

(a) Find the change-of-coordinates matrix from F to D .

(b) Find $[x]_D$ for $x = f_1 - 2f_2 + 2f_3$.

(a) Find the change-of-coordinates matrix from B to C .

(b) Find $[x]_C$ for $x = 5b_1 + 3b_2$.

Feed two different signals into the filter

$$0.35y_k + 2 + 0.5y_k + 1 + 0.35y_k = z_k$$

± 2 so that

$$\{y_k\} = \{\dots, 1, 0.7, 0, -0.7, -1, -0.7, 0, 0.7, 1, 0.7, 0, \dots\}$$

$$y_{k+2}$$

$$0.35y_k + 0.5y_{k+1} + 0.35y_{k+2} = z_k$$

$$0.35(1) + 0.5(0.7) + 0.35(0)$$

$$0.35(1) + 0.5(0.7) + 0.35(0) = 0.7$$

$$\{w_k\}$$

$$\{w_k\} = \{\dots, 1, -0.7, 0, 0.7, -1, 0.7, 0, -0.7, 1, -0.7, 0, \dots\}$$

$y_k = rk$ for some r . Find some solutions of the equation

$$y_{k+3} - 2y_{k+2} - 5y_{k+1} + 6y_k = 0 \text{ for all } k$$

$$r_{k+3} - 2r_{k+2} - 5r_{k+1} + 6r_k$$

$$r_{k+3} - 2r_{k+2} - 5r_{k+1} + 6r_k = 0$$

By synthetic division

$$rk(r^3 - 2r^2 - 5r + 6) = 0$$

So we get

$$rk[(x-1)(x^2 - x - 6)] = 0$$

$$rk[(x-1)(x^2 - 3x + 2x - 6)]$$

$$rk[(x-1)(x^2 - 3x + 2x - 6)] = 0$$

$$rk[(x-1)\{x(x-3) + 2(x-3)\}]$$

$$rk[(x-1)\{x(x-3) + 2(x-3)\}] = 0$$

$$rk(r-1)(r+2)(r-3)$$

$$rk(r-1)(r+2)(r-3) = 0$$

$$y_{k+3} - 2y_{k+2} - 5y_{k+1} + 6y_k$$

$$y_{k+3} - 2y_{k+2} - 5y_{k+1} + 6y_k = 0 \text{ for all } k$$

$$y_{k+2} - 4y_{k+1} + 3y_k$$

$$y_{k+2} - 4y_{k+1} + 3y_k = -4k \text{ for all } k$$

$$y_{k+2} - 4y_{k+1} + 3y_k$$

$$y_{k+2} - 4y_{k+1} + 3y_k = 0$$

$$y_{k+3} - 2y_{k+2} - 5y_{k+1} + 6y_k$$

$$y_{k+3} - 2y_{k+2} - 5y_{k+1} + 6y_k = 0 \text{ for all } k$$

In general, the equation $y_{k+n} + a_1 y_{k+n-1} + \dots + a_n$

In general, the equation $y_{k+n} + a_1 y_{k+n-1} + \dots + a_n - 1 y_{k+1} + a_n y_k$
= 0 for all k

1. $2k, (-4)k; y_{k+2} + 2 y_{k+1} - 8 y_k$

1. $2k, (-4)k; y_{k+2} + 2 y_{k+1} - 8 y_k = 0$

2. $3k, (-3)k; y_{k+2} - 9 y_k$

2. $3k, (-3)k; y_{k+2} - 9 y_k = 0$

3. $2k, (-4)k; y_{k+2} + 2 y_{k+1} - 8 y_k$

3. $2k, (-4)k; y_{k+2} + 2 y_{k+1} - 8 y_k = 0$

4. $3k, (-3)k; y_{k+2} - 9 y_k$

4. $3k, (-3)k; y_{k+2} - 9 y_k = 0$

5. $(-3)k, k(-3)k; y_{k+2} + 6 y_{k+1} + 9 y_k$

5. $(-3)k, k(-3)k; y_{k+2} + 6 y_{k+1} + 9 y_k = 0$

$k; y_{k+2} + 25 y_k$

$k; y_{k+2} + 25 y_k = 0$

$k; y_{k+3} - y_{k+2} + 9 y_{k+1} - 9 y_k$

$k; y_{k+3} - y_{k+2} + 9 y_{k+1} - 9 y_k = 0$

8. $(-1)k, k(-1)k, 5k; y_{k+3} - 3 y_{k+2} - 9 y_{k+1} - 5 y_k$

8. $(-1)k, k(-1)k, 5k; y_{k+3} - 3 y_{k+2} - 9 y_{k+1} - 5 y_k = 0$

9. $(-1)k, 3k; y_{k+3} + y_{k+2} - 9 y_{k+1} - 9 y_k$

9. $(-1)k, 3k; y_{k+3} + y_{k+2} - 9 y_{k+1} - 9 y_k = 0$

10. $1k, (-1)k; y_{k+4} - 2 y_{k+2} + y_k$

10. $1k, (-1)k; y_{k+4} - 2 y_{k+2} + y_k = 0$

11. $y_{k+2} - 7 y_{k+1} + 12 y_k$

11. $y_{k+2} - 7 y_{k+1} + 12 y_k = 0$

12. $16 y_{k+2} + 8 y_{k+1} - 3 y_k$

12. $16 y_{k+2} + 8 y_{k+1} - 3 y_k = 0$

15. $y_{k+4} - 6 y_{k+3} + 8 y_{k+2} + 6 y_{k+1} - 9 y_k$

15. $y_{k+4} - 6 y_{k+3} + 8 y_{k+2} + 6 y_{k+1} - 9 y_k = 0$

$$16. y_{k+3} - (3/4)y_{k+2} + (1/16)y_k$$

$$16. y_{k+3} - (3/4)y_{k+2} + (1/16)y_k = 0$$

Thus is an eigenvalue of A if and only if the equation
 $(A - I)x = 0$

$= -$, and

(i) Case

(i) Case $= -2$

(ii) Case

(ii) Case $= 5$

or equivalently,

$$A = PBP^{-1}.$$

Replacing Q by P^{-1} , we have

$$Q^{-1}BQ = A.$$

Proof If B

Proof If $B = P^{-1}AP$, then

Since

$$\det(P^{-1}) \cdot \det(P) = \det(P^{-1}P)$$

Analyze the long term behavior of the dynamical
 system defined by $x_{k+1} = Ax_k$ ($k = 0, 1, 2, \dots$), with

$$= c_1 v_1 + c_2 (.92)v_2$$

Continuing in the same way, we get the general equation as

$$x_k = c_1 v_1 + c_2 (.92)^k v_2$$

to construct D . By Eqs. (1) – (3). AP

to construct D . By Eqs. (1) – (3). $AP = PD$. This

A

$$A = PDP^{-1}, \text{ where}$$

Let $p(t)$

Let $p(t) = a_0 + a_1 t + a_2 t^2$ and compute

So $T(p)$

$$\text{So } T(p) = (3a_0 + 4a_1)t + (5a_1 - a_2)t^2 + (a_0 - 2a_1 + 7a_2)t^3.$$

(a) A

(a) $A = (I - 1A)I$, so A is similar to A .

$\det(A - I)$

$$\det(A - I) = 0$$

corresponding to

$$\text{corresponding to } = 0.8 - 0.6i \text{ is}$$

Example 2

Plot several trajectories of the dynamical system $x_{k+1} = Ax_k$, when

If x_0

$$\text{If } x_0 = c_1 v_1 + c_2 v_2, \text{ then}$$

A

$$A = PDP^{-1}, \text{ we find that}$$

$P_{y_{k+1}}$

$$P_{y_{k+1}} = AP_{y_k} = (PDP^{-1})P_{y_k} = PD_{y_k}$$

$<$, the origin is a saddle point of the dynamical system. If $x_0 = c_1 v_1 + c_2 v_2$, then

eigen vector for first value of

$$\text{will be } v = (10, 3)$$

out to be

$$1 = 6 \text{ and}$$

z

$$z = y - y y$$

Compute

$$, p, q \text{ where } p(t) = 4 + t \quad q(t) = 5 - 4t^2$$

orthogonal to y

$$\text{orthogonal to } y = (5, -1)$$

inequality holds for x

inequality holds for $x=(3,-2)$ and $y=(-2,1)$

Good Luck!

Keep practicing!