MTH201

Multivariable Calculus

Formula Sheet

1 Chapters, 320 Formulas

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< 0 < , this will be a cone opening up or open

< 0 < , this will be a cone opening up or opening down. If 0 = 2

x-axis (y

x-axis (y = 0)

x2 + y2

x2 + y2 = 0 (along y = 0)

x2 + y2

x2 + y2 = 0.5 (along y = x)

(Along x-axis, y

(Along x-axis, y = 0)

(Along y-axis, x

(Along y-axis, x = 0)

z

z = ln(x2 + y2) ln (x + y)

= 2x2 + 2xy x2 y2

= x2 + 2xy y2

w

w = x2 +3y2+4z2-xyz

dy

dy = 6y - xz

dz

dz = 8z - xy

w

w = x2 + y + z + 4

x

x = et, y = cost, z = t + 4

= (2x) (et) + (1) . (Sint) + (1) (1)

= 2 (et) (et) Sint + 1

= 2 e2t Sint + 1

r

r = x i+ y j + z k

Let a

Let a = a1i + a2j + a3k

and b

and b = b1i + b2j + b3k

a + b

a + b = (a1i + a2j + a3k) + (b1i + b2j + b3k)

= (a1 + b1)i + (a2 + b2)j + (a3 + b3)k

a - b

a - b = (a1i + a2j + a3k) - (b1i + b2j + b3k)

= (a1 - b1)i + (a2 - b2)j + (a3 - b3)k

parallel to the vector v

parallel to the vector v = 4i + 0j – 7k.

4(x – 3) + 2 (y + 1) – 5 (z – 7)

4(x – 3) + 2 (y + 1) – 5 (z – 7) = 0

4x + 2y – 5z + 25

4x + 2y – 5z + 25 = 0

v

v = (x2 x1)i + (y2 y1)j

(x, y)

(x, y) = ax + by

= ai + bj = n

is ax + by + c

is ax + by + c = 0

ax1 + by1 + c

ax1 + by1 + c = 0

ax2 + by2 + c

ax2 + by2 + c = 0

a(x2 x1) + b (y2 y 1)

a(x2 x1) + b (y2 y 1) = 0

ax + by + cz + d

ax + by + cz + d = 0

ax1 + by1 + cz1 + d

ax1 + by1 + cz1 + d = 0

a x2 + by2 + cz2 + d

a x2 + by2 + cz2 + d = 0

= ax + by + cz

= ai + bj + ck

fx (P)

fx (P) = 36, fy (P) = 24, fz (P) = -12

36(x – 2) + 24(y – 3) –12(z – 6)

36(x – 2) + 24(y – 3) –12(z – 6) = 0

3x + 2y – z – 6

3x + 2y – z – 6 = 0

f z(0,0, 0)

f z(0,0, 0) = - 1

f x (0,0,0)(x 0)+f y (0,0,0)(y 0) +

f x (0,0,0)(x 0)+f y (0,0,0)(y 0) + f z (0,0,0)(z 0)=0

z

z = z0+fz(P 0)t

f(x,y,z)

f(x,y,z) = 4x 2 y 2 + 3z 2 10

16(x 2) + 6 (y+3) + 6 (z 1)

16(x 2) + 6 (y+3) + 6 (z 1) = 0

8x + 3y + 3z

8x + 3y + 3z = 10

16

16 = y + 3

8

8 = y + 3

2 (2)6 (4)-2

2 (2)6 (4)-2 = 14

fz (2, 4, 4)

fz (2, 4, 4) =-1

fx (2,4,4)(x 2)+ f y (2,4,4)(y 4)+ fz (2,4,4)(z

fx (2,4,4)(x 2)+ f y (2,4,4)(y 4)+ fz (2,4,4)(z 4) = 0

14 (x 2) + (2) (y 4) (z 4)

14 (x 2) + (2) (y 4) (z 4) = 0

g (x, y, z)

g (x, y, z) = 0 -----(2)

f x g x + f y gy + fzgz

f x g x + f y gy + fzgz = 0

/(x0)

/(x0) = f(x0)(x x0) or

y

y = f(x 0) + f

y

y = x + x x

y+ y

y+ y = cos (x+x) = cos (x+dx)

= cos (60+1)=cos61

cos61

cos61 = y+ y = cosx + y

df

df = fx (x 0 , y 0) dx + fy (x0, y0) dy

dV

dV = V xd x + Vydy + Vzdx

dV

dV = yzdx + xzdy + zydz (1)

dA

dA = A x dx + A y dy = ydx + xdy

f (x, y)

f (x, y) = x3 + y3 3axy, a > 0.

f (x , y)

f (x , y) = x 2 + y2

z

z = f(x, y) = x 2 + y2 (Paraboloid)

f(x,y)

f(x,y) = 2x4 + y2 x2 2y

2 , f xx (1,1)

2 , f xx (1,1) = 12 < 0

2

2 = 128 > 0

2 , f xx (1, 1)

2 , f xx (1, 1) = 12 < 0

2

2 = 128 > 0

f /(x)

f /(x) =3 x2+2x-1

(3x 1) (x + 1)

(3x 1) (x + 1) = 0

f (x)

f (x) = (x 2) 2/3

h(x)

h(x) = x 2/3 on [2, 3].

f(x,y)

f(x,y) = 2 + 2x +2y-x2-y2

U(x)

U(x) = f(x, 0)=2+2x-x2

U/(x)

U/(x) = 2-2x=0 Then x=1

V(y)

V(y)=f(0,y)= 2+2y-y2

With y

With y = 9 - x, we have

f (x, y)

f (x, y) = 2+2x+2(9 - x)-x2-(9- x)2

At this value of x, y

At this value of x, y = 9 – 9/2

f(x,y)

f(x,y)=3xy-6x-3y+7

f(x, y)

f(x, y) = 3xy 6x 3y + 7

v(y)

v(y) =f (0, y) = 3y + 7, 0

w (x)

w (x) = 10 x + 14

10x + 14

10x + 14 = 0

f(x,y)

f(x,y)=xy(48-x-y)

fy

fy=48x-2xy-x2

f xx (16, 16)

f xx (16, 16) = - 32 < 0

f yy (16, 16)

f yy (16, 16) = - 32

2

2 = 768 > 0

f(x,y)

f(x,y) = x 2 + y 2 + z 2

C

C=10(xy+xy)+5(xz+xz)+5(yz+yz)----------(1)

C

C=20xy+10(x+y)z

xyz

xyz=16----------(2)

C

C = 20xy + 10 (x + y) 16

C

C = 20 xy + 160

C x

C x =20y - 160

8

8 = 40 > 0

fxx (1, 2, 2)

fxx (1, 2, 2) = 24 < 0

= 320 > 0

y

y = 2x Or x = y/2

R

R = {(x,y):0< x < 1, 0 < y < 2}

z

z=4-4x-2y ----------(1)

y + z

y + z = 4 and z = 0.

the y-axis, and the line y

the y-axis, and the line y = 2 2x [set

y

y= - 4 - x 2

= 6 (27) + 8

y

y = r sin

P(x, y)

P(x, y) =P(r,)

x

x = r cos

y

y = r sin

x2 + y2

x2 + y2 = r2

y/ x

y/ x =tan

x

x = r cos a = r cos --------(1)

r (Acos + B sin) + C

r (Acos + B sin) + C = 0

r

r=a+b sin , r = absin

r

r=a+b cos , r = abcos

r

r = a (1 – cos)

r

r = a (1 + cos)

r

r = a (1 + sin)

r

r = a (1 – sin)

through the origin when

through the origin when = /4, so the line = /4 is

(> 0) or r

(> 0) or r = a

Sketch the curve r

Sketch the curve r = (> 0) in polar coordinates.

x-axis and the curve y

x-axis and the curve y = 1 x2

ex2+y2dy dx

ex2+y2dy dx =

e-(x2+y2)dxdy

e-(x2+y2)dxdy = lim

-1 432x dx

-1 432x dx = 216x2]

0 (x 2y + z) dz dy dx

0 (x 2y + z) dz dy dx=

0 (x 2x3 + x5) dx

0 (x 2x3 + x5) dx= 1

r (t)

r (t) = (x (t), y (t)) = x (t) i + y (t) j

r (t)

r (t) = (x (t), y (t), z (t)) = x (t) i + y(t)j + z (t)k

x

x = x(t), y = y (t) ---------(1)

x i + y j

x i + y j = x (t)i + y (t) j ---------(2)

x

x = x(t), y = y (t), z = z (t) ---------(3)

xi + yj + zk

xi + yj + zk = x(t)i + y(t)j + z(t)k ---------(4)

if we let r

if we let r = x i +y j and

r(t)

r(t) = x(t) i + y(t) j in 2-sapce

x i + y j

x i + y j = t2 i + 3t j

xi + yj + zk

xi + yj + zk = (cos t)i + (sin t)j + tk

x i + y j + z k

x i + y j + z k = (t3 + 1) i + 3 j + et k

x

x = t3 + 1, y = 3, z = et

xi+yj

xi+yj = (cos t)i + (sin t)j, 0 < t < 2

x

x = cos t, y = sin t (0 < t < 2)

xi + yj + zk

xi + yj + zk = (cost)i + (sint)j + 2k, 0 < t < 2

x

x=cos t, y = sin t, z=2 (0 < t < 2)

The corresponding parametric equations are x

The corresponding parametric equations are x = 2 + t, y = 3t, z = 5 4t

If c

If c = x0i + y0j (in 2-space), then the graph is the point (x0, y0), and if

= 1 + 4 = 5

and for a function r(t)

and for a function r(t) = x(t)i + y(t)j + z(t)k

r/(t)

r/(t) = lim

differentiable, then r/(t)

differentiable, then r/(t) = x/(t)i + y/(t)j

r/(t)

r/(t) = x/(t)i + y/(t)j + z/(t)k

r/(t)

r/(t) = lim

= x/(t)i + y/(t)j

Let r(t)

Let r(t) = t2i +t3j. Find r/(t) and r/(1)

r/(t)

r/(t) = d

= 2t i + 3t2 j

r/(1)

r/(1) = 2i+3j.

r

r = r (t0) + t r/(t0)

xi + yj + zk

xi + yj + zk = (cost)i + (sin t)j + tk

Thus, r(t)

Thus, r(t) = (cos t)i + (sin t)j + tk

r/(t)

r/(t) = (sin t)i + (cos t)j + k

r/(t)

r/(t) = 2ti + 3t2j

of the tangent line at this point is r

of the tangent line at this point is r = r(1) + t r/(1)

r

r = (i + j) + t (2i + 3j)

dt [r1(t) + r2(t)]

dt [r1(t) + r2(t)] = d

r(t). r/(t)

r(t). r/(t) = 0

[r1(t)+r2(t)] dt

[r1(t)+r2(t)] dt = r1(t)dt + r2(t) dt (4)

r(t) dt

r(t) dt = R(t) + C

r

r = (3 cos t)i + (3 sin t)j, 0 < t < 2 -----(1)

x

x = x(t), y = y(t) (a < t < b)

x

x = x(t), y = y(t), z = z(t) (a < t < b)

and in 3-space by x

and in 3-space by x = x(s), y = y(s), z = z (s)

x

x = 2t + 1, y = 3t 2

x

x = 2u + 1, y = 3u 2

13s + 1

13s + 1 = 2

t0 (a sin u)2 + (a cos u)2 du

t0 (a sin u)2 + (a cos u)2 du =

1 9t4 + 1 + 6t2 dt

1 9t4 + 1 + 6t2 dt =

= /t3 + t/

dt

dt = eti 4e-tj

r

r = eui + 4e-u2j

du

du = 2u eu2i 8ue-u2j

1. z

1. z = x2 + y2

4. z

4. z = x2 + 2y2 + 3w2

1. dz

1. dz = 2 (x dx + y dy)

2. dz

2. dz = x2 (3 sin 2y dx + 2x cos 2y dy)

3. dz

3. dz = e3y {2dx + (6x 3) dy}

4. dz

4. dz = 2 (xdx + 2ydy + 3wdw)

5. dz

5. dz = x2y (3ywdx + 2xwdy + xydw)

dz

dz = (3x2 + 4y2) dx + 8xy dy.

P

P = 3x2+4y2 P

dz

dz = (1 + 8xy) dx + 5x2 dy.

for dz

for dz = (1 + 8xy) dx + 5x2 dy

P

P = 1 + 8xy P

1. dz

1. dz = 4x3y3dx + 3x4y2 dy

2. dz

2. dz = (4x3y+2xy3) dx+(x4+3x2y2) dy

3. dz

3. dz=(15y2e3x+2xy2)dx+(10ye3x+x2y)dy

4. dz

4. dz=(3x2e2y2y2e3x)dx+(2x3e2y2ye3x)dy

5. dz

5. dz=(4y3cos4x+3x2cos2y)dx+(3y2sin4x2x3 sin 2y) dy.

dz

dz = Pdx+Qdy where P=z

dz

dz = (2xy + 6x) dx + (x2 + 2y3) dy.

x

x = 2xy + 6x z= (2xy+6x)dx

z

z = x2y + y4

z

z = x2y + 3x2 + f(y)

z

z = x2y + y4

z

z = x2y + 3x2 + y4

dz

dz = (8e4x + 2xy2) dx+(4 cos4y+2x2y) dy

= 8e4x + 2xy2

z

z = (8e4x + 2xy2) dx

z

z = 2e4x + x2y2 + f(y) (i)

y

y = 4 cos 4y + 2x2y

z

z = (4cos 4y + 2x2y) dy

z

z = sin 4y + x2y2 + F(x)

z

z = 2e4x + x2y2 + sin 4y

y

y = 4x for x > 0.

AB Ft ds

AB Ft ds = AB (P dx + Qdy)

along the curve y

along the curve y = 1 + x2.

is the curve y

is the curve y = 1 + x2.

C (Pdx+Qdy)

C (Pdx+Qdy) =

Also y

Also y = 2 + x

0 {9x2dx+32x4dx}

0 {9x2dx+32x4dx}= 47

0 {(1 + 2y) (0) + ydy}

0 {(1 + 2y) (0) + ydy} =

3 and Ic1+c2

3 and Ic1+c2 = 81

AB {P dx+Q dy}

AB {P dx+Q dy} =

3. (a) For a path of integration parallel to the

3. (a) For a path of integration parallel to the y-axis, i.e. x = k, dx = 0

Ic

Ic = IAB = IAK + IKB.

C (x + y) dx from A(0, 1) to B (0, 1) along the

C (x + y) dx from A(0, 1) to B (0, 1) along the semi-circle x2+y2=1

C (Pdx + Qdy) and in this particular case, Q

C (Pdx + Qdy) and in this particular case, Q = 0

0 (x + 1 x2 x+ 1 x2 dx

0 (x + 1 x2 x+ 1 x2 dx = 2

0 (1+cos2)d

0 (1+cos2)d=

Finally, I

Finally, I = I1+I2+I3 = 1

3 + 0

3 + 0 = 1

y dx when c is the circle x2+y2

y dx when c is the circle x2+y2 = 4.

-2 4 x2 dx

-2 4 x2 dx = 4

I

I = 0 {xydx + (1 + y2) dy}

0 (1+y2)dy

0 (1+y2)dy =

2 (1+y2) dy

2 (1+y2) dy=

x

x=0 (4x+3xy)ds =

0 (4x+3xy)(5) dx. But y

0 (4x+3xy)(5) dx. But y = 2x

0 (4x+6x2)(5) dx

0 (4x+6x2)(5) dx = 2 5

0 (2x+3x2) dx

0 (2x+3x2) dx = 4 5

= cos2t+sin2t =1 ds = dt

0 {3x2x4dx+2x3x22xdx}

0 {3x2x4dx+2x3x22xdx}=

I3

I3 = 0+128 = 128

x

x = 3y z = 3ydz=3xy+f(y) -------- (i)

Hence z

Hence z = 3xy + y2

C{3ydx + (3x+2y)dy}

C{3ydx + (3x+2y)dy}=

(1,2) d(3xy+y2)

(1,2) d(3xy+y2)=[3xy+y2]

x

x =x2+yex z =x3

z

z = yex + y2

Ic1 + Ic2

Ic1 + Ic2 = 0

If (P dx + Q dy) is an exact differential, 0 (

If (P dx + Q dy) is an exact differential, 0 (P dx + Q dy) = 0

dz

dz = Pdx + Q dy + R dw is an exact differential of z = f(x, y, w)

z

z = (x3y+1)dw = y3yw+w+f(y)+g(x)

z

z = x3yw + 3x2 4y2 + w

I

I = [x3yw + 3x24y2+w]

I

I = [x3yw + 3x24y2+w]

x2 + 9y2

x2 + 9y2 = 16.

y

y = 1 and Q = 2y + x Q

x2+9y2

x2+9y2 = 16 i.e. x2

P

P = 2x + y

C {Pdx+ Qdy}

C {Pdx+ Qdy}=

x

x = 0 for x > 0.

1 {3x3 dx (x3 + 1) dx}

1 {3x3 dx (x3 + 1) dx} =

2 + 0

2 + 0 = 11

Path of integration parallel to y-axis, dx

Path of integration parallel to y-axis, dx = 0 Ic =

Path of integration parallel to x-axis, dy

Path of integration parallel to x-axis, dy = 0 Ic =

C (P dx+Q dy)

C (P dx+Q dy)=

If A

If A = a1i + a2j + a3k , then

= 2xy xz + 2yz

If A

If A= a1i + a2j + a3k then curl A=A.

F

F = i{2 2x cos y} j{1 e2xy}+ k ({2z cos y 2xze2xy}

(e) With a vector function A

(e) With a vector function A = ax i + ay j + az k

= 2xy + z3 -- x3 = (say)

zk

zk = (2y -- 3x2)i+(2x)j+(3z2)k

= xyz 2y2z + x2z2

z k

z k =(yz+2xz2)i + (xz4yz)j + (xy2y2+2x2z) k

= (2yz2xyz)i+x2yj+(yz2x2z)k

= z2i(2xz 2y+2xy)j+(2xy2z+2xz)k

= i 0 j 0+ k 0= 0

A

A = axi + ayj + azk

= x(-cosx)-(-cosx)(1)dx= -x(cosx) +cosx d x = -x(cosx)+sin x

r + dr, then PQ

r + dr, then PQ = r

integrand is expressed in terms of x, y, z with d

integrand is expressed in terms of x, y, z with dr = dx i + dy j + dz k

integrand is expressed in terms of x, y, z with d

integrand is expressed in terms of x, y, z with dr =dx i + dy j + dz k

dr

dr= dxi+ dy j+ dz k dr= 3du i +4udu j +3u2du k

12u (3 +4u +3u)du

12u (3 +4u +3u)du= 36

z

z= t+5 between A(0,0,5) and B(4,4,7) . As before, expressing V and dr in term of the

= (t2)(2t)+(4t2)(t+5)

= 6t3 + 20t2.

z

z = t+5 dz = dt }

dr

dr = dxi + dy j + dzk

= 2t dt i + 2 dt j + dtk

F

F= F1 i + F2 j +F3 k

And dr

And dr = dx i +dy j +dz k

If F(r)

If F(r) = x2y i + xz j + 2yz k , Evaluate

F

F = 32 t 4 i + 4 t4 j – 4 t5 k

dr

dr = 4dt i + 4t dt j + 3t2 k

= (128t4+16t5 + 12t7) dt

If F(r)

If F(r) = x2y i + 2yzj + 3z2x k

F. d r

F. d r = (x2y i + 2yzj + 3z2xk).

F. d r

F. d r = x2y dx + 2yz dy + 3z 2x dz

F.d r

F.d r = x 2ydx + 2yzdy+3z 2xdz

F. d r

F. d r=0+0+0 = 0

F.dr

F.dr = 0 + 0 + 0 = 0

F. dr

F. dr = 0 + 0 +

F

F = 2t 3 i + 12t2 j + 27t3 k

d r

d r = dxi +dyj +kdz = dt i + 2dt j +3dt k

0 (83t3 + 24t 2) dt

0 (83t3 + 24t 2) dt =

when x

when x = 0, y+z=2 i.e. z = 2 y

So 2x + y + z

So 2x + y + z = 2 cuts the axes at

z

z=2(1 - x) - y

v FdV

v FdV =3 (2i + k)

y

y = 1/ 1+x2

Good Luck!

Keep practicing!