

MTH301

Calculus II

Formula Sheet

1 Chapters, 945 Formulas

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The area of a circle depends on its radius r by th

The area of a circle depends on its radius r by the equation $A = r^2$ so, we say that

$$f(xy, y/x, xz)$$

$$f(xy, y/x, xz) = (xy)(y/x)(xz) + xy = xy z + xy.$$

er the function $g(x,)$

er the function $g(x,) = y \sin(x^2 y)$ and $u(x, y) = x^2 y^3$ $v(x, y) = xy$

Then

Example: Sketch the parabola y

Example: Sketch the parabola $y = -x^2 + 4x$

Graph of y

Graph of $y = -x^2 + 4x$

Example y

Example $y = x^2 - 4x + 3$

Graph of y

Graph of $y = x^2 - 4x + 3$

$$= (z_1 + z_2)/2$$

$$= (9/2, 6, 3/2)$$

$|AC|$

$$|AC| = (9 - 3)^2 + (4 - 2)^2 + (1 - 4)^2$$

$|BC|$

$$|BC| = (9 - 6)^2 + (4 - 10)^2 + (1 + 1)^2$$

$\cos$

$$\cos = x/r$$

$\cos$

$$\cos = y/r$$

$ax + by + cz + d$

$$ax + by + cz + d = 0$$

$$x^2 + y^2$$

$$x^2 + y^2 = r^2$$

$$y/x$$

$$y/x = \tan$$

$$P(x, y)$$

$$P(x, y) = P(r, \theta)$$

$$r$$

$$r = \sqrt{x^2 + y^2}, \tan \theta = y/x$$

$$x$$

$$x = r \cos \theta, y = r \sin \theta, z = z$$

$$x^2 + y^2 + z^2$$

$$x^2 + y^2 + z^2 = (r \cos \theta)^2 + (r \sin \theta)^2 + (z)^2$$

$$r$$

$$r = \sqrt{x^2 + y^2}, \theta = \tan^{-1}(y/x), z = z$$

$$\sin \theta,$$

$$\sin \theta = y/r, z = z$$

$$r^2 + z^2$$

$$r^2 + z^2 = \rho^2, \tan \phi = r/z$$

$$= 2\{\sin^2(\cos^2 + \sin^2) + \cos^2\}$$

$$f(x, y)$$

$$f(x, y) = \sin^{-1}(x+y)$$

$$f(x, y)$$

$$f(x, y) = x^2 + y^2 - 4$$

$$f(x, y)$$

$$f(x, y) = x^3 + 2x^2y - xy - 2y^2$$

$$f(x, y)$$

$$f(x, y) = 4 - x^2$$

$$x^2 + y^2$$

$$x^2 + y^2 = 0 \text{ (along } y = 0)$$

$$x^2 + y^2$$

$$x^2 + y^2 = 0.5 \text{ (along } y = x \text{)}$$

$$x^2 + y^2$$

$$x^2 + y^2 = x \cdot x$$

$$z$$

$$z = 0, x \neq 0.$$

$$x$$

$$x = r \cos, y = r \sin$$

$$x^2 + y^2$$

$$x^2 + y^2 = \lim$$

= (Along y-axis). Now we will find out the limit of the

$$(x, y)$$

$$(x, y) = \ln(2x - y + 1)$$

$$f_y(x, y)$$

$$f_y(x, y) = 4x^3y + 2$$

$$(x, y)$$

$$(x, y) = 6x^2y^2 + 4$$

$$z$$

$$z = 4x^2 - 2y + 7x^4y^5$$

$$z$$

$$z = \ln(x^2 + y^2) - \ln(x + y)$$

$$z$$

$$z = y^2 + 2xy - x^2$$

$$x$$

$$x = x^2 + y^2$$

$$= 2x^2 + 2xy - x^2 - y^2$$

$$= x^2 + 2xy - y^2$$

x

$$x = 2x - yz$$

dy

$$dy = 6y - xz$$

dz

$$dz = 8z - xy$$

w

$$w = x^2 + 3y^2 + 4z^2 - xyz$$

z

$$z = f(x + x, y) - f(x, y)$$

z

$$z = f(x, y + y) - f(x, y)$$

x

$$x = 2f$$

$x(f_x)$

$$x(f_x) = (f_x)x = f_{xx} = f_x$$

y

$$y = 2f$$

y^2

$$y^2 =$$

$y(f_y)$

$$y(f_y) = (f_y)y = f_{yy} = f_y$$

$y \ x$

$$y \ x =$$

x

$$x = -1$$

x

$$x = 2f$$

$$y_x$$

$$y_x =$$

$$y(f_x)$$

$$y(f_x) = (f_x)y = f_{xy}$$

$$y$$

$$y = 2f$$

$$x(f_y)$$

$$x(f_y) = (f_y)x = f_{yx}$$

$$x y$$

$$x y =$$

$$= -y^2 + x^2 - x^2$$

$$x y$$

$$x y = 2z$$

$$f(x, y)$$

$$f(x, y) = x \cos y + y e^x$$

$$= -x \sin y + e^x$$

$$= -\sin y + e^x$$

$$y^2$$

$$y^2 =$$

$$= -x \cos y$$

$$z^2$$

$$z^2 = 0$$

$$x$$

$$x = e^x \sin y - e^y \sin x$$

$$x^2$$

$$x^2 = e^x \sin y - e^y \cos x$$

$$y^2$$

$$y^2 = e^x \sin y - e^y \cos x$$

$$= -1440(4x - 3y + 2z)^2$$

w

$$w = \sin t + 4$$

dt

$$dt = 1 \cdot \times \text{Cost} = \text{Cost}$$

z

$$z = 9xy$$

$$= (6xy^3)(4t^3) + 9x^2y^2(2t)$$

$$= 6(t^4)(t^6)(4t^3) + 9(t^8)(t^4)(2t)$$

z

$$z = \ln(2x^2 + y)$$

y

$$y = 2x$$

w

$$w = x^2 + y + z + 4$$

x

$$x = e, y = \cos t, z = t + 4$$

x

$$x = 2x, w$$

y

$$y = 1, w$$

$$= (2x)(et) + (1) \cdot (-\sin t) + (1)(1)$$

$$= 2(et)(et) - \sin t + 1$$

$$= 2e^{2t} - \sin t + 1$$

w

$$w = \sin x + x^2, x = 3r + 4s$$

dx

$$dx = \cos x + 2x$$

r

$$\begin{aligned} r &= 3x \\ &= (\cos x + 2x) \cdot 3 \\ &= 3 \cos(3r+4s) + 6(3r+4s) \\ &= (\cos x + 2x) \cdot 4 \\ &= 4 \cos x + 8x \\ &= 4 \cos(3r+4s) + 24r + 32s \end{aligned}$$

$$= x$$

r

$$r = xi + yj + zk$$

Let a

$$\text{Let } a = a_1i + a_2j + a_3k$$

and b

$$\text{and } b = b_1i + b_2j + b_3k$$

$a + b$

$$\begin{aligned} a + b &= (a_1i + a_2j + a_3k) + (b_1i + b_2j + b_3k) \\ &= (a_1 + b_1)i + (a_2 + b_2)j + (a_3 + b_3)k \end{aligned}$$

$a - b$

$$\begin{aligned} a - b &= (a_1i + a_2j + a_3k) - (b_1i + b_2j + b_3k) \\ &= (a_1 - b_1)i + (a_2 - b_2)j + (a_3 - b_3)k \end{aligned}$$

$a \cdot b$

$$a \cdot b = |a| |b| \cos \theta$$

$a \cdot b$

$$\begin{aligned} a \cdot b &= |a| |b| \cos \theta \\ &= |a| |b| \cos \theta \end{aligned}$$

$$= (3)(2) \cos$$

a.b

$$a.b = (1)(6) + (-2)(3) + (-2)(2)$$

$$= \cos - 1$$

a

$$a = a_1 i + a_2$$

$$= \cos - 1$$

$$= a_1 b_1 i + a_1 b_2$$

$$= a_1 b_1 + a_1 b + a$$

OC/

$$OC/ = / / \cos$$

$$= |b| / |a| \cos$$

since $|b| \cos$

$$\text{since } |b| \cos = B \cdot a$$

Vector Projection of b

$$\text{Vector Projection of } b = 6i + 3j + 2k$$

onto a

$$\text{onto } a = i - 2j - 2k \text{ is}$$

$|b| \cos$

$$|b| \cos = b \cdot |a|$$

$$= (6i + 3j + 2k).$$

$a \times b$

$$a \times b = (|a| |b| \sin$$

$\times b$

$$\times b = |a| |b| \sin$$

$$\text{en } a \times b$$

$$\text{en } a \times b = 0$$

$$\times i$$

$$\times i = j \times j = k \times k = 0$$

$$\text{en } a \times b$$

$$\text{en } a \times b = |a| |b| \wedge n$$

$$i \times j$$

$$i \times j = k, j \times i = -k$$

$$j \times k$$

$$j \times k = i, k \times j = -i$$

$$k \times i$$

$$k \times i = j, i \times k = -j$$

$$\times b|$$

$$\times b| = |a| |b| |\sin$$

$$|a$$

$$|a = |A| |b| \sin$$

$$a$$

$$a = 2i + j + k \text{ and}$$

$$b$$

$$b = -4i + 3j + k. \text{ Then}$$

$$= b_1 i + b_2$$

$$b$$

$$b = (a_1 i + a_2$$

$$s / (a \times b).c /$$

$$s / (a \times b).c / = |a \times b| |c| |\cos /$$

$$\text{product is commutative, } (a \times b).c$$

$$\text{product is commutative, } (a \times b).c = a.(b \times c)$$

$$(a * b).c$$

$$(a * b).c = (b \times c).a = (c \times a).b$$

b

$$b = b_1i + b_2j + b_3k.$$

c

$$c = c_1i + c_2j + c_3k.$$

$$a \cdot (b \times c)$$

$$a \cdot (b \times c) = a.$$

$$= -21 - 16 + 14$$

$$|a \cdot (b \times c)|$$

$$|a \cdot (b \times c)| = 23.$$

grand

$$\text{grand} =$$

grad

$$\text{grad} =$$

$$D u f(x_0, y_0)$$

$$D u f(x_0, y_0) = f_x(x_0, y_0)u_1 + f_y(x_0, y_0)u_2$$

a

$$a = 3i + 4$$

$$D u f(x, y)$$

$$D u f(x, y) = f(x, y) \cdot \hat{u}$$

$$D u f(x, y)$$

$$D u f(x, y) = f(x, y) \cdot \hat{u}$$

u

$$u = u_1i + u_2j$$

u_1

$$u_1 = \cos$$

u_2

$$u_2 = \sin$$

$$D u f(x_0, y_0)$$

$$D u f(x_0, y_0) = f_x(x_0, y_0)u_1 + f_y(x_0, y_0)u_2$$

$$Duf(-2, 0)$$

$$Duf(-2, 0) = f_x(-2, 0) \cos$$

$$f(x, y)$$

$$f(x, y) = f_x(x, y)i + f_y(x, y)j$$

$$4^2 + 3^2$$

$$4^2 + 3^2 = 5$$

$$Duf(5, 5)$$

$$Duf(5, 5) = f \cdot u$$

$$f(2, 0)$$

$$f(2, 0) = x(2, 0)i + y(2, 0)j$$

$$f(x, y)$$

$$f(x, y) = xe^{y+\cos(xy)} \text{ at the point}$$

$$uf$$

$$uf = f \cdot u = |f|$$

$$)$$

$$) = -|f|.$$

$$Duf$$

$$Duf = |f| \cos(\pi/2) = |f| \cdot 0 = 0$$

$$(f)(1, 1)$$

$$(f)(1, 1) = (xi + y$$

$$j)(1, 1)$$

$$j)(1, 1) = i + j.$$

$$Ax + By + C$$

$$Ax + By + C = 0$$

$$y$$

$$y = mx + c$$

$$x$$

$$x = x_0 + at, y = y_0 + bt$$

z

$$z = z_0 + ct$$

and a

$$\text{and } a = 4, b = 0, c = -7.$$

or v

$$\text{or } v = 4i + 0j - 7k.$$

$z + y^2$

$$z + y^2 = 9, -3 \leq x \leq 3, 0 \leq y \leq 3$$

x

$$x = 3 \cos, y = 3 \sin$$

y^2

$$y^2 = 9 \cos^2 + 9 \sin^2$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0)$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$ax + by + cz - ax_0 - by_0 - cz_0$$

$$ax + by + cz - ax_0 - by_0 - cz_0 = 0$$

$$ax + by + cz + d$$

$$ax + by + cz + d = 0$$

ector n

$$\text{ector } n = 4i + 2j - 5k.$$

uation of plane $ax + by + cz + d$

$$\text{uation of plane } ax + by + cz + d = 0$$

$$+ b(y - y_0) + c(z - z_0)$$

$$+ b(y - y_0) + c(z - z_0) = 0$$

is $ax + by + c$

$$\text{is } ax + by + c = 0$$

$$(x_2 - x_1) + b(y_2 - y_1)$$

$$(x_2 - x_1) + b(y_2 - y_1) = 0$$

$$ax_1 + by_1 + c$$

$$ax_1 + by_1 + c = 0$$

$$ax_2 + by_2 + c$$

$$ax_2 + by_2 + c = 0$$

$$x + by + cz + d$$

$$x + by + cz + d = 0$$

$$x_1 + by_1 + cz_1 + d$$

$$x_1 + by_1 + cz_1 + d = 0$$

$$y_2 + cz_2 + d$$

$$y_2 + cz_2 + d = 0$$

$$= (x_2 - x_1)i + (y_2 - y_1)j$$

$$(x, y)$$

$$(x, y) = ax + by$$

$$x$$

$$x = a, y = b$$

$$= ai + bj = n$$

$$= ax + by + cz$$

$$= ai + bj + ck$$

$$x$$

$$x = g(t), y = h(t), r = g(t)i + h(t)j$$

$$f_x(P)$$

$$f_x(P) = 36, f_y(P) = 24, f_z(P) = -12$$

$$36(x - 2) + 24(y - 3) - 12(z - 6)$$

$$36(x - 2) + 24(y - 3) - 12(z - 6) = 0$$

$$x + 2y - z - 6$$

$$x + 2y - z - 6 = 0$$

$$f_z(0, 0, 0)$$

$$f_z(0, 0, 0) = -1$$

$$f_x(0, 0, 0)(x - 0) + f_y(0, 0, 0)(y - 0) + f_z(0, 0, 0)(z - 0) = 0$$

z

$$z = z_0 + f_z(P_0)t$$

$$x - 2y + 3z = 14$$

$$x - 2y + 3z - 14 = 0$$

$$2(x - 1) - 4(y + 2) + 6(z - 3)$$

$$2(x - 1) - 4(y + 2) + 6(z - 3) = 0$$

$$f(x, y, z)$$

$$f(x, y, z) = 4x^2 - y^2 + 3z^2 - 10$$

$$16(x - 2) + 6(y + 3) + 6(z - 1)$$

$$16(x - 2) + 6(y + 3) + 6(z - 1) = 0$$

$$8x + 3y + 3z$$

$$8x + 3y + 3z = 10$$

$$8$$

$$8 = y + 3$$

$$f_z(2, 4, 4)$$

$$f_z(2, 4, 4) = -1$$

$$f_x(2, 4, 4)(x - 2) + f_y(2, 4, 4)(y - 4) + f_z(2, 4, 4)(z - 4)$$

$$f_x(2, 4, 4)(x - 2) + f_y(2, 4, 4)(y - 4) + f_z(2, 4, 4)(z - 4) = 0$$

$$896(x - 2) + (-2)(y - 4) - (z - 4)$$

$$896(x - 2) + (-2)(y - 4) - (z - 4) = 0$$

$$(x, y, z)$$

$$(x, y, z) = x^2 + y^2 - 638(2)$$

$$2 + y^2$$

$$2 + y^2 = 63$$

$$g(x, y, z)$$

$$g(x, y, z) = 0 \text{ -----}(2)$$

$$f_x g_x + f_y g_y + f_z g_z$$

$$f_x g_x + f_y g_y + f_z g_z = 0$$

$$x$$

$$x = 3 \quad y = 3$$

$$y$$

$$y = x + x - x$$

$$= 7 - 4 \approx .65$$

$$y \approx dy$$

$$y \approx dy = -\sin x \, dx = -\sin 60^\circ (1^\circ)$$

$$y + y$$

$$y + y = \cos(x+x) = \cos(x+dx)$$

$$= \cos(60^\circ + 1^\circ) = \cos 61^\circ$$

$$\cos 61^\circ$$

$$\cos 61^\circ = y + y = \cos x + y$$

$$df$$

$$df = f_x(x_0, y_0) \, dx + f_y(x_0, y_0) \, dy$$

$$dA$$

$$dA = A_x \, dx + A_y \, dy = y \, dx + x \, dy$$

$$dV$$

$$dV = V_x \, dx + V_y \, dy + V_z \, dz$$

$$dV$$

$$dV = yz \, dx + xz \, dy + zy \, dz \quad (1)$$

$$\times 10$$

$$\times 10 = 3.25\%$$

$$f(x, y)$$

$$f(x, y) = x + y - 3axy, \quad a > 0.$$

$$f_x(x, y)$$

$$f_x(x, y) = x^2 + y^2$$

z

$$z = f(x, y) = x^2 + y^2 \text{ (Paraboloid)}$$

$$f(x, y)$$

$$f(x, y) = 2x^2 - 4x + xy^2 - 1$$

$$f_x(x, y)$$

$$f_x(x, y) = 4x - 4 + y$$

D

$$D = f_{xx}(0, 2) \cdot f_{yy}(0, 2) - [f_{xy}(0, 2)]^2$$

$$= -16 < 0.$$

D

$$D = f_{xx}(0, -2) \cdot f_{yy}(0, -2) - [f_{xy}(0, -2)]^2$$

$$= -16 < 0.$$

$$f(x, y)$$

$$f(x, y) = e^{-(x^2 + y^2 + 2x)}$$

$$f_x(x, y)$$

$$f_x(x, y) = -2(x+1)e$$

$$f_x(x, y)$$

$$f_x(x, y) = 0, x + 1 = 0, x = -1 \text{ and}$$

$$= (-2e)(-2e) > 0$$

$$f(x, y)$$

$$f(x, y) = 2x^4 + y^2 - x^2 - 2y$$

2, 1

$$2, 1 = 4 > 0, \text{ so } f \text{ is minimum at}$$

$$2, f_{xx}(1, 1)$$

$$2, f_{xx}(1, 1) = -12 < 0$$

2

$$2 = 128 > 0$$

$$2, f_{xx}(-1, -1)$$

$$2, f_{xx}(-1, -1) = -12 < 0$$

2

$$2 = 128 > 0$$

$$f'(x)$$

$$f'(x) = 3x^2 + 2x - 1$$

$$f(x)$$

$$f(x) = x^3 + x^2 - x + 1 \text{ on } [-2, 1/2]$$

$$-1)(x+1)$$

$$-1)(x+1) = 0$$

$$h(x)$$

$$h(x) = x^{2/3}$$

$$f(x, y)$$

$$f(x, y) = 2 + 2x + 2y - x^2 - y^2$$

$$U(x)$$

$$U(x) = f(x, 0) = 2 + 2x - x^2$$

$$V(y)$$

$$V(y) = f(0, y) = 2 + 2y - y^2$$

$$U'(x)$$

$$U'(x) = 2 - 2x = 0 \text{ The}$$

$$, y)$$

$$, y) = 2 + 2x + 2(9 - x)$$

$$(x)$$

$$(x) = f(x, 9 - x) = -61 + 18x$$

$$f(x, y)$$

$$f(x, y) = 3xy - 6x - 3y + 7$$

$$v(y)$$

$$v(y) = f(0, y) = -3y + 7, 0$$

$$0x + 14$$

$$0x + 14 = 0$$

$$w'(x)$$

$$w'(x) = -10x + 14$$

$$f(x, y)$$

$$f(x, y) = xy(48 - x - y)$$

$$f_x$$

$$f_x = 48y - 2xy - y^2$$

$$f$$

$$f = 48x - 2xy - x^2$$

$$f_{xx}(16, 16)$$

$$f_{xx}(16, 16) = -32 < 0$$

$$f_{yy}(16, 16)$$

$$f_{yy}(16, 16) = -32$$

$$z$$

$$z = 768 > 0$$

$$f(x, y)$$

$$f(x, y) = x^2 + y^2 + z^2$$

$$C$$

$$C = 10(xy + xy) + 5(xz + xz) + 5(yz + yz) \text{-----} (1)$$

$$C$$

$$C = 20xy + 10(x + y)z$$

$$xyz$$

$$xyz = 16 \text{-----} (2)$$

$$C$$

$$C = 20xy + 10(x + y)xy$$

C

$$C = 20xy + y$$

C x

$$C_x = 20y - 160$$

8

$$8 = 40 > 0$$

have z

have $z = 4$, so when its dimensions are $2 \times 2 \times 4$.

e $f(x, y, z)$

e $f(x, y, z) = xyz$ and we are given the plane $x + y + z = 5$ from this

$f_{xx}(1, 2, 2)$

$$f_{xx}(1, 2, 2) = -24 < 0$$

$$= 320 > 0$$

2, z

$$2, z = 2 \quad x + y + z = 5$$

solid V

$$\text{solid } V =$$

Volume

$$\text{Volume} = \int_0^1 \int_0^{3x} \int_0^{5-x-y} dz \, dy \, dx$$

$$= \int_0^1 60y + 13y \, dy$$

$\int_R f(x, y) \, dA$

$$\int_R f(x, y) \, dA =$$

$\text{Vol}(S)$

$$\text{Vol}(S) =$$

$\text{ol}(S)$

$$\text{ol}(S) =$$

$\text{Vol}(S)$

$$\text{Vol}(S) =$$

Vol (S)

$$\text{Vol}(S) =$$

$$\iint_R f(x, y) dA$$

$$\iint_R f(x, y) dA =$$

$$\iint_R f(x, y) dA$$

$$\iint_R f(x, y) dA =$$

so, A(x)

$$\text{so, } A(x) =$$

Vol (S)

$$\text{Vol}(S) = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) dy dx$$

$$\iint_R f(x, y) dA$$

$$\iint_R f(x, y) dA =$$

$$\iint_R f(x, y) dA$$

$$\iint_R f(x, y) dA =$$

$$\iint_R f(x, y) dA$$

$$\iint_R f(x, y) dA =$$

$$\iint_R f(x, y) dx dy$$

$$\iint_R f(x, y) dx dy =$$

y

$$y = 2x \text{ Or } x = y/2$$

R

$$R = \{(x, y): 0 < x < 1, 0 < y < 2\}$$

nes y + z

$$\text{nes } y + z = 4$$

e y-axis, and the line y

$$\text{e y-axis, and the line } y = 2 - 2x \text{ [set}$$

y

$$y = -4 - x^2$$

$$= 6(27) + 8$$

Volume

$$\text{Volume} =$$

$$= 8(2) = 16$$

$$y - 1$$

$$y - 1 = -2x + 4$$

$$= (3)3 + 3(3)^2$$

Volume

$$\text{Volume} = 8$$

y

$$y = x^2/2 \, dx =$$

area of R

$$\text{area of R} =$$

y

$$y = x^2/2 \, dx$$

(r, θ) , then for any integer n

(r, θ) , then for any integer $n=0, 1, 2, 3, \dots$

nd

nd = 45 in (2) it follows that $(-3, 45^\circ)$ and $(-3, 225^\circ)$ represent the same

r

$$r = 6 \text{ and } \theta = 135^\circ \text{ i.e. } x = \cos$$

y

$$y = r \sin$$

$P(x, y)$

$$P(x, y) = P(r, \theta)$$

x

$$x = r \cos$$

y

$$y = r \sin \theta$$

y/x

$$y/x = \tan \theta$$

substitute x

$$\text{substitute } x = r \cos \theta \quad a = r \cos \theta \text{ -----(1)}$$

r sin

$$r \sin \theta = b$$

$$r (A \cos \theta + B \sin \theta) + C$$

$$r (A \cos \theta + B \sin \theta) + C = 0$$

e pairs from

$$e \text{ pairs from } =$$

x + y

$$x + y = y \text{ or } x + y$$

r

$$r = a (1 - \cos \theta)$$

r

$$r = a (1 - \cos \theta)$$

r

$$r = a (1 + \cos \theta)$$

r

$$r = a (1 + \sin \theta)$$

r

$$r = a (1 - \sin \theta)$$

r

$$r = a + b \sin \theta, \quad r = a - b \sin \theta$$

$$r = a + b \cos \theta, \quad r = a - b \cos \theta$$

r²

$$r^2 = 4 \cos^2 \theta$$

n

$n = \pi/4$, so the line $\theta = \pi/4$ is

r

$$r = a \sin n$$

r

$$r = a \cos n$$

0) or r

$$0) \text{ or } r = a$$

$$= 0, \pi, 2\pi, 3\pi, \dots$$

the curve r

the curve $r = r_2(\theta)$. These intersections determine the r -limits of integration.

le is

le is $\theta = \pi/4$. This yields the θ -limits of the integration.

from

$$\text{from } \theta = -\pi/4$$

r

$$r = 1 + \cos \theta$$

2 to

$$\theta \text{ to } \theta = 2\pi$$

$\sin \theta dA$

$$\sin \theta dA =$$

A

$$A = \int_0^{2\pi} \int_{-1}^1 r dr d\theta = 4$$

$r dr d\theta$

$$r dr d\theta =$$

0 $\sin^2 \theta d\theta$

$$\int_0^{2\pi} \sin^2 \theta d\theta = 4$$

$$r \, dr \, d$$

$$r \, dr \, d = A$$

$$0 \, 2 \cos 2 \, d$$

$$0 \, 2 \cos 2 \, d = 4 \sin 2]$$

$$f(x,y) \, dx \, dy$$

$$f(x,y) \, dx \, dy =$$

$$x^2 + y^2$$

$$x^2 + y^2 = 1, \, r = 1$$

$$0 \, r^3 \, dr \, d$$

$$0 \, r^3 \, dr \, d =$$

$$4 \, d$$

$$4 \, d =$$

$$\text{Evaluate } I$$

$$\text{Evaluate } I =$$

$$a \, d$$

$$a \, d =$$

$$\text{Evaluate } I$$

$$\text{Evaluate } I = 0$$

$$2 \cdot r \, dr \, d$$

$$2 \cdot r \, dr \, d =$$

$$0 \, 64 \sin^4 \, d$$

$$0 \, 64 \sin^4 \, d = 64.3.1$$

$$+ y$$

$$+ y = 4y \cdot \ln p$$

$$m \, r^2$$

$$m \, r^2 = 4r \sin, \, r = 4 \sin$$

$$2$$

$$2 = 12 \text{ (using Walli's formula)}$$

$$0e^r dr d\theta$$

$$0e^r dr d\theta =$$

$$(e-1)d$$

$$(e-1)d = 2$$

$$2+y^2)dx dy$$

$$2+y^2)dx dy = \lim_{a \rightarrow \infty} \int_0^a \int_0^a e^{-(x^2+y^2)} dx dy$$

$$- \exp \{-(x^2 + y^2)\} dx dy$$

$$- \exp \{-(x^2 + y^2)\} dx dy = \lim_{a \rightarrow \infty}$$

$$(1 - \exp \{-a^2\})d\theta$$

$$(1 - \exp \{-a^2\})d\theta = \lim_{a \rightarrow \infty}$$

$$= - \lim_{a \rightarrow \infty} \exp \{$$

$$= 1/4$$

$$- \exp \{-(x^2+y^2)\} dx dy$$

$$- \exp \{-(x^2+y^2)\} dx dy =$$

$$- \exp \{-x^2\} dx] dy$$

$$- \exp \{-x^2\} dx] dy = [$$

$$- \exp \{-t^2\} dt]$$

$$- \exp \{-t^2\} dt] = \lim_{a \rightarrow \infty}$$

$$-a \exp \{-t^2\} dt]$$

$$-a \exp \{-t^2\} dt] = 4 \lim_{a \rightarrow \infty}$$

If f is continuous on the region G , then $f(x$

If f is continuous on the region G , then $f(x, y, z) dV =$

, $y, z) dx dz dy$

, $y, z) dx dz dy =$

$a \int_0^a \int_0^a \int_0^a f(x, y, z) dy dz dx$

$a \int_0^a \int_0^a \int_0^a f(x, y, z) dy dz dx =$

$-1/4 \int_0^2 32x dx$

$-1/4 \int_0^2 32x dx = 216x^2]$

$$12xy2z3 dV$$

$$12xy2z3 dV=$$

$$0 12xy2z3dzdydx$$

$$0 12xy2z3dzdydx =$$

$$y dx$$

$$y dx =$$

$$R xyz dx dy dz$$

$$R xyz dx dy dz=$$

$$0 (x - 2x^3 + x^5) dx$$

$$0 (x - 2x^3 + x^5) dx= 1$$

$$r(t)$$

$$r(t) = (x(t), y(t), z(t))i + y(t)j + z(t)k$$

$$r(t)$$

$$r(t) = (\ln t) i + t^2 + 2j + (\cos t)k$$

$$xi + yj$$

$$xi + yj = x(t)i + y(t)j$$

$$xi + yj + zk$$

$$xi + yj + zk = x(t)i + y(t)j + z(t)k$$

if we let r

if we let $r = xi + yj$ and

and let r

and let $r = xi + yj + zk$ and $r(t) = x(t)i + y(t)j + z(t)k$

in 3-space, then both (2) and (4) can be written a

in 3-space, then both (2) and (4) can be written as $r = r(t)$

$$xi + yj + zk$$

$$xi + yj + zk = (\cos t)i + (\sin t)j + tk$$

$$xi + yj$$

$$xi + yj = t^2i + 3tj$$

$$x i + y j + z k$$

$$x i + y j + z k = (t^3 + 1) i + 3 j + e t k$$

Sketch the graph of the vector-valued function $r(t)$

Sketch the graph of the vector-valued function $r(t) = (\cos t)i + (\sin t)j$, $0 < t < 2$

$$x i + y j$$

$$x i + y j = (\cos t)i + (\sin t)j, 0 < t < 2$$

$$x$$

$$x = \cos t, y = \sin t (0 < t < 2)$$

Sketch the graph of the vector-valued function $r(t)$

Sketch the graph of the vector-valued function $r(t) = (\cos t)i + (\sin t)j + 2k$, $0 < t < 2$

$$x i + y j + z k$$

$$x i + y j + z k = (\cos t)i + (\sin t)j + 2k, 0 < t < 2$$

$$x$$

$$x = \cos t, y = \sin t, z = 2 (0 < t < 2)$$

Describe the graph of the vector equation r

Describe the graph of the vector equation $r = (-2 + t)i + 3tj + (5 - 4t)k$

The corresponding parametric equations are x

The corresponding parametric equations are $x = -2 + t$, $y = 3t$, $z = 5 - 4t$

If $r(t)$

$$\text{If } r(t) = t i + (t - 1) j$$

The graph of $r(t)$

The graph of $r(t) = (\cos t)i + (\sin t)j + 2k$, $0 < t < 2$

$$= x(t)i + y(t)j + z(t)k$$

$t r(t)$

$$t r(t) = (\lim$$

$$r(t)$$

$$r(t) = \lim$$

$$r(t)$$

$$r(t) = L$$

$$r'(t)$$

$$r'(t) = \lim$$

$$r'(t)$$

$$r'(t) = x'(t)i + y'(t)j$$

$$r'(t)$$

$$r'(t) = x'(t)i + y'(t)j + z'(t)k$$

$$= x'(t)i + y'(t)j$$

$$\text{Let } r(t)$$

$$\text{Let } r(t) = t^2i + t^3j. \text{ Find } r'(t) \text{ and } r'(1)$$

$$r'(t)$$

$$r'(t) = d$$

$$r'(1)$$

$$r'(1) = 2i + 3j.$$

$$r$$

$$r = r(t_0) + t r'(t_0)$$

$$xi + yj + zk$$

$$xi + yj + zk = (\cos t)i + (\sin t)j + tk$$

$$\text{Thus, } r(t)$$

$$\text{Thus, } r(t) = (\cos t)i + (\sin t)j + tk$$

$$r'(t)$$

$$r'(t) = (-\sin t)i + (\cos t)j + k$$

$$\text{At the point where } t$$

$$\text{At the point where } t = \pi/6, \text{ these vectors are}$$

$$\text{so from (5) with } t_0$$

$$\text{so from (5) with } t_0 = \pi/6 \text{ a vector equation of the tangent line is}$$

$$2t, z$$

$$2t, z =$$

$$r/(t)$$

$$r/(t) = 2ti + 3t2j$$

of the tangent line at this point is r

$$\text{of the tangent line at this point is } r = r(1) + t r/(1)$$

$$r$$

$$r = (i + j) + t (2i + 3j)$$

3-space $r(t)$

$$3\text{-space } r(t) = x(t)i + y(t)j + z(t)k$$

$$dt [r_1(t) + r_2(t)]$$

$$dt [r_1(t) + r_2(t)] = d$$

$$dt [r_1(t) \times r_2(t)]$$

$$dt [r_1(t) \times r_2(t)] = r_1 \times dr_2$$

$$t, \text{ then } r(t) \cdot r/(t)$$

$$t, \text{ then } r(t) \cdot r/(t) = 0$$

$$r(t) \cdot r/(t)$$

$$r(t) \cdot r/(t) = 0$$

(b) If $r(t)$

(b) If $r(t) = x(t)i + y(t)j + z(t)k$ is a

$$r(t)dt$$

$$r(t)dt = (x(t)dt)i + (y(t)dt)j + (z(t)dt)k \quad (2a)$$

$$ar(t)dt$$

$$ar(t)dt =$$

$$cr(t)dt$$

$$cr(t)dt = c \cdot r(t)dt$$

$$[r_1(t) - r_2(t)]dt$$

$$[r_1(t) - r_2(t)]dt = r_1(t)dt - r_2(t)dt$$

$$[r(t) + r(t)] dt$$

$$[r(t) + r(t)] dt = r_1(t) dt + r(t) dt$$

$$[r(t) dt]$$

$$[r(t) dt] = r(t)$$

$$/(t)$$

$$/(t) = r(t), \text{ then}$$

$$r(t) dt$$

$$r(t) dt = R(t) + C$$

$$r$$

$$r = (3 \cos t)i + (3 \sin t)j, 0 < t < 2$$

by substituting t

by substituting $t = s/3$ in (10).

$$x$$

$$x = x(t), y = y(t) \quad (a < t < b)$$

from t

$$\text{from } t = 0 \text{ to } t =$$

$$dt$$

$$dt =$$

$$2 dt$$

$$2 dt = 2$$

and in 3-space by x

$$\text{and in 3-space by } x = x(s), y = y(s), z = z(s)$$

$$x$$

$$x = 2t + 1, y = 3t - 2$$

$$x$$

$$x = 2u + 1, y = 3u - 2$$

$$13s + 1$$

$$13s + 1 = 2$$

$$\int_0^t (-a \sin u)^2 + (a \cos u)^2 du$$

$$\int_0^t (-a \sin u)^2 + (a \cos u)^2 du =$$

x

$$x = a \cos(s/a), y = a \sin(s/a) \quad (0 < s < 2a)$$

Arc length

$$\text{Arc length} =$$

$$\int_1^9 9t^4 + 1 + 6t^2 dt$$

$$\int_1^9 9t^4 + 1 + 6t^2 dt =$$

$$= [t^3 + t]$$

r

$$r = e^t i + 4e^{-t} j$$

d

$$d = e^t i - 4e^{-t} j$$

$$= e^{u_1} i + 4e^{-u_2} j$$

du

$$du = 2u e^{u^2} i - 8u e^{-u^2} j$$

If z

$$\text{If } z = f(x, y, w), dz = x dx + y$$

1. z

$$1. z = x^2 + y^2$$

4. z

$$4. z = x^2 + 2y^2 + 3w^2$$

1. dz

$$1. dz = 2(x dx + y dy)$$

2. dz

$$2. dz = x^2 (3 \sin 2y dx + 2x \cos 2y dy)$$

3. dz

$$3. dz = e^{3y} \{ 2dx + (6x -$$

5. dz

$$5. dz = x y (3y w dx + 2x w dy + xy dw)$$

P

$$P = x \text{ and } Q =$$

yx

$$yx = 2z$$

dz

$$dz = (3x^2 + 4y^2) dx + 8xy dy.$$

P

$$P = 3x^2 + 4y^2 \quad P$$

Q

$$Q = 8xy \quad Q$$

dz

$$dz = (1 + 8xy) dx + 5x^2 dy.$$

for dz

$$\text{for } dz = (1 + 8xy) dx + 5x^2 dy$$

P

$$P = 1 + 8xy \quad P$$

3. dz

$$3. dz = (15y^2 e^{3x} + 2xy^2) dx + (10y e^{3x} + x^2 y) dy$$

4. dz

$$4. dz = (3x^2 e^{2y} - 2y^2 e^{3x}) dx + (2x^3 e^{2y} - 2y e^{3x}) dy$$

1. dz

$$1. dz = 4x^3 y^3 dx + 3x^4 y^2 dy$$

dz

$$dz = P dx + Q dy \text{ where } P = z$$

x and Q

$$x \text{ and } Q = z$$

z

$$z = Pdx \text{ and also } z = Qdy$$

dz

$$dz = (2xy + 6x) dx + (x^2 + 2y^3) dy.$$

P

$$P = x = 2xy + 6x \quad z = (2xy + 6x) dx$$

z

$$z = x^2y + y^4$$

z

$$z = x^2y + 3x^2 + f(y)$$

z

$$z = x^2y + y^4$$

z

$$z = x^2y + 3x^2 + y^4$$

z

$$z = 2e^{4x} + x^2y^2 + \sin 4y$$

dz

$$dz = (8e^{4x} + 2xy^2) dx + (4 \cos 4y + 2x^2y) dy$$

$$= 8e^{4x} + 2xy^2$$

z

$$z = (8e + 2xy) dx$$

z

$$z = 2e^{4x} + x^2y^2 + f(y)$$

$$= 4 \cos 4y + 2x^2y$$

z

$$z = (4 \cos 4y + 2x^2y) dy$$

z

$$z = \sin 4y + x^2y^2 + F(x)$$

z

$$z = 2e^{4x} + x^2y^2 + \sin 4y$$

$$= A_1 - A_2 \quad A =$$

y

$$y = 4x \text{ for } x > 0.$$

$$= - \int_0^4 y \, dx = A = 4 \text{ square units}$$

or A

$$\text{or } A = - \int_0^4 y \, dx = -$$

A

$$A = - \int_0^4 y \, dx = -$$

0 $\int_C F \, ds$

$$\int_C F \, ds = \int_C F \, ds \text{ from } A \text{ to } B$$

AB $\int_C F \, dx$

$$\int_C F \, dx =$$

AB $\int_C F \, ds$

$$\int_C F \, ds = \int_C (P \, dx + Q \, dy)$$

C $\int_C F \, ds$

$$\int_C F \, ds =$$

0 $\int_C F \, ds$

$$\int_C F \, ds = \int_C F \, ds \text{ from } A \text{ to } B$$

AB $\int_C F \, dx$

$$\int_C F \, dx =$$

is the curve y

$$\text{is the curve } y = 1 + x.$$

C $\int_C (P \, dx + Q \, dy)$

$$\int_C (P \, dx + Q \, dy) =$$

C $\int_C (x + 3y) \, dx$

$$\int_C (x + 3y) \, dx =$$

along the curve y

along the curve $y = 1 + x^2$.

Also y

Also $y = 2 + x$

Evaluate I

Evaluate $I = \int (x$

Evaluate I

Evaluate $I =$

$\int_0^1 \{9x^2 dx + 32x^4 dx\}$

$\int_0^1 \{9x^2 dx + 32x^4 dx\} =$

Evaluate I

Evaluate $I =$

For IAB

For $IAB =$

$\int_0^1 \{(1 + 2y) (0) + y dy\}$

$\int_0^1 \{(1 + 2y) (0) + y dy\} =$

3 and $Ic1+c2$

3 and $Ic1+c2 = 81$

$AB \int ds$

$AB \int ds = -$

$AB \int \{Pdx + Qdy\}$

$AB \int \{Pdx + Qdy\} =$

3. (a) For a path of integration parallel to the y

3. (a) For a path of integration parallel to the y -axis, i.e. $x = k$, $dx = 0$

$\int_C Qdy$

$\int_C Qdy = 0 \quad I_C =$

I_C

$I_C = IAB = IAK + IKB.$

As usual, I

As usual, $I =$

for I

for $I = 2$

dx

$$dx = \cos d 1 - x^2 = \cos$$

Limits : x

$$\text{Limits : } x = 0, = 0; x = 1, =$$

$$0 \cos 2d$$

$$0 \cos 2d =$$

$$0 (1 + \cos 2)d$$

$$0 (1 + \cos 2)d =$$

$$0 \{x^2 dx + 2x(1-x)dx\}$$

$$0 \{x^2 dx + 2x(1-x)dx\} =$$

$$0 (x^2 + 2x - 2x^2)dx$$

$$0 (x^2 + 2x - 2x^2)dx =$$

I_3

$$I_3 = 0 dy = 0$$

$$3 + 0$$

$$3 + 0 = -1$$

$y dx$ when c is the circle $x^2 + y^2$

$y dx$ when c is the circle $x^2 + y^2 = 4$.

$$x^2 + y^2$$

$$x^2 + y^2 = 4$$

$$4 - x^2 dx$$

$$4 - x^2 dx = -2$$

$$-2 \int 4 - x^2 dx$$

$$-2 \int 4 - x^2 dx = -4$$

tegral, substitute x

tegral, substitute $x = 2 \sin$ and finish it off. $I = -4$

I

$$I = \int_0^1 \{xy dx + (1 + y^2) dy\}$$

$$\int_0^1 (1 + y^2) dy$$

$$\int_0^1 (1 + y^2) dy =$$

$$\int_0^1 2 (1 + y^2) dy$$

$$\int_0^1 2 (1 + y^2) dy =$$

$$\int_0^1 C f(x, y) dx$$

$$\int_0^1 C f(x, y) dx =$$

AB Ftds

$$AB Ftds = \{P dx + Q dy\}$$

$$\int_0^1 C f(x, y) dx$$

$$\int_0^1 C f(x, y) dx =$$

Evalaute I

$$Evalaute I =$$

$$\int_0^1 5) dx$$

$$\int_0^1 5) dx = 2.5$$

$$\int_0^1 (2x + 3x^2) dx$$

$$\int_0^1 (2x + 3x^2) dx = 4.5$$

$$\int_0^1 C f(x, y) ds$$

$$\int_0^1 C f(x, y) ds =$$

$$= \int_0^1 4xy ds w$$

$$\int_0^1 4 \sin t \cos t dt$$

$$\int_0^1 4 \sin t \cos t dt = 2$$

Evaluate I

$$Evaluate I = 0$$

$$\int_0^1 \{3x^2x^4 dx + 2x^3x^2x dx\}$$

$$\int_0^1 \{3x^2x^4 dx + 2x^3x^2x dx\} =$$

$$= 3x^2 y^2 \text{ and } z$$

y

$$y = 2x^3 y$$

Evaluate I

$$\text{Evaluate } I = \{3ydx + (3x+2y)dy\} \text{ from } A(1, 2) \text{ to } B$$

y

$$y = 3 = Q$$

x

$$x = 3y \quad z = 3ydz = 3xy + f(y) \text{ ----- (i)}$$

Hence z

$$\text{Hence } z = 3xy + y^2$$

$$C\{3ydx + (3x+2y)dy\}$$

$$C\{3ydx + (3x+2y)dy\} =$$

$$(1, 2) d(3xy + y^2)$$

$$(1, 2) d(3xy + y^2) = [3xy + y^2]$$

Evaluate I

$$\text{Evaluate } I =$$

x

$$x = x^2 + yex \quad z = x^3$$

y

$$y = ex + y \quad z = yex + y^2$$

(a) I

(a) $I = (Pdx + Qdy)$ is independent of the path of integration

(b) I

(b) $I = 0$ ($P dx + Q dy$) is zero.

$$I_{c1} + I_{c2}$$

$$I_{c1} + I_{c2} = 0$$

dz

$$dz = Pdx + Q dy + R dw \text{ is an}$$

Verify that dz

Verify that $dz = (3x^2yw + 6x)dx + (x^3w - 8y)dy + (x^3y + 1)dw$ is inexact differential and hence

y

$$y = 3x^2w; Q$$

x

$$x = 3x^2w P$$

P

$$P = 3x^2y; x$$

R

$$R = 3x^2y P$$

y

$$y = x^3; Q$$

w

$$w = x^3 R$$

x; Q

$$x; Q = z$$

y; R

$$y; R = z$$

z

$$z = (x^3y + 1)dw = y^3yw + w + f(y) + g(x)$$

z

$$z = x^3yw + 3x^2 - 4y^2 + w$$

I

$$I = [x^3yw + 3x^2 - 4y^2 + w]$$

$$-(8 + 3 - 16 + 4)$$

$$-(8 + 3 - 16 + 4) = 36$$

I

$$I = [x^3yw + 3x$$

$$= (24+12 \ 4+3)$$

$$x \, dx \, dy$$

$$x \, dx \, dy = -0$$

Evaluate I

Evaluate $I = \oint_C \{(2x - y)dx + (2y + x)dy\}$ around the boundary C of the ellipse

$$x^2 + 9y^2$$

$$x^2 + 9y^2 = 16.$$

$$x \, dx \, dy$$

$$x \, dx \, dy = -$$

$$R(-1-1)dx \, dy$$

$$R(-1-1)dx \, dy = 2$$

$$x^2 + 9y^2$$

$$x^2 + 9y^2 = 16 \text{ i.e. } x^2$$

$$A$$

$$A = 16$$

$$I$$

$$I = 2A = 32$$

evaluate I

$$\text{evaluate } I = 0$$

$$P$$

$$P = 2x + y$$

$$= 2 \times \text{the area of the triangle} = 2 \times 1 = 2$$

For I

$$\text{For } I = - (1-3) \, dx \, dy = 2 \, dx \, dy$$

$$x \, dx \, dy$$

$$x \, dx \, dy = -0 (P \, dx + Q \, dy)$$

Evaluate the line integral I

$$\text{Evaluate the line integral } I = \oint_C \{xy \, dx + (2x$$

I

$$I = \int_C \{xy \, dx + (2 - y) \, dy\}$$

$$\int_C \{P \, dx + Q \, dy\}$$

$$\int_C \{P \, dx + Q \, dy\} = -$$

P

$$P = xy \quad P$$

y

$$y = x; \quad Q = 2x - y \quad Q$$

$$\int_C (x - 2) \, dx \, dy$$

$$\int_C (x - 2) \, dx \, dy = -$$

I

$$I = -$$

$$\int_C x^2 \, dx$$

$$\int_C x^2 \, dx = -$$

P

$$P = 1 \text{ and } x$$

$$\int_C \{1 - (-1)\} \, dx \, dy$$

$$\int_C \{1 - (-1)\} \, dx \, dy = -0$$

$$\int_C R \, dx \, dy$$

$$\int_C R \, dx \, dy = -0$$

$$(y \, dx - x \, dy)$$

$$(y \, dx - x \, dy) = 0$$

Therefore, the area of the closed region A

Therefore, the area of the closed region $A =$

$$\int_C R \, dx \, dy$$

$$\int_C R \, dx \, dy = 1$$

I_1

$$I_1 =$$

$$0 (6x^2 dx - 3x^2 dx)$$

$$0 (6x^2 dx - 3x^2 dx) =$$

$$I_2$$

$$I_2 =$$

$$x$$

$$x = 0 \text{ for } x > 0.$$

$$(x dy - y dx) \quad 2A$$

$$(x dy - y dx) \quad 2A = 0$$

$$(x dy - y dx)$$

$$(x dy - y dx) =$$

$$0 (6x^3 dx - 2x^3 dx)$$

$$0 (6x^3 dx - 2x^3 dx) =$$

$$1 \{3x^3 dx - (x^3 + 1) dx\}$$

$$1 \{3x^3 dx - (x^3 + 1) dx\} =$$

$$\text{ration parallel to x-axis, } dy$$

$$\text{ration parallel to x-axis, } dy = 0 \quad I_c =$$

$$(c) I$$

$$(c) I = 0$$

$$(P dx + Q dy)$$

$$(P dx + Q dy) = -$$

$$(x dy - y dx)$$

$$(x dy - y dx) = 2$$

$$R dx dy$$

$$R dx dy = 2A$$

$$= \text{grad} =$$

$$z$$

$$z = i$$

grad

$$\text{grad} = =$$

If A

$$\text{If } A = a_1i + a_2j + a_3k$$

div A

$$\text{div } A = \nabla \cdot A = a_1$$

If A

$$\text{If } A = x^2yi - xyzj + yz^2k \text{ then}$$

$$= 2xy - xz + 2yz$$

If A

$$\text{If } A = 2x^2yi - 2(xy^2 + y^3z)j + 3y^2z^2k \text{ determine } \nabla \cdot A \text{ i.e. div } A.$$

A

$$A = 2x^2yi - 2(xy^2 + y^3z)j + 3y^2z^2k$$

$\times A$

$$\times A = i$$

If A

$$\text{If } A = (y^4 - x^2z^2)i + (x$$

Curl A

$$\text{Curl } A = \nabla \times A =$$

$\times F$

$$\times F = i\{2 - 2x \cos y\} - j\{1 - e^{2xy}\} + k\{2z \cos y - 2xe^{2xy}\}$$

Grad

$$\text{Grad} =$$

(i) div A

$$(i) \text{ div } A = \nabla \cdot A = ax$$

If A

$$\text{If } A = x^2yi + yz^3j - zx^3k$$

$$= 2xy + z^3 + x^3 = (\text{say})$$

zk

$$zk = (2y+3x^2)i + (2x)j + (3z^2)k$$

div grad

$$\text{div grad} = \nabla \cdot (\nabla f)$$

$$= xyz - 2y^2z + x^2z^2$$

grad

$$\text{grad} f = \nabla f$$

zk

$$zk = (yz+2xz^2)i + (xz-4yz)j + (xy-2y^2+2xz^2)k$$

div grad

$$\text{div grad} f = \nabla \cdot (\nabla f) = 2z - 4z + 2xz$$

grad

$$\text{grad} f = x i + y j$$

zk

$$zk = 2$$

div grad

$$\text{div grad} f = \nabla \cdot (\nabla f) = 2$$

Curl curl F

$$\text{Curl curl } F = \nabla \times (\nabla \times F) = i + 2j + 6k$$

$$= (2yz - 2xyz)i + x^2yzj + (yz^2 - x^2z)k$$

$$= z^2i - (-2xz - 2y + 2xy)j + (2xy - 2z + 2xz)k$$

grad

$$\text{grad} f = \nabla f$$

curl grad

$$\text{curl grad} f = \nabla \times (\nabla f) = 0$$

A

$$A = axi + ayj + azk$$

div curl A

$$\text{div curl } A = \nabla \cdot (\nabla \times A) = 0$$

grad

$$\text{grad} =$$

z k

$$z k = 2$$

div grad

$$\text{div grad} = \nabla \cdot (\nabla f) = 2$$

(a) curl grad

$$(a) \text{ curl grad} = \nabla \times (\nabla f) = 0$$

) div curl A

$$) \text{ div curl } A = \nabla \cdot (\nabla \times A) = 0$$

) div grad

$$) \text{ div grad} = \nabla \cdot (\nabla f) = 2$$

x ln x dx

$$x \ln x dx = \ln x \cdot x dx -$$

x sin x dx

$$x \sin x dx = x \sin x dx - [\sin x dx \cdot d(x)]$$

cos x dx

$$\cos x dx = -x(\cos x) + \sin x$$

integrand is expressed in terms of x, y, z with

$$\text{integrand is expressed in terms of } x, y, z \text{ with } dr = dx + dy j + dz k$$

integrand is expressed in terms of x, y, z with

$$\text{integrand is expressed in terms of } x, y, z \text{ with } dr = dx + dy j + dz k$$

dr

$$dr = dx i + dy j + dz k \quad dr = 3du i + 4udu j + 3u^2 du k$$

$$12u (3i + 4uj + 3uk) du$$

$$12u (3i + 4uj + 3uk) du = 36$$

along the curve c defined by x

along the curve c defined by $x = t^2; y = 2t; z = t+5$

$$= (t^2)(2t) + (4t^2)(t+5)$$

$$= 6t^3 + 20t^2.$$

z

$$z = t+5 \quad dz = dt$$

$$= 2t \, dt \, i + 2 \, dt \, j + dt \, k$$

V_{dr}

$$V_{dr} = (6t + 20t)(2t \, i + 2 \, j + k) \, dt$$

V_{dr}

$$V_{dr} = 2 \{ 6t^4$$

F

$$F = F_1 i + F_2 j + F_3 k$$

And dr

$$\text{And } dr = dx \, i + dy \, j + dz \, k$$

If $F(r)$

If $F(r) = x^2y \, i + xz \, j + 2yz \, k$, Evaluate

F

$$F = 32t^4 \, i + 4t^4 \, j - 4t^5 \, k$$

dr

$$dr = 4dt \, i + 4t \, dt \, j + 3t^2 \, k$$

along the stated curve

along the stated curve =

$$= (128t^4 + 16t^5 + 12t^7) \, dt$$

$$t^4 + 16t^5 + 12t^7) \, dt$$

$$t^4 + 16t^5 + 12t^7) \, dt = 128$$

$$F(r)$$

$$F(r) = x^2y \mathbf{i} + 2yz \mathbf{j} + 3z^2x \mathbf{k}$$

$$F \cdot dr$$

$$F \cdot dr = (x^2y \mathbf{i} + 2yz \mathbf{j} + 3z^2x \mathbf{k}) \cdot (dx \mathbf{i} + dy \mathbf{j} + dz \mathbf{k})$$

$$F \cdot dr$$

$$F \cdot dr = x^2y dx + 2yz dy + 3z^2x dz$$

$$F \cdot dr$$

$$F \cdot dr = x^2y dx + 2yz dy + 3z^2x dz$$

$$F \cdot dr$$

$$F \cdot dr = 0 + 0 + 0 = 0$$

$$F \cdot dr$$

$$F \cdot dr = 0 + 0 + 0 = 0$$

$$F \cdot dr$$

$$F \cdot dr = 0 + 0 +$$

$$F$$

$$F = 2t^3 \mathbf{i} + 12t^2 \mathbf{j} + 27t^3 \mathbf{k}$$

$$dr$$

$$dr = dx \mathbf{i} + dy \mathbf{j} + dz \mathbf{k} = dt \mathbf{i} + 2dt \mathbf{j} + 3dt \mathbf{k}$$

$$0 (83t^3 + 24t^2) dt$$

$$0 (83t^3 + 24t^2) dt =$$

$$F \cdot dr \text{ via } c_1, c_2 \text{ and } c_3$$

$$F \cdot dr \text{ via } c_1, c_2 \text{ and } c_3 = 27$$

$$F \cdot dr \text{ via } c_4$$

$$F \cdot dr \text{ via } c_4 = 28.75$$

$$\text{when } x$$

$$\text{when } x = 0, y+z=2 \text{ i.e. } z = 2 - y$$

$$\text{So } 2x + y + z$$

$$\text{So } 2x + y + z = 2 \text{ cuts the axes at}$$

$$\nabla F dV$$

$$\nabla F dV = 0 \ 0$$

$$z$$

$$z = 2(1 - x) - y$$

$$\nabla F dV$$

$$\nabla F dV = 3(2i + k)$$

$$x^2 + y^2$$

$$x^2 + y^2 = 9.$$

$$r$$

$$r = 0[2zi + 2zj + r \sin z k]$$

$$+ r^2 \sin k) dr d$$

$$+ r^2 \sin k) dr d = 4$$

$$z$$

$$z = 0(2i + 2zj + r \sin k) r dz dr d =$$

$$0(8i + 16j + 4r \sin k) r dr d$$

$$0(8i + 16j + 4r \sin k) r dr d = 4$$

$$k) d$$

$$k) d = 36[i + 2j - \cos k]$$

$$0(9i + 18j + 9 \sin k) d$$

$$0(9i + 18j + 9 \sin k) d = 36$$

$$dS$$

$$dS = \hat{n} dS \text{ where } \hat{n} = S$$

$$S k$$

$$S k = 2xi + 2yj$$

$$\hat{n}$$

$$\hat{n} = S$$

$$|S|$$

$$|S| = xi + yj$$

$$dS$$

$$dS = \hat{n} dS = xi + yj$$

$$SF dS$$

$$SF dS = SF \hat{n} dS = 2$$

$$\int_S xyz(xi + yj) dS$$

$$\int_S xyz(xi + yj) dS = 2$$

$$dS$$

$$dS = 2 dz$$

$$SF dS$$

$$SF dS = 8$$

$$\int_0^2 \int_0^2 (\cos 2s \sin t + \cos \sin 2j) 2 dz ds$$

$$\int_0^2 \int_0^2 (\cos 2s \sin t + \cos \sin 2j) 2 dz ds = 4$$

$$= 12(i + j)$$

$$S: x^2 + y^2 + z^2 = 9$$

$$S: x^2 + y^2 + z^2 - 9 = 0$$

$$z k$$

$$z k = 2xi + 2yj + 2zk$$

$$\hat{n}$$

$$\hat{n} = 1$$

$$6(2xi + 2yj + 2zk)$$

$$6(2xi + 2yj + 2zk) = 1$$

$$SF \cdot dS$$

$$SF \cdot dS = SF \cdot \hat{n} dS = S(yi + 2j + k) \cdot 1$$

$$(xi + yj + zk) dS$$

$$(xi + yj + zk) dS = 1$$

Limits of and are

Limits of and are $= 0$ to 2 ; $= 0$ to 2

$$SF \cdot dS$$

$$SF \cdot dS = 1$$

$$\vec{F} \cdot d\vec{S}$$

$$\vec{F} \cdot d\vec{S} = 1$$

$$xy$$

$$xy = 3 \sin \cos \cdot 3 \sin \sin = 9 \sin^2 \cos$$

$$dS$$

$$dS = 9 \sin \theta d\theta$$

$$0 \sin^3 \theta$$

$$0 \sin^3 \theta = 0$$

$$0 \sin^2 \theta$$

$$0 \sin^2 \theta = 0$$

$$2 \theta$$

$$2 \theta = 9$$

$$S: x^2 + y^2 = 4$$

$$S: x^2 + y^2 - 4 = 0$$

$$z$$

$$z = 2x + 2y$$

$$\hat{n}$$

$$\hat{n} = \frac{2x}{\sqrt{4x^2 + 4y^2 + 4}} \hat{i} + \frac{2y}{\sqrt{4x^2 + 4y^2 + 4}} \hat{j} + \frac{2}{\sqrt{4x^2 + 4y^2 + 4}} \hat{k}$$

$$|S|$$

$$|S| = 2x \hat{i} + 2y \hat{j} + 2 \hat{k}$$

$$\vec{F} \cdot d\vec{S}$$

$$\vec{F} \cdot d\vec{S} = \vec{F} \cdot \hat{n} dS$$

$$\vec{F} \cdot \hat{n} dS$$

$$\vec{F} \cdot \hat{n} dS = 1$$

$$2 \int_S (2y^2) dS$$

$$2 \int_S (2y^2) dS = 4 \int_S y^2 dS$$

$$dS$$

$$dS = 2 dx dz$$

$$\int S \cdot F \cdot dS$$

$$\int S \cdot F \cdot dS = \int S_y^2 dS =$$

$$\int 4 \sin^2 z \, dz$$

$$\int 4 \sin^2 z \, dz = 8$$

Limits:

$$\text{Limits: } z = 0 \text{ to } z = 5$$

$$\int S \cdot F \cdot dS$$

$$\int S \cdot F \cdot dS = 4$$

$$\int (1 - \cos 2z) \, dz$$

$$\int (1 - \cos 2z) \, dz = 4$$

$$\int_0^5 dz$$

$$\int_0^5 dz = 4 [z]$$

$$\int_1^{10} F \cdot dr$$

$$\int_1^{10} F \cdot dr = \int_1^{10} c_2(AB) F \cdot dr$$

$$\int_1^{10} F \cdot dr + \int_{10}^1 F \cdot dr$$

$$\int_1^{10} F \cdot dr + \int_{10}^1 F \cdot dr = \int_1^{10} F \cdot dr - \int_1^{10} F \cdot dr$$

$$= \int_1^{10} F \cdot dr - \int_1^{10} F \cdot dr = 0$$

$$\int_1^{10} F \cdot dr$$

$$\int_1^{10} F \cdot dr = \int_{10}^1 F \cdot dr \text{ But } \int_{10}^1 F \cdot dr = - \int_1^{10} F \cdot dr$$

$$= \int_1^{10} F \cdot dr + \int_1^{10} F \cdot dr$$

If F

If $F = 2xyzi + x^2zj + x^2yk$, evaluate the line integral $\int F \cdot dr$ between $A(0,0,0)$ and $B(2,4,6)$

F

$$F = 2xyzi + x^2zj + x^2yk$$

F.dr

$$F \cdot dr = (2xyzi + x^2zj + x^2yk) \cdot (dx i + dy j + dz k)$$

$$= 2xyz \, dx + x^2z \, dy + x^2y \, dz$$

$$C F \cdot dr$$

$$C F \cdot dr =$$

$$F \cdot dr$$

$$F \cdot dr = (2xyz \, dx + x^2z \, dy + x^2y \, dz)$$

$$F \cdot dr$$

$$F \cdot dr = (2xyz \, dx + x^2z \, dy + x^2y \, dz)$$

$$dr$$

$$dr = 0 + 0 +$$

$$C_3 F \cdot dr$$

$$C_3 F \cdot dr = 96$$

$$C_1 F \cdot dr$$

$$C_1 F \cdot dr = 0 + 0 + 0$$

$$O F \cdot dr$$

$$O F \cdot dr = 0$$

$$V \operatorname{div} F \, dV$$

$$V \operatorname{div} F \, dV = \int_S F \cdot dS$$

$$z(y)$$

$$z(y) = 2x + 0 + 0 = 2x$$

$$V \operatorname{div} F \, dV$$

$$V \operatorname{div} F \, dV = \int_V 2x \, dv = \int_V 2x \, dz \, dy \, dx$$

$$V \operatorname{div} F \, dV$$

$$V \operatorname{div} F \, dV =$$

$$0 \int_V 2x \, dz \, dy \, dx$$

$$0 \int_V 2x \, dz \, dy \, dx =$$

$$F$$

$$F = x^2i + zj + yk$$

$$(i) S_1 (\text{base}): z$$

$$(i) S_1 (\text{base}): z = 0; \hat{n} = -k (\text{outwards and downwards})$$

F

$$F = x^2i + yk$$

$$S_1 F \cdot \hat{n} dS$$

$$S_1 F \cdot \hat{n} dS = S_1 (x^2i + yk) \cdot (-k) dy dx =$$

$$0 (-y) dy dx$$

$$0 (-y) dy dx =$$

$$S_2 (\text{top}): z$$

$$S_2 (\text{top}): z = 2; \hat{n} = k \quad dS_2 = dx \, dy$$

$$S_2 F \cdot \hat{n} dS$$

$$S_2 F \cdot \hat{n} dS = S_2 (x^2i + zj + yk) \cdot (k) dy dx =$$

$$\hat{n}$$

$$\hat{n} = j \quad dS_3 = dx \, dz$$

F

$$F = x^2i + zj + yk$$

$$S F \cdot \hat{n} dS$$

$$S F \cdot \hat{n} dS = S (x^2i + zj + 3k) \cdot (j) dz dx = z \, dz \, dx =$$

$$S_4 F \cdot \hat{n} dS$$

$$S_4 F \cdot \hat{n} dS = -2$$

$$\text{for } S_4 F \cdot \hat{n} dS$$

$$\text{for } S_4 F \cdot \hat{n} dS = S_4 (x^2i + zj + yk) \cdot (-j) dz dx =$$

$$0 (-z) dz dx$$

$$0 (-z) dz dx =$$

$$S_5 F \cdot \hat{n} dS$$

$$S_5 F \cdot \hat{n} dS = 6; \quad S_5 F \cdot \hat{n} dS = 0$$

$$(v) S_5 (\text{front}): x$$

$$(v) S_5 (\text{front}): x = 1; \hat{n} = i \quad dS_5 = dy \, dz$$

$$(vi) S_6 (\text{back}): x$$

$$(vi) S_6 (\text{back}): x = 0; \hat{n} = -i \quad dS_6 = dy \, dz$$

$$2+2 - 2+6+0$$

$$2+2 - 2+6+0 = 6$$

and from our previous work in section (a) $\int_V \text{div } F$

and from our previous work in section (a) $\int_V \text{div } F \, dV = 6$

$$\int_S \text{curl } F \cdot d\mathbf{S}$$

$$\int_S \text{curl } F \cdot d\mathbf{S} = 0$$

A hemisphere S is defined by $x^2 + y^2 + z^2 = 4$ ($z > 0$). A vector

field

$$\int_S \text{curl } F \cdot d\mathbf{S}$$

$$\int_S \text{curl } F \cdot d\mathbf{S} = 0$$

$$S: x^2 + y^2 + z^2 = 4$$

$$S: x^2 + y^2 + z^2 = 4 \quad (z > 0)$$

C is the circle $x^2 + y^2 = 4$ ($z = 0$).

C is the circle $x^2 + y^2 = 4$.

$$= \int_C (2y \, dx - x \, dy + xz \, dz)$$

x

$$x = -2 \sin \theta; \, dy = 2 \cos \theta; \, \text{Limits} = 0 \text{ to } 2\pi$$

$$\int_0^{2\pi} (4 \sin \theta [-2 \cos \theta] - 2 \cos \theta \cdot 2 \cos \theta) \, d\theta$$

$$\int_0^{2\pi} (4 \sin \theta [-2 \cos \theta] - 2 \cos \theta \cdot 2 \cos \theta) \, d\theta = -4$$

$$\int_0^{2\pi} (1 + \sin^2 \theta) \, d\theta$$

$$\int_0^{2\pi} (1 + \sin^2 \theta) \, d\theta = -2$$

$$\int_0^{2\pi} (3 - \cos^2 \theta) \, d\theta$$

$$\int_0^{2\pi} (3 - \cos^2 \theta) \, d\theta = -2$$

$$\int_C F \cdot d\mathbf{r}$$

$$\int_C F \cdot d\mathbf{r} = \int_C (2y \, dx - x \, dy + xz \, dz)$$

$$= -12$$

$$\int_S \text{curl } F \cdot d\mathbf{S}$$

$$\int_S \text{curl } F \cdot d\mathbf{S} = \int_S \text{curl } F \cdot \mathbf{n} \, dS$$

F

$$F = 2yi - xj + xzk$$

$$= 2xi + 2yj + 2zk$$

$$4x^2 + 4y^2 + 4z^2$$

$$4x^2 + 4y^2 + 4z^2 = xi + yj + zk$$

$$\text{hen } S \text{ curl } F \cdot \hat{n} dS$$

$$\text{hen } S \text{ curl } F \cdot \hat{n} dS = S(-zk - 3k).$$

$$S \text{ curl } F \cdot \hat{n} dS$$

$$S \text{ curl } F \cdot \hat{n} dS = 1$$

$$0(2\sin^2 \cos \sin + 3\sin \cos) dd$$

$$0(2\sin^2 \cos \sin + 3\sin \cos) dd = -4$$

$$2d$$

$$2d = -12(2)$$

$$\text{oscillations. } f(x+p)$$

$$\text{oscillations. } f(x+p) = f(x)$$

$$\text{i.e. } f(x)$$

$$\text{i.e. } f(x) = 3 \quad 0 < x < 4$$

$$(x)$$

$$(x) = f(x+6)$$

$$f(x)$$

$$f(x) = f(x+5)$$

$$f(x)$$

$$f(x) = f(x+13)$$

$$f(x)$$

$$f(x) = f(x+8)$$

$$f(x)$$

$$f(x) = f(x+3)$$

$$f(x)$$

$$f(x) = f(x+2).$$

$$f(x)$$

$$f(x) = f(x + 8).$$

$$n \{-\cos n + \cos n\}$$

$$n \{-\cos n + \cos n\} = 0$$

$$n \{\sin n + \sin n\}$$

$$n \{\sin n + \sin n\} = 0$$

$$= (n \neq 0)$$

$$(1 + \cos 2nx) dx$$

$$(1 + \cos 2nx) dx = 1$$

$$= (n \neq 0)$$

$$\{\sin(n+m)x + \sin(n-m)x\} dx$$

$$\{\sin(n+m)x + \sin(n-m)x\} dx = 1$$

$$2 \{0 + 0\}$$

$$2 \{0 + 0\} = 0$$

$$1 \{0 + 0\}$$

$$1 \{0 + 0\} = 0$$

$$\cos^2 nx dx$$

$$\cos^2 nx dx = \text{from (d) above.}$$

$$\sin nx \sin mx dx$$

$$\sin nx \sin mx dx = -\frac{1}{2}$$

$$(-2) \sin nx \sin mx dx$$

$$(-2) \sin nx \sin mx dx = -\frac{1}{2}$$

$$\sin^2 nx dx$$

$$\sin^2 nx dx =$$

$$\sin^2 nx dx$$

$$\sin^2 nx dx = (\neq 0)$$

$$\cos^2 nx dx$$

$$\cos^2 nx dx = (n \neq 0) (e)$$

$$\cos nx \cos mx \, dx$$

$$\cos nx \cos mx \, dx = 0 \quad (n \neq m)$$

$$\sin nx \sin mx \, dx$$

$$\sin nx \sin mx \, dx = 0 \quad (n \neq m)$$

$$\cos nx \cos mx \, dx$$

$$\cos nx \cos mx \, dx =$$

$$\sin nx \sin mx \, dx$$

$$\sin nx \sin mx \, dx = \quad (n = m)$$

$$x)$$

$$x) = A_0 + c_1 \sin(x+1) + c_2 \sin(2x+2) + c_3 \sin(3x+3) + \dots + c_n \sin(nx+n) + \dots$$

$$f(x)$$

$$f(x) = A_0 + a_1 \cos x + a_2 \cos 2x + \dots + a_n \cos nx + \dots + b_1 \sin x + b_2 \sin 2x + \dots + b_n \sin nx + \dots$$

$$f(x)dx$$

$$f(x)dx =$$

$$\{0 + 0\}$$

$$\{0 + 0\} = 2A_0$$

$$2A_0$$

$$2A_0 =$$

$$f(x) \cos mx \, dx$$

$$f(x) \cos mx \, dx =$$

$$\{a_n(0) + b_n(0)\}$$

$$\{a_n(0) + b_n(0)\} = 0 \text{ for } n \neq m$$

$$n + 0$$

$$n + 0 = a_n \text{ for } n = m$$

$$f(x) \sin mx \, dx$$

$$f(x) \sin mx \, dx = A$$

$$) + b_n(0)\}$$

$$) + b_n(0)\} = 0 \text{ for } n \neq m$$

$$= 0 + 0 + b_n = b_n \text{ for } n = m$$

$$f(x) dx$$

$$f(x) dx = 2 \times \text{mean value of } f(x) \text{ over a period}$$

$$f(x) \cos nx dx$$

$$f(x) \cos nx dx = 2 \times \text{mean value of } f(x) \cos nx \text{ over a period.}$$

$$f(x)$$

$$f(x) = f(x + 2)$$

$$\text{period}$$

$$\text{period} = 2.$$

$$0 \times \cos nx dx$$

$$0 \times \cos nx dx = 2$$

$$2n [2 - 0]$$

$$2n [2 - 0] = -1$$

$$\{b_n \sin nx\}$$

$$\{b_n \sin nx\} =$$

Consider one cycle between x

Consider one cycle between $x=0$ and $x=$.

$$f(x)$$

$$f(x) = f(x + 2)$$

$$-1/2 \sin nx dx$$

$$-1/2 \sin nx dx = 4$$

$$f(x)$$

$$f(x) = 2 + 8$$

$$f(x)$$

$$f(x) = f(x + 2)$$

$$f(x)$$

$$f(x) = 2 + \{\cos x - 3\}$$

1) $f(x)$

$$f(x) = 2 + 8$$

(2) $f(x)$

$$(2) f(x) = 2 + 8$$

(3) $f(x)$

$$(3) f(x) = 2 + 8$$

(4) $f(x)$

$$(4) f(x) = 2 +$$

$f(x)$

$$f(x) = f(x + 2)$$

$$= -n^2 \{1 - \cos n\}$$

But $\cos n$

But $\cos n = 1$ (n even) and $\cos n = -1$ (n odd)

n^2 (n odd) and a_n

$$n^2$$
 (n odd) and $a_n = 0$

(n even) and b_n

$$(n \text{ even}) \text{ and } b_n = -1$$

(odd) $\times$ (even)

$$(odd) \times (even) = (odd) (-) \times (+) = (-)$$

6. $(2x + 3) \sin 4x$ neither (N) (O)

$$6. (2x + 3) \sin 4x \text{ neither (N) (O) } = (N)$$

9. $(x + 4) \sin 2x$ odd (E) (O)

$$9. (x + 4) \sin 2x \text{ odd (E) (O) } = (O)$$

$-a f(x) dx$

$$-a f(x) dx = 2$$

$-f(x) dx$

$$-f(x) dx = 2$$

$-f(x) \cos nx dx$

$$-f(x) \cos nx dx = 2$$

$$- \int f(x) \sin nx \, dx$$

$$- \int f(x) \sin nx \, dx = 0.$$

$$- \int f(x) \, dx$$

$$- \int f(x) \, dx = 2$$

$$- \int f(x) \cos nx \, dx$$

$$- \int f(x) \cos nx \, dx = 2$$

$$f(x)$$

$$f(x) = 2 + 8 \{\cos x - 1\}$$

$$- \int f(x) \, dx$$

$$- \int f(x) \, dx = -$$

$$- \int (\text{even function}) \, dx$$

$$- \int (\text{even function}) \, dx = 2$$

$$f(x)$$

$$f(x) = f(x + 2)$$

$$2 \, dx$$

$$2 \, dx = 1$$

$$\cos nx \, dx$$

$$\cos nx \, dx = 2$$

$$2n^2 \{\cos n - \cos 0\}$$

$$2n^2 \{\cos n - \cos 0\} = 2$$

$$n (\cos 2n - \cos n)$$

$$n (\cos 2n - \cos n) = 2$$

$$n \cos 2n$$

$$n \cos 2n = -2$$

$$\text{But } \cos 2n$$

$$\text{But } \cos 2n = 1.$$

$$n$$

$$n=1 \{a_n \cos nx + b_n \sin nx\}$$

y

$$y = f(x + 0)$$

$f(x)$

$$f(x) = f(x + 2)$$

$f(x) dx$

$$f(x) dx = 1$$

$f(x) \sin nx dx$

$$f(x) \sin nx dx =$$

$a \sin nx dx$

$$a \sin nx dx =$$

$\cos n$

$$\cos n = -1 \text{ (n odd)}$$

But $\cos n$

$$\cos n = 1 \text{ (n even) and } c$$

$f(x)$

$$f(x) = 2 +$$

Note also that at x

Note also that at $x =$, another finite

x

$x =$ in the series gives the same result.

to be 'undefined' at x

to be 'undefined' at $x = 0, x =$, etc.

of how the function behave between x

of how the function behave between $x = -$ and

$f(x)$

$$f(x) = f(x + 2)$$

a_0

$$a_0 = 2$$

$$- f(x) \cos nx \, dx$$

$$- f(x) \cos nx \, dx = 2$$

$$\cos n$$

$$\cos n = 1$$

$$(n \text{ even}) \text{ and } \cos n$$

$$(n \text{ even}) \text{ and } \cos n = -1 \text{ (n odd)}$$

$$x)$$

$$x) = 1 + x$$

$$f(x)$$

$$f(x) = f(x + 2)$$

$$= - \text{ and } x = 0 \text{ so that the graph is symmetrical}$$

$$0 (1+x) \sin nx \, dx$$

$$0 (1+x) \sin nx \, dx = 2$$

$$\cos n$$

$$\cos n = 2$$

$$(n \text{ even}) \text{ and } \cos n$$

$$(n \text{ even}) \text{ and } \cos n = -1$$

$$= 4+2$$

$$f(t)$$

$$f(t) = f(t + T)$$

angular velocity, radians per seconds, is define
angular velocity, radians per seconds, is defined by $= 2\pi f$,
then

$$T \text{ and } T$$

$$T \text{ and } T = 2$$

$$x$$

$$x=1 B_n \sin (nt + n)$$

$f(t)$

$$f(t) = 2(1+t)$$

$f(t)$

$$f(t) = f(t+2)$$

x

$$x = 1 \{a_n \cos nt + b_n \sin nt\}$$

$f(t) dt$

$$f(t) dt =$$

$(0) dt$

$$(0) dt = [2t + t^2]$$

$-1 t) \cos nt dt$

$$-1 t) \cos nt dt =$$

2

$$2 = a_n = 2$$

$-12(1+t) \sin nt dt$

$$-12(1+t) \sin nt dt = 2$$

before

$$\text{before} =$$

$f(t)$

$$f(t) = f(t+T)$$

$f(t)$

$$f(t) = f(t+T)$$

$f(t)$

$$f(t) = 4-t, 0 < t < 4.$$

But

$$\text{But} = 2$$

$\cos n$

$$\cos n = 1$$

$$(n \text{ even}); \cos n$$

$$(n \text{ even}); \cos n = -1$$

$$f(t)$$

$$f(t) = 2 + 1$$

where

$$\text{where} =$$

$$f(t)$$

$$f(t) = 3 + t$$

$$f(t)$$

$$f(t) = f(t + 4)$$

$$\int_0^1 f(t) \sin nt \, dt$$

$$\int_0^1 f(t) \sin nt \, dt =$$

$$\int_0^1 (3 + t) \sin nt \, dt$$

$$\int_0^1 (3 + t) \sin nt \, dt =$$

$$= (3 + 2) \cos n2$$

$$n^2 \int_0^1 (\sin n)$$

$$n^2 \int_0^1 (\sin n) = -2$$

$$-\frac{T}{2} \int_0^T u(t) \, dt$$

$$-\frac{T}{2} \int_0^T u(t) \, dt = 2$$

$$-\frac{T}{2} \int_0^T u(t) \cos nt \, dt$$

$$-\frac{T}{2} \int_0^T u(t) \cos nt \, dt = T$$

$$-\frac{T}{2} \int_0^T u(t) \sin nt \, dt$$

$$-\frac{T}{2} \int_0^T u(t) \sin nt \, dt = 2$$

$$2 \left(-\frac{1}{2} \right)$$

$$2 \left(-\frac{1}{2} \right) = E/2$$

$$\int_0^1 [\cos(1+n)t - \cos(1-n)t] \, dt$$

$$\int_0^1 [\cos(1+n)t - \cos(1-n)t] \, dt = -E$$

$$s^4 \mathcal{L}\{2\sin 3t + \cos 3t\}$$

$$s^4 \mathcal{L}\{2\sin 3t + \cos 3t\} = s + 6$$

e- between the limits t

e- between the limits $t=0$ and $t =$

The first Shift theorem states that if $\mathcal{L}\{F(t)\}$

The first Shift theorem states that if $\mathcal{L}\{F(t)\} = f(s)$ then
 $\mathcal{L}\{e(-at) F(t)\} = f(s+a)$

$\sin 2t\}$

$$\sin 2t\} = (s+3)$$

If $\mathcal{L}\{F(t)\}$

If $\mathcal{L}\{F(t)\} = f(s)$ then $\mathcal{L}\{e(-at) F(t)\} = f(s+a)$

$$-8 A(-2) + B(0)$$

$$-8 A(-2) + B(0) = 4$$

$$10$$

$$10 = (0) + (0) B = 5$$

$\mathcal{L}\{F(t)\}$

$$\mathcal{L}\{F(t)\} = - (0)$$

$\mathcal{L}\{F(t)\}$

$$\mathcal{L}\{F(t)\} = - (0)$$

x

$x = -2 + 3e$, To verify it we find dx

x

$x = -2 + 3e$ is solution of

Given $x(0)$

$$\text{Given } x(0) = 5, xP/$$

So $x(0)$

$$\text{So } x(0) = 5, xP/$$

Good Luck!

Keep practicing!