Virtual University	**MIDTERM EXAMINATION(Solution File)** **SEMESTER SPRING 2004** **MTH501- Linear Algebra**	**Total Marks:40** **Duration: 90min**

Instructions

1. **The duration of this examination is 90 minutes.**
2. **This examination is closed book, closed notes, closed neighbors.**
3. **Answer all questions.**
4. **Do not ask any questions about the contents of this examination from anyone.**
5. **All the questions are descriptive in that paper, you can't answer yes or no only and if in certain questions your answer is yes or no then you have to justify your answer.**
6. **There is no mistake in the question paper and if you feel that there is something wrong with the question then made the best assumptions which you think and then give the answer.**

Question No: 1 **Marks: 5**

Determine the value(s) of h such that the matrix $\begin{bmatrix} 1 & 4 & -2 \\ 3 & \boldsymbol{h} & -6 \end{bmatrix}$ is augmented matrix of a consistent system.

Solution:

$$\begin{bmatrix} 1 & 4 & -2 \\ 3 & \boldsymbol{h} & -6 \end{bmatrix} \square \begin{bmatrix} 1 & 4 & -2 \\ 0 & -12+\boldsymbol{h} & 0 \end{bmatrix} \text{ By } \boldsymbol{R}_2 - 3\boldsymbol{R}_1$$

It implies, $x_1 + 4\,x_2 = -2$

$(-12 + h)\,x_2 = 0$

Therefore, the above system of equations is consistent for all h.

Question No: 2 **Marks: 5**

Let $\boldsymbol{u} = \begin{bmatrix} 8 \\ 2 \\ 3 \end{bmatrix}$ and $\boldsymbol{A} = \begin{bmatrix} 4 & 3 & 5 \\ 0 & 1 & -1 \\ 1 & 2 & 0 \end{bmatrix}$. Is $\boldsymbol{u}$ in the subset of $\boldsymbol{R}^3$ spanned by the columns of $\boldsymbol{A}$? Why or why not?

Solution:

Consider $\boldsymbol{u} = a_1\,\boldsymbol{c_1} + a_2\,\boldsymbol{c_2} + a_3\,\boldsymbol{c_3}$

Where a_1, a_2 and a_3 are any scalars.

It implies that

$$\begin{bmatrix} 8 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4a_1 + 3a_2 + 5a_3 \\ a_2 - a_3 \\ a_1 + 2a_2 \end{bmatrix}$$

$$\Rightarrow \quad \begin{aligned} 4a_1 + 3a_2 + 5a_3 &= 8 \\ a_2 - a_3 &= 2 \qquad (1) \\ a_1 + 2a_2 &= 3 \end{aligned}$$

Now $\begin{vmatrix} 4 & 3 & 5 \\ 0 & 1 & -1 \\ 1 & 2 & 0 \end{vmatrix} = 4(0+2) - 0 + 1(-3-5) = 8 - 8 = 0$

Therefore, the system (1) is inconsistent. Hence $\boldsymbol{u}$ is not in the subset of $\boldsymbol{R}^3$ spanned by the columns of A.

Question No: 3 **Marks: 5**

Assume that T is a linear transformation. Find the standard matrix of T.

T: $\boldsymbol{R}^2 \rightarrow \boldsymbol{R}^4$, $T(\boldsymbol{e_1}) = (1, 2, 0, 5)$ and $T(\boldsymbol{e_2}) = (3, -6, 1, 0)$, where $\boldsymbol{e_1}$ and $\boldsymbol{e_2}$ are the columns of 2 x 2 identity matrix.

Solution:

The standard matrix is:

$$A = \begin{bmatrix} T(\boldsymbol{e_1}) & T(\boldsymbol{e_2}) \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & -6 \\ 0 & 1 \\ 5 & 0 \end{bmatrix}$$

Question No: 4 **Marks: 5**

Find the entries in the second row of $\boldsymbol{AB}$, where

$$\boldsymbol{A} = \begin{bmatrix} 2 & -5 & 0 \\ -1 & 3 & -4 \\ 6 & -8 & -7 \\ -3 & 0 & 9 \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} 4 & -6 \\ 7 & 1 \\ 3 & 2 \end{bmatrix}$$

Solution:

$$\rightarrow \begin{bmatrix} 2 & -5 & 0 \\ -1 & 3 & -4 \\ 6 & -8 & -7 \\ -3 & 0 & 9 \end{bmatrix} \begin{bmatrix} \downarrow & \downarrow \\ 4 & -6 \\ 7 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} * & * \\ -4+21-12 & 6+3-8 \\ * & * \\ * & * \end{bmatrix} = \begin{bmatrix} * & * \\ 5 & 1 \\ * & * \\ * & * \end{bmatrix}$$

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Question No: 5 **Marks: 5**

Solve the equation $\boldsymbol{Ax} = \boldsymbol{b}$ by using the $\boldsymbol{LU}$ factorization given for $\boldsymbol{A}$.

$$\boldsymbol{A} = \begin{bmatrix} 4 & 3 & -5 \\ -4 & -5 & 7 \\ 8 & 6 & -8 \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & -5 \\ 0 & -2 & 2 \\ 0 & 0 & 2 \end{bmatrix}$$

Solution:

Consider

$$\begin{cases} Ly = b \\ Ux = y \end{cases}$$

$$Ly = b \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix} \Rightarrow y_1 = 2, y_2 = -2, y_3 = 2$$

$$Ux = y \Rightarrow \begin{bmatrix} 4 & 3 & -5 \\ 0 & -2 & 2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} \Rightarrow x_1 = 1/4, x_2 = 2, x_3 = 1$$

Question No: 6 **Marks: 5**

Determine the matrix $\begin{bmatrix} 9 & -5 & 2 \\ 5 & -8 & -1 \\ -2 & 1 & 4 \end{bmatrix}$ is strictly diagonally dominant.

Solution:

Consider

$$|9| > |-5| + |2|$$
$$|-8| > |5| + |-1|$$
$$|4| > |-2| + |1|$$

Therefore, the given matrix is strictly diagonally dominant.

Question No: 7 **Marks: 5**

Let S be the parallelogram determined by the vectors $\boldsymbol{b}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\boldsymbol{b}_2 = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$, and let $\boldsymbol{A} = \begin{bmatrix} 1 & -.1 \\ 0 & 2 \end{bmatrix}$. Compute the area of the image of S under the mapping $\boldsymbol{x} \rightarrow \boldsymbol{Ax}$.

Solution:

The area of S is $\left| \det \begin{bmatrix} 1 & 5 \\ 3 & 1 \end{bmatrix} \right| = 14$, and det A = 2. Therefore, by definition, the area of image of S under the mapping $\boldsymbol{x} \rightarrow \boldsymbol{Ax}$ is |det A|.{area of S}= 2 . 14 = 28

Question No: 8 **Marks: 5**

Let $\boldsymbol{A} = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}$ and $\boldsymbol{w} = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$. Determine if $\boldsymbol{w}$ is in Col $\boldsymbol{A}$?

Solution:

Consider $\boldsymbol{w} = a_1 \boldsymbol{c_1} + a_2 \boldsymbol{c_2} + a_3 \boldsymbol{c_3}$
Where a_1, a_2 and a_3 are any scalars.
The Augmented matrix is:

$$\begin{bmatrix} -8 & -2 & -9 & 2 \\ 6 & 4 & 8 & 1 \\ 4 & 0 & 4 & -2 \end{bmatrix} \square \begin{bmatrix} 1 & 0 & 1 & -1/2 \\ 0 & 1 & 1/2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

It implies, $a_1 + a_3 = -1/2$
$a_2 + (1/2)a_3 = 1$
Hence $\boldsymbol{w}$ is in Col A.

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