MTH501 Linear Algebra

Mid Term Examination - November 2004

Time Allowed: 90 Minutes

Instructions

Please read the following instructions carefully before attempting any of the questions:

1. Attempt all questions. Marks are written adjacent to each question.
2. Do not ask any questions about the contents of this examination from anyone.

a. If you think that there is something wrong with any of the questions, attempt it to the best of your understanding.
b. If you believe that some essential piece of information is missing, make an appropriate assumption and use it to solve the problem.

****WARNING: Please note that Virtual University takes serious note of unfair means. Anyone found involved in cheating will get an `F` grade in this course.**

Total Marks: 45 Total
Questions: 11

Question No. 1 **Marks : 1**

Consider the system of Linear equations

$$x_1 + 4x_2 \square 5x_3 = 0$$

$$2x_1 \square x_2 + 8x_3 = 9$$

$$x_1 + x_2 \square 8x_3 = 2$$

Then determinant of the Matrix of Coefficients of the above system is

(a) 91
(b) 123
(c) 141
(d) **81**

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Question No. 2 **Marks : 2**

Consider the matrix $A = \begin{pmatrix} 2 & 3 & 4 \\ 2 & 3x & y \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 1 \\ 0 & 2 \\ 3x & f \end{pmatrix}$ such that

$$A + B = \begin{pmatrix} 4 & 4 & 5 \\ \leq 2 & 5 & 12 \end{pmatrix}$$

Then the values of x and y are

(a) x = - 2 and y = 4
(b) x = 2 and y = - 6
(c) x = 0 and y = 0
(d) x = 2 and y = -4

Question No. 3 **Marks : 1**

Which statement about the set S is false where S = {(1, 1, 3), (2, 3, 7),(2, 2, 6)}

(a) The set S contain an element which is solution of the equation 5x – y- z = 0
(b) The Set S is linearly independent.
(c) The set S contain two elements which are multiple of each other.
(d) The Set S is linearly dependent.

Question No. 4 **Marks : 2**

Consider a linear transformation T such that $T\begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \\ \leq 3 \end{pmatrix}$ Then $T\begin{pmatrix} 5 \\ 25 \\ \leq 15 \end{pmatrix} =$

(a) $\begin{pmatrix} 9 \\ \leq 18 \end{pmatrix}$

(b) $\begin{pmatrix} 1 \\ 0 \\ \leq 5 \end{pmatrix}$

(c) $\begin{pmatrix} 30 \\ \leq 45 \end{pmatrix}$

(d) $\begin{pmatrix} 12 \\ \leq 9 \end{pmatrix}$

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Question No. 5 **Marks : 1**

How many subspaces R_2 have?

(a) only two: {0} and R_2
(b) Only four: {0} *x*- axis and *y* -axis and R_2
(c) **Infinitely many.**
(d) None of the above.

Question No. 6 **Marks : 8**

An oil refinery produces low-sulfur and high-sulfur fuel. Each ton of low-sulfur requires 5 minutes in the blending plant and 4 minutes in the refining plant; each ton of high sulfur fuel requires 4 minutes in the blending plant and 2 minutes in the refining plant. If the blending plant is available for 3 hours and the refining plant is available for 2 hours, how many tons of each type of fuel should be manufactured so that the plants are fully utilized?

Solution:

The data given in the question can be formed into system of linear equations as

	Low-sulfur	High-sulfur
Blending plant	5	4
Refining Plant	4	2

As we are given in the question that we have blending plant and refining plant available for 3 and 2 hours respectively. Let x tons of low-sulfur and y tons of high sulfur be the amount should be manufactured so that plants are fully utilized. Then from the above data we must have the system,

$$5x + 4y = 180$$
$$4x + 2y = 120$$

Augmented matrix for the above system

$$\begin{pmatrix} 5 & 4 & 180 \\ 4 & 2 & 120 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 60 \\ 2 & 120 & \end{pmatrix} \quad by\ R_1 \square R_2$$

$$\sim \begin{pmatrix} 1 & 2 & 60 \\ 0 \square 6 \square & 120 \end{pmatrix}$$

is

So we have x + 2y = 60 and -6y = - 120, thus we have y= 20 tons and x = 20tons are the required manufactured tons of each low-sulfur and high-sulfur so that we can utilize both plants for the given time.

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$$A = \begin{pmatrix} 3 & 1 & \\ 4 & 1 & \\ 0 & & \end{pmatrix}$$

Consider the matrix then find the entry a_{33} of the matrix A^2 without calculating the matrix A^2.

Solution:

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$$A = \begin{bmatrix} 3 & 1 & \\ 4 & 1 & \\ 7 & 0 & \end{bmatrix}$$

Find LU decomposition for the matrix .

Solution:

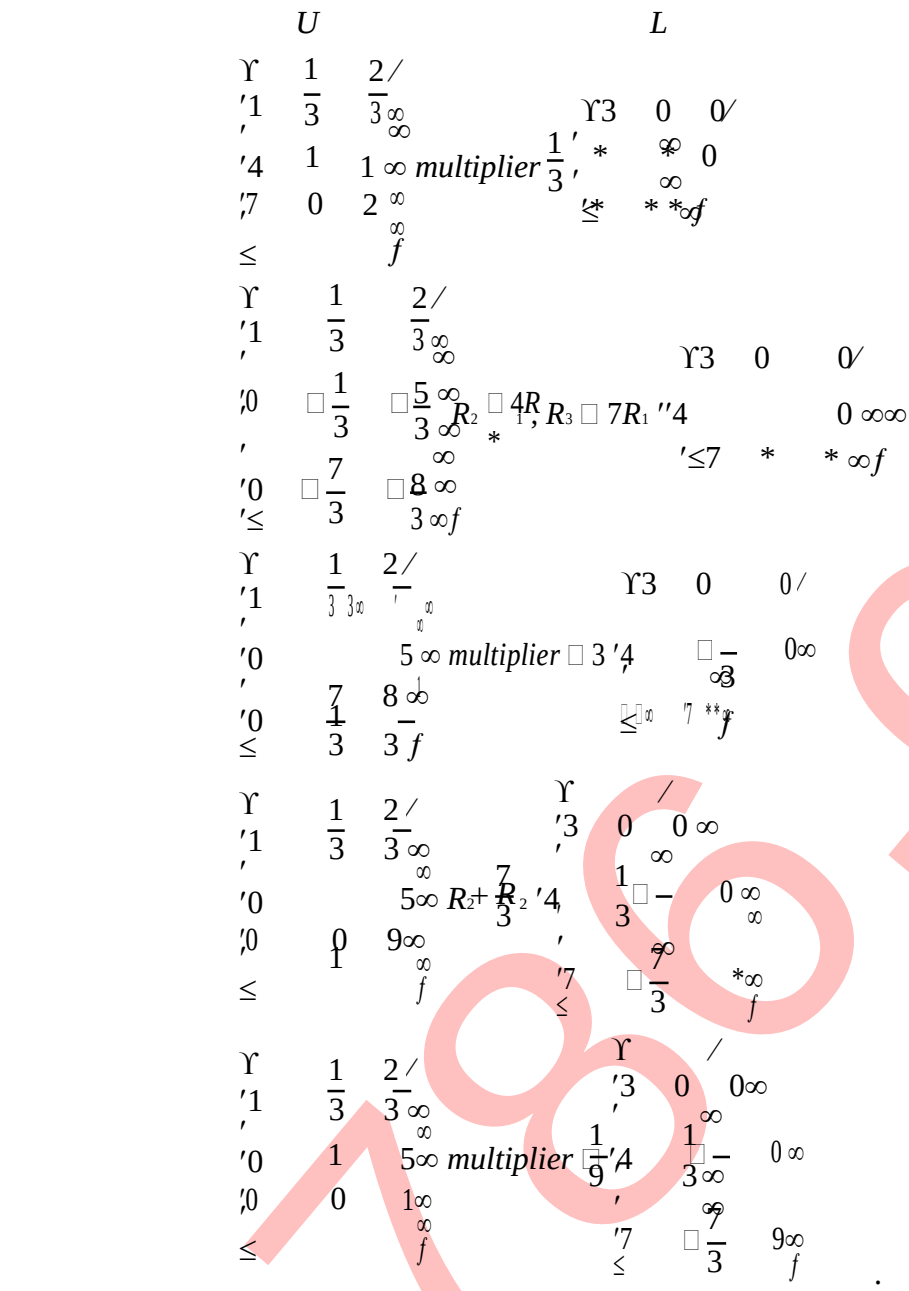
[redacted]

Required entry a_{33} of $A =$ [redacted]

marks : 3 = [redacted]

[redacted]

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The required LU factorization is

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$$L = \begin{bmatrix} 3 & 0 & 0 \\ 4 & -\frac{1}{3} & 0 \\ 7 & -\frac{7}{3} & 9 \end{bmatrix} \quad U = \begin{bmatrix} 1 & \frac{1}{3} & \frac{2}{3} \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

Question No. 9 **Marks : 8**

$$v = \begin{bmatrix} 4 \\ -9 \\ 2 \end{bmatrix} \qquad u_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, u_2 = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix} \text{ and } u_3 = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$$

Write v as linear combination of u_1, u_2, u_3. (That is you have to find out the constants such that $v = c_1u_1 + c_2u_2 + c_3u_3$)

Solution:

We will try to find out the constants such that we can write $v = c_1u_1 + c_2u_2 + c_3u_3$ and we get the augmented matrix of the system of linear equations correspond to that linear combination as

$$\begin{bmatrix} 1 & 1 & 1 & 4 \\ 2 & 4 & -3 & 9 \\ -1 & 2 & 2 & 2 \end{bmatrix}$$

And echelon form of that matrix

$$\begin{bmatrix} 1 & 1 & 1 & 4 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

is $\Rightarrow x_3 = 3, x_2 = -1, x_1 = 2$

Question No. 10 **Marks : 5**

$$M \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Show that the mapping $M: R3 \quad R_3$ defined by is a linear transformation. Also write the matrix of that linear transformation.

Solution:

We will prove that the mapping satisfies the conditions,

(i) $M(\boldsymbol{u} + \boldsymbol{v}) = M(\boldsymbol{u}) + M(\boldsymbol{v})$ (ii) $M(c\boldsymbol{u})$

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$\Upsilon a + d/\ \Upsilon 1\ 0 \qquad 1/\Upsilon a + d/\ \Upsilon 1\ 0 \qquad 1/\clubsuit\Upsilon a/\ \Upsilon d/\leftrightarrow$

$M' \quad \infty \quad ' \qquad 1\infty' \quad \infty \quad '$

(i) $'c + f\ f\ '1\ 0 \qquad 1\infty'c + f\ f\ '1\ 0 \qquad 1\infty f\ \blacktriangledown\spadesuit'\leq c\ \infty f$ Now as we know that

$'\leq f\infty f\uparrow\spadesuit$

matrices are distributed over addition that is if A, B and C are matrices then A (B + C) = AB + BC, so we can write,

$\Upsilon \qquad \Upsilon 1\ 0\ 1/\Upsilon a/\ \Upsilon 1\ 0\ 1/\Upsilon d/$

$a \qquad 1$
1

$+ \qquad 'c + f\infty\ '1\ 0\ 1f\ \spadesuit'c\infty\ 'f\infty\spadesuit\ '1\ 0$
$1f\ 'c\infty\ '1\ 0\ 1\infty'f\infty$

d

$/$

Υ
1

0

1

$/$

$\clubsuit$
Υ
a

$/$

Υ
d

$/$

$\leftrightarrow$

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also by

$$\Upsilon a/\Upsilon 1\ 0\ 1/\Upsilon a/ \quad \Upsilon d/\Upsilon 1\ 0\ 1/\Upsilon d/$$

$$M'\infty' \quad \infty'\infty \quad '\infty' \quad \infty'e\infty$$

$$'c\infty'1\ 0\ 1f'c\infty \quad 'f\infty'1\ 0\ 1\infty'f\infty$$

definition we have . Hence we have,

$$'b+e\infty='0\ 1 \quad \infty'b+e\infty='0\ 1 \quad 1\infty\infty \blacklozenge''b\infty\infty+''e$$

$$\Upsilon a+d/ \quad \Upsilon a/ \quad \Upsilon d/$$

$$\leq \quad \infty \quad \leq \quad f \leq \quad \infty \quad \leq$$

$$M' \quad \infty \quad '\infty \quad '\infty$$

$$'c+f\,f \quad 'c\infty \quad 'f\infty f$$

.

$$\Upsilon cd/\Upsilon 1\ 0 \quad 1/\Upsilon cd/\Upsilon 1\ 0 \quad 1/\clubsuit\ \Upsilon d/\leftrightarrow \quad \Upsilon 1\ 0 \quad 1/\Upsilon cd/$$

$$M\propto\infty M' \quad ' \quad 1\infty\blacklozenge'\infty''\infty\leftarrow \quad 1\infty\blacklozenge'\infty\leftarrow \quad ' \quad ce$$

$$\leq \quad f\ 'ef\ f'1\ 0\infty\heartsuit\leq f \quad 1\infty f\uparrow'cf\leq'1\ 0 \quad \infty\leq f1\infty\spadesuit'f\infty\leq f'1\ 0 \quad 1\infty'cf\,f$$

As we have A(cB) = cAB where c is a real number.

$$\Upsilon cd/ \quad \Upsilon d$$

$$'ce \quad '\leq\infty f\leq \quad \infty\leq f \quad \leq f\leq \quad f\leq f$$

$$'\leq cf\infty f \quad '\leq f\infty f$$

(ii)

$$\Upsilon 1\ 0 \quad 1/$$

$$'b+e\infty=M'b\infty+M'e\infty \qquad ' \quad 1\infty\infty$$

$$\leq \quad \infty \quad \leq f \quad \leq \qquad '1\ 0 \quad 1\infty f$$

$$'ce\infty='0\ 1 \quad \infty'ce\infty='0\ 1 \quad \infty\spadesuit c'e\infty\spadesuit=c'0\ 1$$

Solve the system 3x + y = 12 by using Jacobi's Method starting x_0 = (0, 0, 0) and only three iterations. $1\infty\infty''\infty\infty$

$$\leq\infty\leq \quad f\leq\infty\leq \quad f\heartsuit\leq f\uparrow \quad \leq \quad f\leq\infty$$

2x – 4y = -6

Solution:

$$M'\infty 1=cM \quad 'e\infty$$

Iteration 1
(12/3 , 3/2)
iteration 2
(3.5 , 3.5)
iteration 3 '0 1

Hence M is a linear transformation and matrix of transformation is ≤

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(2.833, 3.25)

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