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— Qalb-e-Momin

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Practice Questions

Chapter No. 09

Question no 1

Consider a linear transformation $T : R^2 \to R^2$ defined by $T(\vec{x}) = A\ \vec{x}$ then verify that $T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v})$, where

$$A=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix},\ \vec{u}=\begin{bmatrix}2\\ 0\end{bmatrix} \text{ and } \vec{v}=\begin{bmatrix}6\\ -1\end{bmatrix}.$$

Solution

$$\vec{u}+\vec{v}=\begin{bmatrix}2\\ 0\end{bmatrix}+\begin{bmatrix}6\\ -1\end{bmatrix}=\begin{bmatrix}8\\ -1\end{bmatrix}$$

$$T(u+v)=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}\begin{bmatrix}8\\ -1\end{bmatrix}=\begin{bmatrix}24\\ -3\end{bmatrix}$$

now

$$T(\vec{u})=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}\begin{bmatrix}2\\ 0\end{bmatrix}=\begin{bmatrix}6\\ 0\end{bmatrix}$$

and

$$T(\vec{v})=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}\begin{bmatrix}6\\ -1\end{bmatrix}=\begin{bmatrix}18\\ -3\end{bmatrix}$$

$$\Rightarrow\ T(\vec{u})+T(\vec{v})=\begin{bmatrix}6\\ 0\end{bmatrix}+\begin{bmatrix}18\\ -3\end{bmatrix}=\begin{bmatrix}24\\ -3\end{bmatrix}$$

$$\therefore\ T(\vec{u}+\vec{v})=T(\vec{u})+T(\vec{v}) \text{ verified.}$$

Question no 2

Consider a linear transformation $T: R^2 \to R^2$ defined by $T(\vec{x})=A\ \vec{x}$ then find the value of $\vec{x}$ such that $A\ \vec{x}=\vec{b}$, where

$$A=\begin{bmatrix}3 & 3\\ 1 & 4\end{bmatrix} \text{ and } \vec{b}=\begin{bmatrix}4\\ 6\end{bmatrix}.$$

Solution

We have to find $\vec{x}$ such that $T(x)=\vec{b}$ or $A\vec{x}=\vec{b}$. That is,

$$\begin{bmatrix}3 & 3\\ 1 & 4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}4\\ 6\end{bmatrix}$$

$$\Rightarrow 3x_1+3x_2=4$$

$$x_1+4x_2=6$$

Solving them simultaneously, it gives

$x_1 = -\frac{2}{9}$, $x_2 = \frac{14}{9}$

Hence $\vec{x} = \begin{bmatrix} -\frac{2}{9} \\ \frac{14}{9} \end{bmatrix}$

Note: $\vec{x}$ can also be calculated by reducing the augmented matrix of the system $Ax = \vec{b}$.

Question no 3

Consider a linear transformation $T : R^2 \to R^2$ defined by $T(\vec{x}, \vec{y}) = (2x - y, 0)$ then find $T(\vec{u})$, where $\vec{u} = (-1 , 4)$.

Solution

$$T(-1 , 4) = (-1 - 4 , 0)$$
$$= (-5 , 0)$$

Question no 4

Consider a linear transformation $T : R^2 \to R^2$ defined by

$$T(\vec{u}) = A\vec{u} = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + 3x_2 \\ 2x_1 - x_2 \end{bmatrix}$$

Find $T(\vec{u})$, where

$\vec{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

Solution

$$T(\vec{u}) = A\vec{u} = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 + 3 \\ 2(3) + 1(-1) \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

Question no 5

Show that the transformation $L : R^3 \to R^2$ defined by $L(x,\ y,\ z) = (3y - z,\ x)$ is linear by proving $L(\vec{u} + \vec{v}) = L(\vec{u}) + L(\vec{v})$ and $L(c\,\vec{u}) = c\,L(\vec{u})$, *where c is scalar*.

Solution

Let $\vec{u} = (x_1,\ y_1,\ z_1)$ and $\vec{v} = (x_2, y_2, z_2)$, then

$$L(\vec{u} + \vec{v}) = L((x_1,\ y_1, z_1) + (x_2, y_2, z_2))$$

$$= L(x_1 + x_2,\ y_1 + y_2,\ z_1 + z_2)$$

$$= (3(y_1 + y_2) - (z_1 + z_2)\ ,\ x_1 + x_2)$$

$$= (3y_1 - z_1 + 3y_2 - z_2\ ,\ x_1 + x_2)$$

$$= (3y_1 - z_1,\ x_1) + (3y_2 - z_2, x_2)$$

$$= L(\vec{u}) + L(\vec{v})$$

$$L(c\,\vec{u}) = L(c\,(x_1,\ y_1, z_1))\ \ ;\ \textit{where c is scalar}$$

$$= L(c\,x_1,\ c\,y_1, c\,z_1))$$

$$= (3c\,y_1 - c\,z_1,\ c\,x_1)$$

$$= c\,(3y_1 - z_1,\ x_1)$$

$$= c\,L\,(x_1, y_1, z_1)$$

$$= c\,L\,(\vec{u})$$

Question no 6

Consider a linear transformation $L : R^2 \to R^2$ for which $L(i) = L\,(1\ ,\ 0) = (1\ ,\ 2)$ and $L(j) = L\,(0\ ,\ 1) = (3\ ,\ -2)$

are given. Find $L\,(-3\ ,\ 4)$, where $(-3\ ,\ 4) = -3i + 4\,j$.

Since $(-3\ ,\ 4) = -3i + 4\,j$,

$$
\begin{aligned}
L(-3\ ,\ 4) &= L(-3i + 4j) \\
&= -3L(i) + 4L(j) \\
&= -3(1\ ,\ 2) + 4(3\ ,\ -2) \\
&= (-3\ ,\ -6) + (12\ ,\ -8) \\
&= (\ 9\ ,\ -14)
\end{aligned}
$$