ELECTRIC POTENTIAL.

ELECTRIC POTENTIAL ENERGY DIFFERENCE.

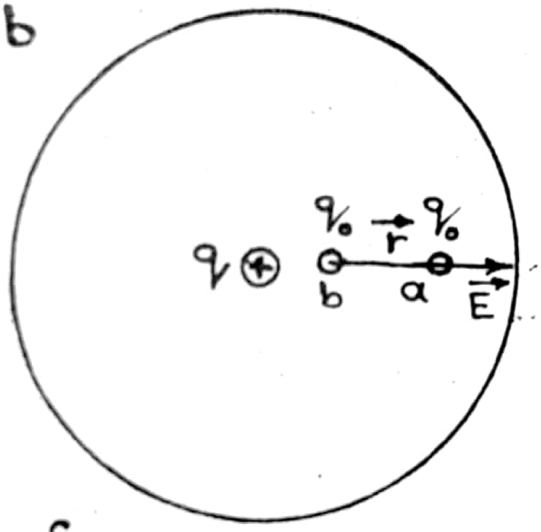
The electric potential energy difference between two points is equal to negative of work done by electrostatic force (conservative) in moving charge q_0 between two points, in an electric field with constant velocity.

Change in electrostatic P·E $= \Delta U = -W \longrightarrow (1)$

If test charge q_0 moves in electric field $\vec{E}$ from point a to b under the action of electrostatic force

$\vec{F} = q_0 \vec{E}$

then by equation (1)



$$\Delta U = -\int_a^b \vec{F} \cdot d\vec{S}$$

$$U_b - U_a = -q_0 \int_a^b \vec{E} \cdot d\vec{S} = -q_0 \int \vec{E} \cdot d\vec{r} \longrightarrow (2)$$

Because, electric force is conservative, the integral is independent of path and depends only on initial position 'a' and final position 'b'.

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If electric field is produced due to positive point charge q, then by eq. (2)

$$U_b - U_a = -q_0 \int_{r_a}^{r_b} \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r} \cdot \hat{r}\, dr$$

Dot prod of unit vectors $\hat{i}\cdot\hat{i}=1$

$$U_b - U_a = -\frac{1}{4\pi\epsilon_0} q q_0 \int_{r_a}^{r_b} r^{-2}\, dr = -\frac{q q_0}{4\pi\epsilon_0} \left[\frac{-1}{r}\right]_{r_a}^{r_b}$$

$$U_b - U_a = \frac{q q_0}{4\pi\epsilon_0} \left[\frac{1}{r_b} - \frac{1}{r_a}\right] \longrightarrow (3)$$

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ELECTRIC POTENTIAL, V :- Electric potential at a point is defined as the negative of work done by electric field in moving a unit positive test charge from infinity, where potential is taken to be zero, to given point.

If point "a" is at infinity then $V_a = V_\infty = 0$

Also, if 'b' is at distance r from the source charge then by equation ② $V_b = V_r$ is given by

$$V_{(r)} - V_\infty = -\int_\infty^r \vec{E} \cdot d\vec{s}$$

Taking $d\vec{s} = d\vec{r}$

OR

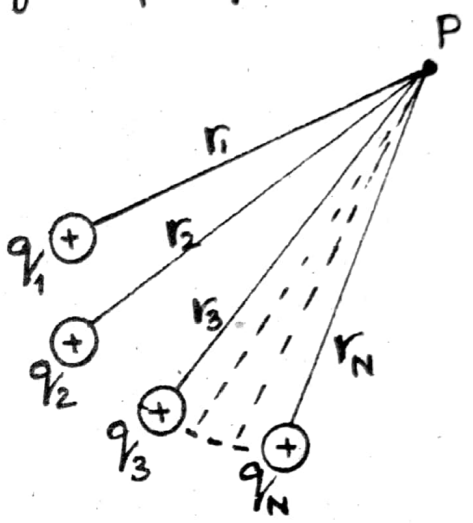
$$V_{(r)} = -\int_\infty^r \vec{E} \cdot d\vec{r} \longrightarrow ③$$

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POTENTIAL DUE TO COLLECTION OF CHARGES.

To find the potential at field point P, due to system of N- charges $q_1, q_2 \dots q_N$ located at distances $r_1, r_2 \dots r_N$ from point P, use principle of superposition.

Find the potential at point P, due to each charge turn by turn, keeping all other charges absent except one due to which potential is being determined.



Add algebric, potentials due to all the charges to get total potential at point P.

If $V_1, V_2, V_3 \dots V_N$ is potential at P due to respective charges, then

$$V = V_1 + V_2 + \dots + V_N$$

$$V = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\epsilon_0}\frac{q_2}{r_2} + \dots + \frac{1}{4\pi\epsilon_0}\frac{q_N}{r_N}$$

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \dots + \frac{q_N}{r_N}\right]$$

$$V = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N} q_i / r_i$$

Similarly,

POTENTIAL due to volume charge distribution $= \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho\, dV}{r}$

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POTENTIAL DUE TO ELECTRIC DIPOLE.

Two equal and opposite charges separated by small distance (d) constitute an electric dipole.

ELECTRIC DI-POLE MOMENT, $\vec{P}$:- (Vector)

Magnitude of Electric dipole moment is equal to the product of magnitude of either charge and distance between the two charges. It is directed from negative to positive charge. Thus if charges $+q$ and $-q$ are separated by small distance d, then, Magnitude of dipole moment $= P = qd$

FORMULA FOR POTENTIAL DUE TO DIPOLE.

Consider a dipole consisting of charges $+q$ and $-q$ separated by distance d, and placed along Z-axis.

Field point P where potential due to dipole is to be determined at a distance r and at an angle θ from center of dipole. Also, distance of P from $+q$ is r_1 and from $-q$ is r_2.

Potential at P due to $+q = V_1 = \frac{1}{4\pi\epsilon_0} \times \frac{q}{r_1}$

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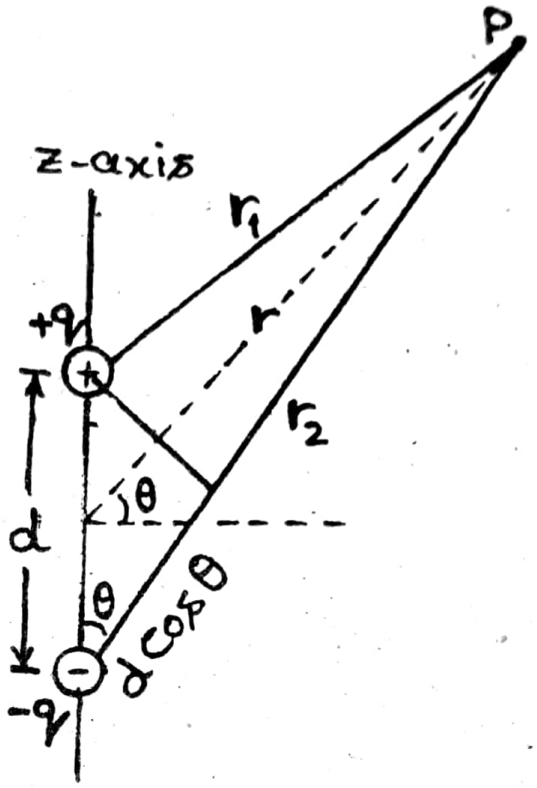
Potential at P due to $-q = V_2 = \frac{1}{4\pi\epsilon_0} \times \frac{-q}{r_2}$

Resultant potential at P $= V = V_1 + V_2$

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r_1} - \frac{1}{4\pi\epsilon_0}\frac{q}{r_2}$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{r_2 - r_1}{r_1 r_2}\right] \longrightarrow (1)$$



As $r >> d$ then

$r_2 - r_1 = d\cos\theta$

$r_1 r_2$ = Geometric mean between r_1 and $r_2 = r^2$

Put values of $r_2 - r_1$ and $r_1 r_2$ in equ. (1)

$$V = \frac{q}{4\pi\epsilon_0} \times \frac{d\cos\theta}{r^2} = \frac{1}{4\pi\epsilon_0} \times \frac{qd}{r^2}\cos\theta$$

$$\boxed{V = \frac{1}{4\pi\epsilon_0} \times \frac{P\cos\theta}{r^2}} \longrightarrow (4)$$

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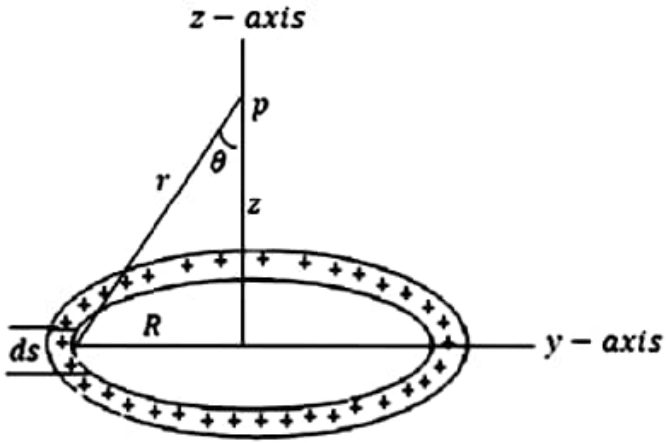
This is the expression of electric potential at any point 'P' due to a dipole.

30.7 Electric Potential of Continuous Charge Distribution

30.7.1 Electric Potential due to a Ring of Charge

Consider a ring of positive charge of radius 'R' as shown in the figure.We want to find out the electric potential at point 'P' which is at the distance 'z' from the plane of ring.

As the charge is distributed uniformly over it, so it has constant linear charge density λ. For an infinitesimal length element 'ds' of ring,



$$\lambda = \frac{dq}{ds}$$

$$dq = \lambda\, ds$$

The electric potential due to the charge dq at point 'P' is given by;

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$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r}$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{\sqrt{z^2+R^2}}$$

$\therefore From\ Figure, r = \sqrt{z^2 + R^2}$

The total value of electric potential is

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda\, ds}{\sqrt{z^2 + R^2}}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{z^2 + R^2}}\int \lambda\, ds$$

$\therefore \int \lambda\, ds = q$

Therefore,

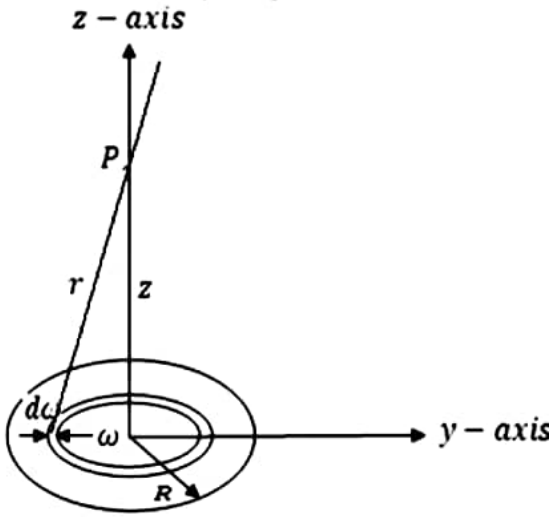
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{z^2 + R^2}}$$

This is the expression of electric potential at any point 'P' due to a ring of charge.

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30.7.2 Electric Potential due to a Disk of Charge

Consider a circular disk of uniform surface charge density as shown in the figure below. We want to find out the electric potential at point 'P' which is at the distance 'z' from the plane of disk.



Consider a small element of the disk in the ring shape of radius 'ω' and width '$d\omega$'. If 'dq' is the charge on this element of ring, then

$$\sigma = \frac{dq}{dA}$$

$$\therefore \textit{where } dA \textit{ is the is the area of length element}$$

$$dq = \sigma\, dA$$

$$dq = \sigma\,(2\pi\omega\, d\omega)$$

$$\therefore dA = 2\pi\omega\, d\omega$$

We know the electric potential due to the ring of charge which has the radius ω is given by;

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\sigma\,(2\pi\omega\, d\omega)}{z^2 + \omega^2}$$

The electrical potential at point 'P' due to whole disk is

$$V = \frac{1}{4\pi\varepsilon_0}\int_{\omega=0}^{\omega=R} \frac{\sigma\,(2\pi\omega\, d\omega)}{z^2 + \omega^2}$$

$$V = \frac{\sigma}{4\varepsilon_0}\int_{\omega=0}^{\omega=R} (z^2 + \omega^2)^{-\frac{1}{2}}\ (2\omega)\, d\omega$$

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30.8 Equipotential Surfaces

If all the points on a surface have the same value of electric potential, then, it is known as equipotential surface. The examples of some equipotential surfaces are given below

- In case of uniform electric field, the equipotential surfaces are the planes as shown by the dashed line in fig. (a). The potential at points B_1, B_2 and B_3 is same. So, no work will be done in moving a test charge from B_1 to B_2 or B_3.

- The concentric spheres about a point charge '$+q$' are the equipotential surfaces as shown in the fig. (b) by dashed lines.

- The equipotential surfaces of a dipole are shown by the dashed lines in fig. (c)

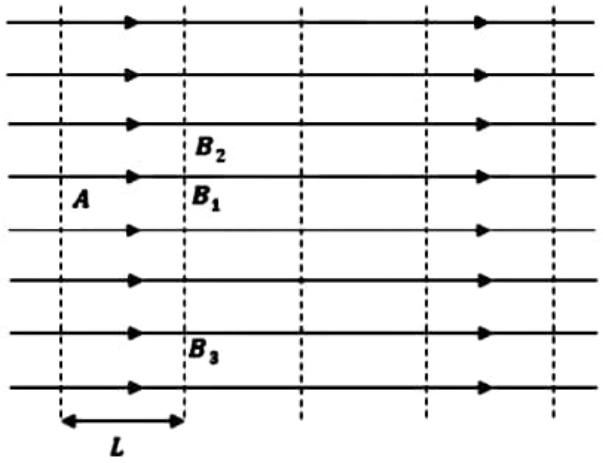


Fig. (a)

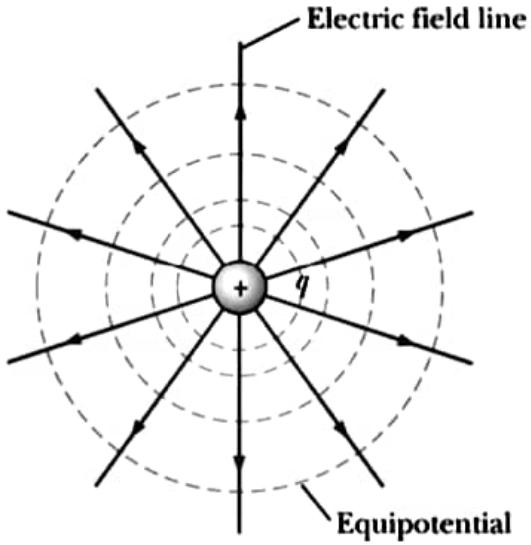


Fig. (b)

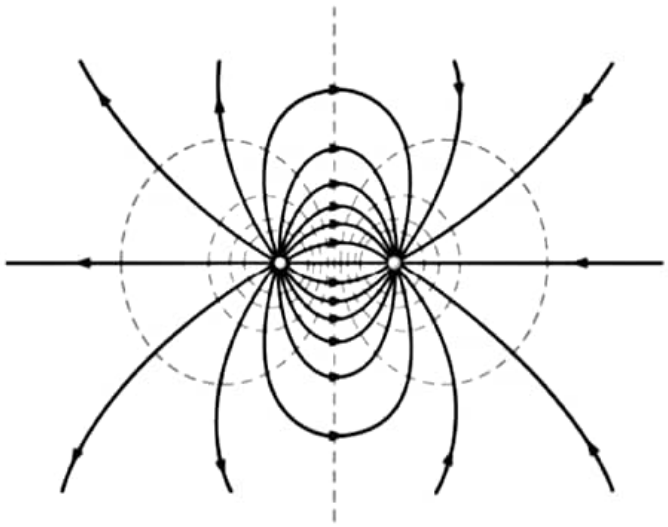


Fig. (c)

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