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— Qalb-e-Momin

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Practice Questions

Chapter No. 10 to 12

Question

Let $L : R^2 \to R^2$ be a linear operator defined by $L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x + 2y \\ x - y \end{bmatrix}$. Find the standard matrix A representing L *i.e*; $L = [L(\mathrm{e}_1), L(\mathrm{e}_2)]$.

Solution

The standard matrix A representing L is the 2×2 matrix whose columns are $L(e_1)$ and $L(e_2)$ respectively.

So,

$$L(e_1) = L\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$$
$$= \begin{bmatrix} 2+2(0) \\ 1-0 \end{bmatrix}$$
$$= \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Similarly

$$L(e_2) = L\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$
$$= \begin{bmatrix} 2(0)+2(1) \\ 0-1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Therefore

$$A = \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix}$$

Question

Show that $L(\vec{x}) = A\vec{x}$ for the linear operator $L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - y \\ 2x + y \end{bmatrix}$, where A is a standard matrix and

$L(e_1)=\begin{bmatrix}1\\2\end{bmatrix}$, $L(e_2)=\begin{bmatrix}-1\\1\end{bmatrix}$

Solution

To show that $L(x)=Ax$, we have to find standard matrix first which is of order 2×2 with columns are $L(e_1)$ and $L(e_2)$ respectively.
So,

Therefore, $A=\begin{bmatrix}1 & -1\\2 & 1\end{bmatrix}$

Now,

$$Ax=\begin{bmatrix}1 & -1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

$$\Rightarrow \quad Ax=\begin{bmatrix}x-y\\2x+y\end{bmatrix}=L(x)$$

Question

If $A=B$, then find the values of x and y, where $A=\begin{bmatrix}1 & 2 & x-2\\3 & 4 & 5\\6 & 5+y & 0\end{bmatrix}$ and

$B=\begin{bmatrix}1 & 2 & 1-x\\3 & 4 & 5\\6 & -y & 0\end{bmatrix}$.

Solution

Since both the matrices are equal, their corresponding entries will also be equal.

That is

$$x-2=1-x$$
$$x+x=1+2$$
$$2x=3$$
$$x=\frac{3}{2}$$

Also,

$$5+y=-y$$
$$2y=-5$$
$$y=\frac{-5}{2}$$

Question

If $A=\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1\\ 6\end{bmatrix}$, then show that $(AB)^t=B^tA^t$.

Solution

$$AB=\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1\\ 6\end{bmatrix}$$
$$=\begin{bmatrix}2+0\\ 0+6\end{bmatrix}$$
$$=\begin{bmatrix}2\\ 6\end{bmatrix}$$

Now

$$(AB)^t=\begin{bmatrix}2 & 6\end{bmatrix}$$

$$B^tA^t=\begin{bmatrix}1 & 6\end{bmatrix}\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}2+0 & 0+6\end{bmatrix}$$
$$=\begin{bmatrix}2 & 6\end{bmatrix}$$
$$=(AB)^t$$

Therefore, $(AB)^t = B^t A^t$

Evaluate the determinant of the matrix $A = \begin{bmatrix} 2 & 3 & 1 & -2 \\ 0 & 4 & 6 & 8 \\ 0 & 2 & 2 & 3 \\ 0 & 1 & 1 & 3 \end{bmatrix}$.

The determinant of the given matrix is calculated as

$$\det(A) = \begin{vmatrix} 2 & 3 & 1 & -2 \\ 0 & 4 & 6 & 8 \\ 0 & 2 & 2 & 3 \\ 0 & 1 & 1 & 3 \end{vmatrix}$$

Expanding it for first column, it gives

$$= 2 \begin{vmatrix} 4 & 6 & 8 \\ 2 & 2 & 3 \\ 1 & 1 & 3 \end{vmatrix}$$

$$= 2[4(6-3) - 6(6-3) + 8(2-2)]$$

$$= 2[12 - 18 + 0]$$

$$= -12$$

So, $\det(A) = -12$

Question

If $A = \begin{bmatrix} -1 & 2 \\ 3 & -7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix}$, then calculate the inverse of AB (if exists) ; where $A^{-1} = \begin{bmatrix} -7 & -2 \\ -3 & -1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}$.

Solution

We know that $(AB)^{-1} = B^{-1}A^{-1}$

Prove it by using simple matrix multiplication.

To find the inverse of AB, use the matrix identity:

$$(AB)^{-1} = B^{-1}A^{-1}$$

$(AB)^{-1} = B^{-1}A^{-1}$

Given

$$A^{-1} = \begin{bmatrix} -7 & -2 \\ -3 & -1 \end{bmatrix}, \qquad B^{-1} = \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}$$

we compute

$$(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix} \begin{bmatrix} -7 & -2 \\ -3 & -1 \end{bmatrix}$$

Now multiply:

$$\begin{aligned} (1,1) &= 4(-7) + (-3)(-3) = -28 + 9 = -19 \\ (1,2) &= 4(-2) + (-3)(-1) = -8 + 3 = -5 \\ (2,1) &= (-5)(-7) + 4(-3) = 35 - 12 = 23 \\ (2,2) &= (-5)(-2) + 4(-1) = 10 - 4 = 6 \end{aligned}$$

Therefore,

$$(AB)^{-1} = \begin{bmatrix} -19 & -5 \\ 23 & 6 \end{bmatrix}$$

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