MTH501-Linear Algebra
Midterm Special 2006

Question # 1:-

Let A= $\begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$ Write 5A.Is det5A=5detA?

$$5A = \begin{pmatrix} 15 & 5 \\ 20 & 10 \end{pmatrix}$$

Let A = $\begin{pmatrix} 3 & 1 \\ 4 & 2 \end{pmatrix}$

$\det 5A = 5^2 \det A$

$\det 5A = 5^2((3 \cdot 2) - (4 \cdot 1))$

$\det 5A = 25(6 - 4)$

$\det 5A = 25(2)$

$\det 5A = 50$

$\det 5A = 5^2 \det A$

$\det 5A = 25(2)$

$\det 5A = 50$

$\det A = ((3 \cdot 2) - (1 \cdot 4))$

$\det A = 6 - 4 = 2$

$5\det A = 5 \cdot 2 = 10$

Hence proved det5A not equal to 5detA

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Question #2:-

Let A = $\begin{pmatrix} 5 & 1 \\ 3 & 2 \end{pmatrix}$ and B = $\begin{pmatrix} 2 & 0 \\ 4 & 3 \end{pmatrix}$ .Show that these matrices do not commute.

Solution:-

$$AB = \begin{pmatrix} 5 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 14 & 3 \\ 14 & 6 \end{pmatrix}$$

$$BA = \begin{pmatrix} 2 & 0 \\ 4 & 3 \end{pmatrix}\begin{pmatrix} 5 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} 10 & 2 \\ 29 & 10 \end{pmatrix}$$

$AB \neq BA$

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Question # 3:-

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Use Cramer's rule to solve the system $3X_1 - 2X_2 = 6$, $-5X_1 + 4X_2 = 8$

Note : In order to get full marks do all necessary steps

Solution:-

$3X_1 - 2X_2 = 6$

$-5X_1 + 4X_2 = 8$

View the System as Ax =b. Using the notation introduced above**.**

$$A = \begin{pmatrix} 3 & -2 \\ -5 & 4 \end{pmatrix}$$

$$A_1(b) = \begin{pmatrix} 6 & -2 \\ 8 & 4 \end{pmatrix}$$

$$A_2(b) = \begin{pmatrix} 3 & 6 \\ -5 & 8 \end{pmatrix}$$

Since detA = 2, the system has a unique solution. By Cramer's rule,

$$x_1 = \frac{\det A_1(b)}{\det A} = \frac{24+16}{2} = 20$$

$$x_2 = \frac{\det A_2(b)}{\det A} = \frac{24+30}{2} = 27$$

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Question # 4:-

Find the matrix of linear transformation $T : R_3 \to R_4$ defined by

$T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_1 - x_3, x_1)$ with respect to the standard basis of R^3 ?

Note : In order to get full marks do all necessary steps

Solution:-

The standard basis of R_3 is $\{e_1, e_2, e_3\}$

$E_1 = (1, 0, 0)$

$E_2 = (0, 1, 0)$

$E_3 = (0, 0, 1)$

$T(e_1) = T(1, 0, 0) = (1, 0, 1, 1)$

$T(e_2) = T(0, 1, 0) = (1, 1, 0, 0)$

$T(e_3) = T(0, 0, 1) = (0, 1, -1, 0)$

The matrix of linear transformation is

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$$A=\begin{pmatrix}1 & 1 & 0\\ 0 & 1 & 1\\ 1 & 0 & 1\\ 1 & 0 & 0\end{pmatrix}$$

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$$A=\begin{pmatrix}3 & 2 & 1\\ 0 & k & 2\\ 0 & 4 & 2\end{pmatrix}$$

If $|A|=0$ such that then the value of k is

8

4

0

None of these

$$x+y=1$$
$$x+y=0$$

The system of linear equations has

No solution

Infinitely many solutions

Infinitely many solutions

d) None of these

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$$A=\begin{pmatrix}4 & 3\\ 7 & 2\\ 9 & 0\end{pmatrix} \text{ and } B=\begin{pmatrix}2 & 5\\ 1 & \end{pmatrix}$$

Let then AB is

$$\begin{pmatrix}11 & 38\\ 16 & 47\\ 18 & 45\end{pmatrix}$$

a)

b) A and B are not comfortable for multiplication

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$$\begin{matrix} \Upsilon 11 & 38 \\ ' 16 & 47 \infty\infty \\ \leq ' 8 & 45 f\infty \end{matrix}$$

c)

d) None of these

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$$\begin{matrix} \Upsilon \square 4 & \Upsilon 2 \\ ' \infty & ' \infty \\ \leq 6 f & \leq \square 3 f \end{matrix}$$

Determine whether the set S={ } is a spanning set for $\mathbb{R}^2$ or not.

Note : In order to get full marks do all necessary steps

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Let T : $R^2 \square R^3$ be a linear transformation defined by T(x)=Ax , where

$$\begin{matrix} \Upsilon 1 & \square 3 \\ ' 3 & 5 \infty \\ ' \leq \square 1 & 7 \infty f \end{matrix} \qquad \begin{matrix} \Upsilon 3 \\ ' 2 \infty \\ ' \leq \square 5 \infty f \end{matrix}$$

A= ,then find an x in $\mathbb{R}^2$ whose image under T is b.Where b=

Note : In order to get full marks do all necessary steps

$$\begin{matrix} \Upsilon 1 & 2 \\ ' 5 & 1 f \infty \end{matrix}$$

The inverse of the matrix is

$$\begin{matrix} \Upsilon 1 \square & 2 \\ ' \square 5 & f 1 \infty \end{matrix}$$

(a)

$$\underline{1} \begin{matrix} \Upsilon 1 \square & 2 \\ 9 ' \leq 5 & 1 \infty f \end{matrix}$$

b)

$$\underline{1} \begin{matrix} \Upsilon \square 1 & 2 \\ 9 ' \leq 5 & \square 1 \infty f \end{matrix}$$

(c)

d) None of these

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The set of vectors {(5,0,0), (7,2,-6), (9,4,-8)} is,

a) Linearly independent

b) Linearly dependent

c) Basis of R_3

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