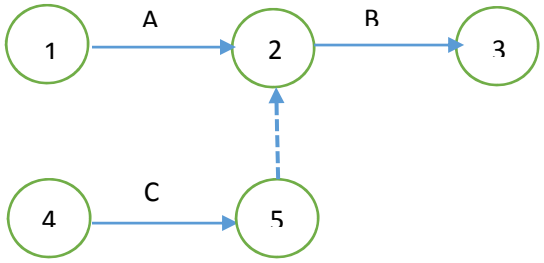
Solution of Practice Questions Lecture 4

Question 1:

Write the relationship between the activities.



Solution

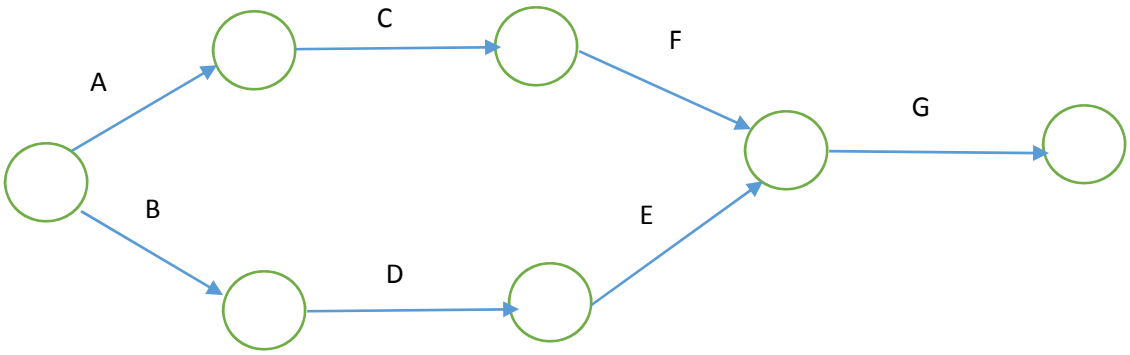
Activity A and activity C are predecessors of activity B.

Question 2:

Construct a network diagram for the following activity data.

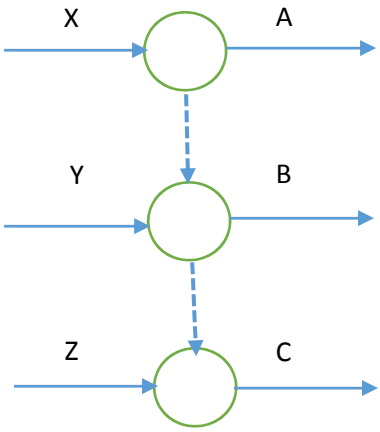
Activity	Predecessor
A	-
B	-
C	A
D	B
E	D
F	C
G	E,F

Solution



Question 3:

Write the relationship between the activities for the following network diagram.



Solution

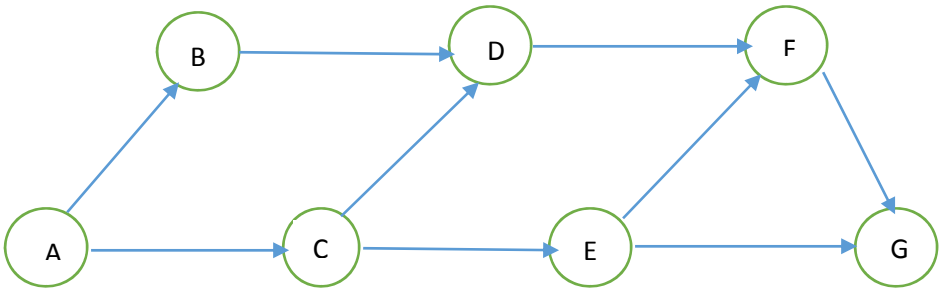
X is predecessor of A, B and C.

Y is predecessor of B and C.

Z is predecessor of C only.

Question 4:

Construct a table that shows the predecessor activities for the following network.



Solution

Event	Predecessor
A	-
B	A
C	A
D	B,C
E	C
F	D,E
G	E,F

Question 5:

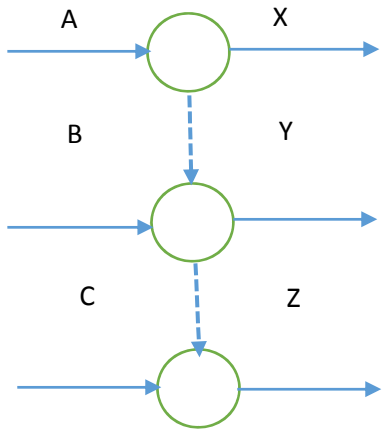
Construct a network diagram for the following relationship.

1) A is predecessor of X, Y and Z.

2) B is predecessor of Y and Z.

3) C is predecessor of Z only

Solution



Practice Questions Lecture#5-6

Question 1:

Which of the following in the highway system has the same analogy as that of the flow branches and nodes of the network flow diagram.

a. Roads
b. Intersection
c. Vehicles

Question 2:

Define the following

a. Dummy activity and its use
b. Looping error in network
c. Dangling error in network.

Question 3:

What is PERT? How it is different from CPM?

Question 4:

Draw a network diagram for the set of activities below

Activity	Predecessor	Duration
A	-	1
B	A	6
C	A	10
D	B	7
E	C	2
F	D,E	4
G	F	5

Question 5:

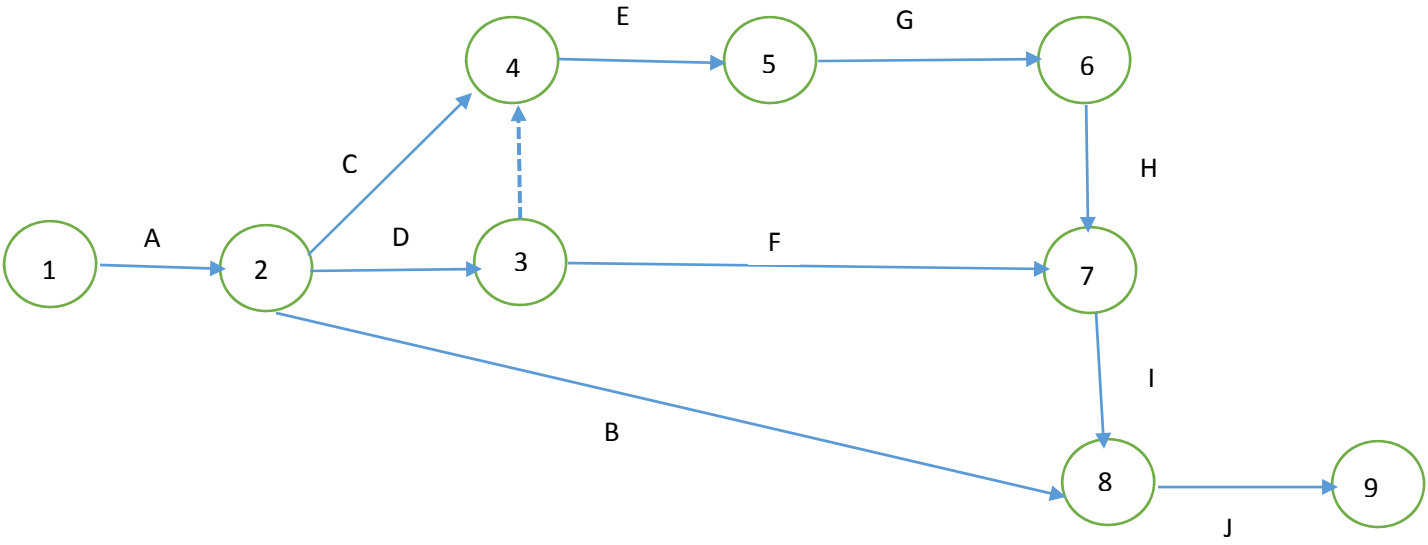
Define the following terms

a. Earliest Start Time

b. Earliest Finish Time
c. Latest Start Time
d. Latest Finish Time

Question 6:

Construct a table for the network diagram which shows the relationship between activities and their predecessors.



Question 7:

Calculate EST and EFT for each job of the following data.

Activity	Predecessor	Duration
P(1-2)	-	8
Q(1-3)	-	10
R(2-3)	P	12
S(2-4)	P	14
T(3-4)	R,Q	18

Question 8:

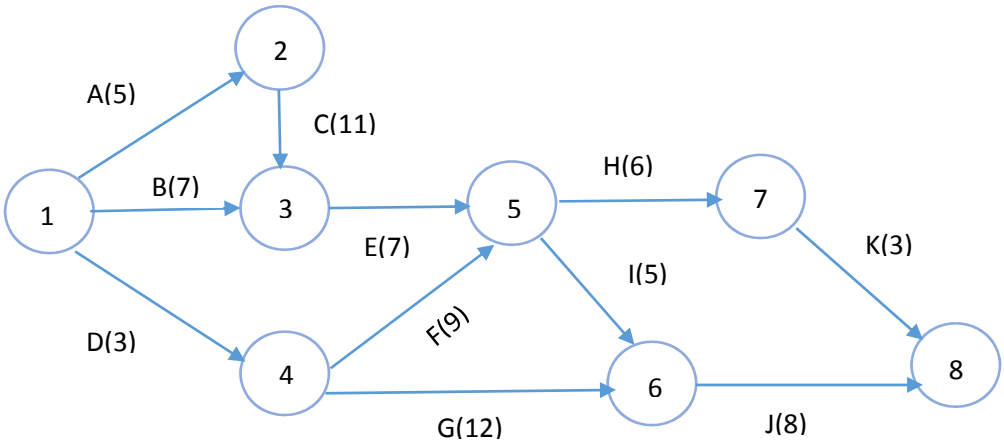
If project completion date is 38 days for the following data then find the LST and LFT for the following data.

Activity	Predecessor	Duration
P(1-2)	-	8
Q(1-3)	-	10
R(2-3)	P	12
S(2-4)	P	14
T(3-4)	R,Q	18

Solution of Practice Questions Lecture#7-9

Question 1:

Find the critical path and the expected duration of the following project.



Solution

There are 7 possible paths to get from the starting point at node 1 and travel through the network to end at node 8.

Possible Paths

1-2-3-5-7-8=A-C-E-H-K=5+11+7+6+3=32

1-2-3-5-6-8=A-C-E-I-J=5+11+7+5+8=36

1-3-5-7-8=B-E-H-K=7+7+6+3=23

1-3-5-6-8=B-E-I-J=7+7+5+8=27

1-4-5-6-8=D-F-I-J=3+9+5+8=25

1-4-5-7-8=D-F-H-K=3+9+6+3=21

1-4-6-8=D-G-J=3+12+8=23

Hence the critical path is the longest distance with the shortest possible time i.e.

=1-2-3-5-6-8

=A-C-E-I-J

=5+11+7+5+8

=36

Question 2:

A project has the following time schedule.

Activity	Time(hours)
1-2	2
1-3	2
1-4	1
2-5	4
3-6	8
3-7	5
4-6	3
5-8	1
6-9	5
7-8	4
8-9	3

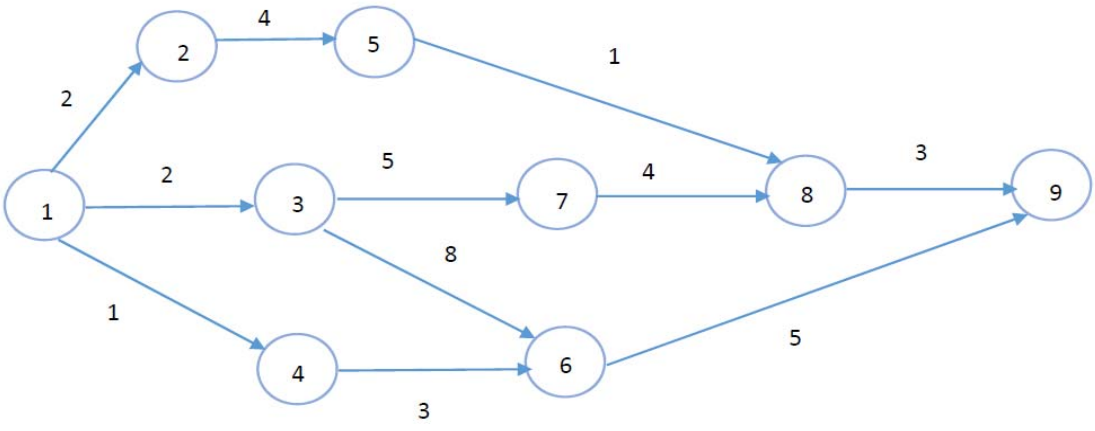
a) Draw the network

b) Calculate the following time estimates.

- Earliest Start Time (EST) of each activity.
- Earliest Finish Time (EFT) of each activity.

Solution

a) Network



b)

Event	EST
1	0
2	2
3	2
4	1
5	6
6	10
7	7
8	11
9	15

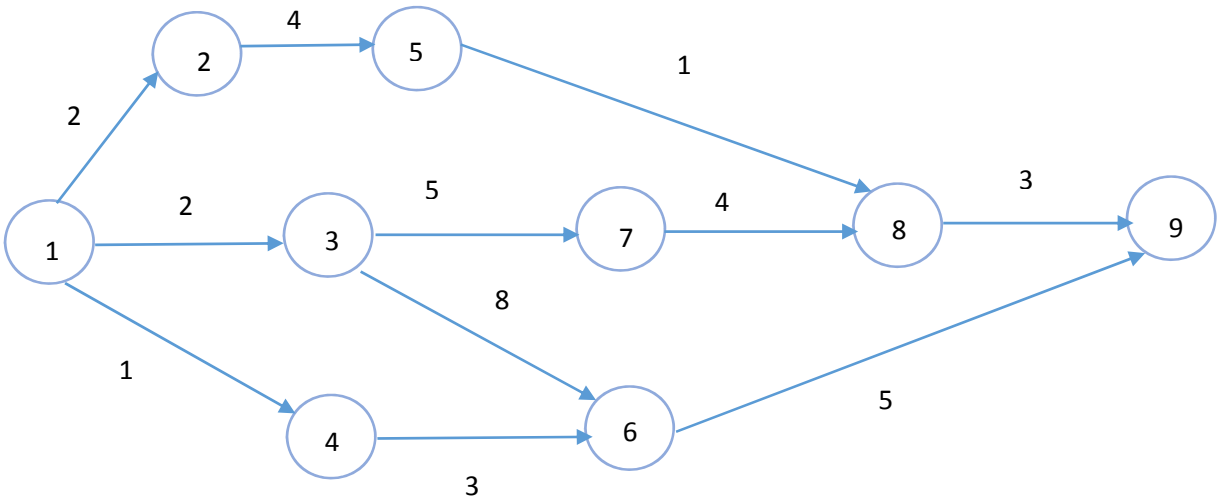
EST of any activity is the EST of its tail event

Activity	Time in Hours	EST	EFT
1-2	2	0	2
1-3	2	0	2
1-4	1	0	1
2-5	4	2	6
3-6	8	2	10
3-7	5	2	7
4-6	3	1	4
5-8	1	6	7
6-9	5	10	15
7-8	4	7	11
8-9	3	11	14

EFT=EST + Duration of the activity

Question 3:

Calculate the LST and LFT for each activity for the network given below.



Find the critical path and the expected duration of the project.

Solution

Event	LFT
1	0
2	7
3	2
4	7
5	11
6	10
7	8
8	12
9	15

LFT of any activity is the LFT of its head event

Activity	Time in Hours	LST	LFT
1-2	2	5	7
1-3	2	0	2
1-4	1	6	7
2-5	4	7	11
3-6	8	2	10
3-7	5	3	8
4-6	3	7	10
5-8	1	11	12
6-9	5	10	15
7-8	4	8	12
8-9	3	12	15

LST=LFT- Duration of the activity

Possible Paths:

- 1-2-5-8-9=2+4+1+3=10
- 1-3-7-8-9-=2+5+4+3=14
- 1-3-6-9=2+8+5=15
- 1-4-6-9=1+3+5=9

Hence 1-3-6-9 is the critical path with duration 15.

Question 4:

A small marketing project consists of the jobs as shown in the table below.

Activity	Predecessor	Duration
X	-	4
Y	-	5
Z	X	6

a) Draw the Network diagram.

b) Calculate the following time estimates.

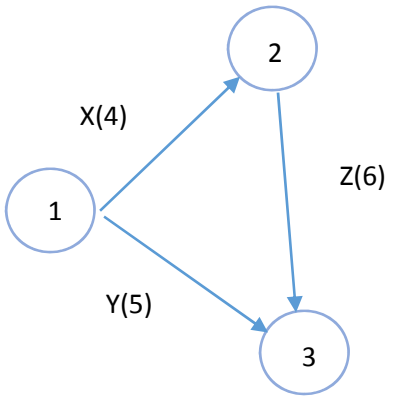
- Earliest Start Time of each activity
- Earliest Finish Time of each activity
- Latest Start Time of each activity
- Latest Finish Time of each activity

c) Critical Path and its duration.

d) Find Total Float, Independent Float and free Float for each activity.

Solution

a) Network:



b) Time Estimates

Activity	Duration	EST	EFT	LST	LFT
1-2	4	0	4	0	4
1-3	5	0	5	5	10
2-3	6	4	10	4	10

c) Possible Paths:

Time taken to complete the activities in two paths are:

- 1-2-3=4+6=10
- 1-3=5

Hence 1-2-3 is the critical path and the duration is 10 days.

d)

Activity	Duration	Total Float	Free Float	Independent Float
1-2	4	0	0	0
1-3	5	5	5	5
2-3	6	0	0	0

Question 5:

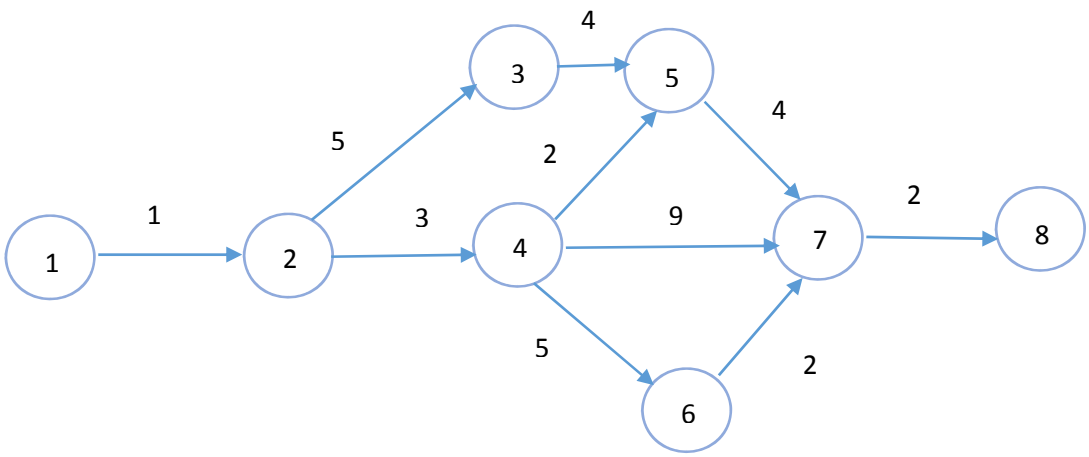
A project with 10 activities is given below

Activity	1-2	2-3	2-4	3-5	4-5	4-6	4-7	5-7	6-7	7-8
Duration	1	5	3	4	2	5	9	4	2	2

a) Construct the Network

b) Find the Critical Path.

Solution a) Network



b) ***Possible Paths:***

- 1-2-3-5-7-8=1+5+4+4+2=16
- 1-2-4-5-7-8=1+3+2+4+2=12
- 1-2-4-7-8=1+3+9+2=15
- 1-2-4-6-7-8-=1+3+5+2+2=13

Hence the critical path is 1-2-3-5-7-8=16

Question 6

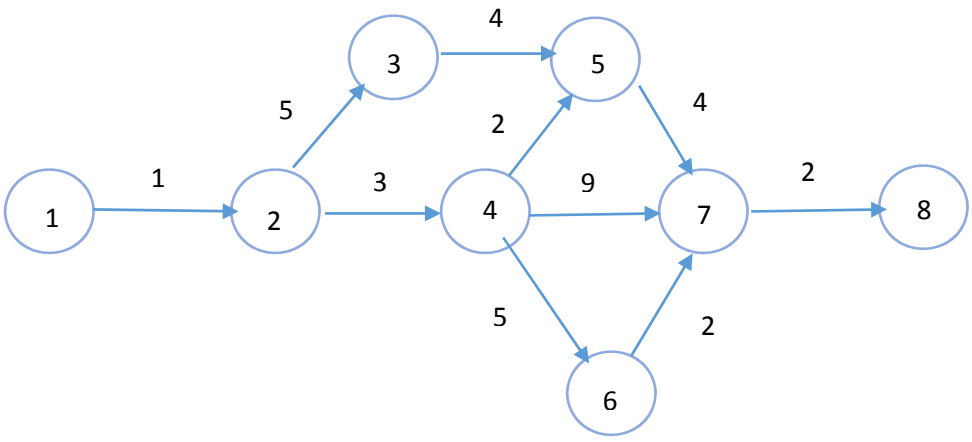
How many types of floats are there? Also mention their formulas.

Solution

1. Independent float
   $I = j_E - i_L - D$
2. Free float
   $F = j_E - i_E - D$
3. Total float
   $I = j_L - i_E - D$.

Question 7:

A project is represented by the network shown below



Find total float and free float.

Solution

Activity	Total Float	Free Float
1-2	0	0
2-3	0	0
2-4	1	0
3-5	0	0
4-5	4	4
4-6	3	0
4-7	1	1
5-7	0	0
6-7	3	3
7-8	0	0

Question 8:

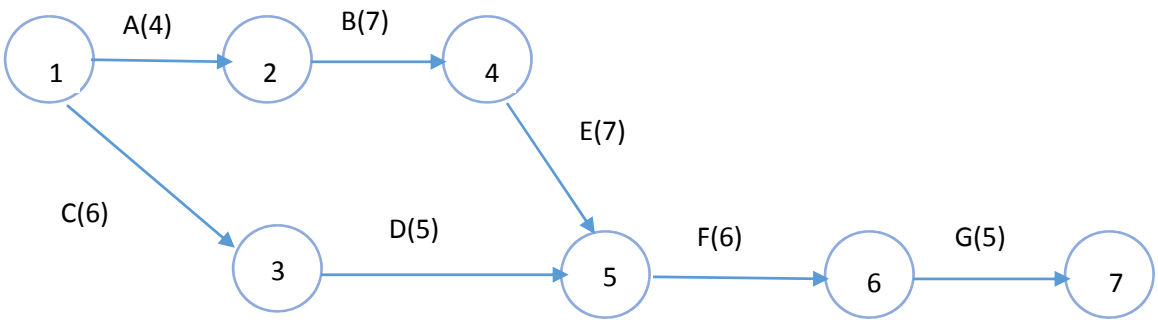
A project consist of the seven activities for which the relevant data are given below:

Activity	Preceding activity	Duration
A	-	4
B	A	7
C	-	6
D	C	5
E	B	7
F	D,E	6
G	F	5

(a) Draw an arrow diagram representing the project and find the project completion time.
(b) Calculate the total float for each activity.

Solution

a)



b)

Activity	Total Float
1-2	0
1-3	7
2-4	0
3-5	7
4-5	0
5-6	0
6-7	0

Critical path:

Path of zero total float through the project is called the critical path. Total float is zero on the activities A, B, E, F and G. Therefore

=A-B-E-F-G

=1-2-4-5-6-7

=4+7+7+6+5

=29

Solution of Practice Questions Lecture 10

Question 1:

If we have 10, 12 and 26 days as optimistic, most likely and pessimistic time for an activity, then calculate the following

a) Expected time of the activity

b) Variance and standard deviation.

Solution

a)

$$T_e = \frac{10 + (12 * 4) + 26}{6}$$

$$= \frac{84}{6} = 14$$

b)

$$V_t = \left[\frac{(t_p - t_o)}{6}\right]^2$$

$$= \left[\frac{(26 - 10)}{6}\right]^2$$

$$= \left[\frac{16}{6}\right]^2 = \left[\frac{8}{3}\right]^2 = \left[2.67\right]^2 = 7.11$$

$$\text{Standard } Deviation = \sqrt{Variance}$$

$$= \sqrt{7.11}$$

$$= 2.67$$

Question 2:

A teacher notes that when her student starts reading the optimistic (t_0), most likely (t_m) and pessimistic (t_p) time estimates are 10, 20 and 60.Calculate the following

a) What will be the expected duration of reading?

b) What would be its variance?

Solution

a)

$$T_e = \frac{t_o + 4t_m + t_p}{6}$$
$$= \frac{10 + (4*20) + 60}{6}$$
$$= \frac{150}{6}$$
$$= 25$$

b)

$$V_t = \left[\frac{(t_p - t_o)}{6}\right]^2$$
$$= \left[\frac{(60-10)}{6}\right]^2$$
$$= \left[\frac{50}{6}\right]^2 = [8.334]^2 = 69.45$$

Question 3:

The time estimates for the pert model of a medical institute for each activity are given as follows:

Activity	Time Estimates		
	t_O	t_m	t_p
A	2	4	12
B	10	15	20
D	5	5	5

Compute the expected time for each activity.

Solution

As $T_e = \frac{t_o + 4t_m + t_p}{6}$

For Activity A:

$$T_e = \frac{2 + (4*4) + 12}{6}$$
$$= \frac{30}{6} = 5$$

For Activity B:

$$T_e = \frac{10 + (15 * 4) + 20}{6}$$

$$= \frac{90}{6} = 15$$

For Activity D:

$$T_e = \frac{5 + (4 * 5) + 5}{6}$$

$$= \frac{30}{6} = 5$$

Question 4:

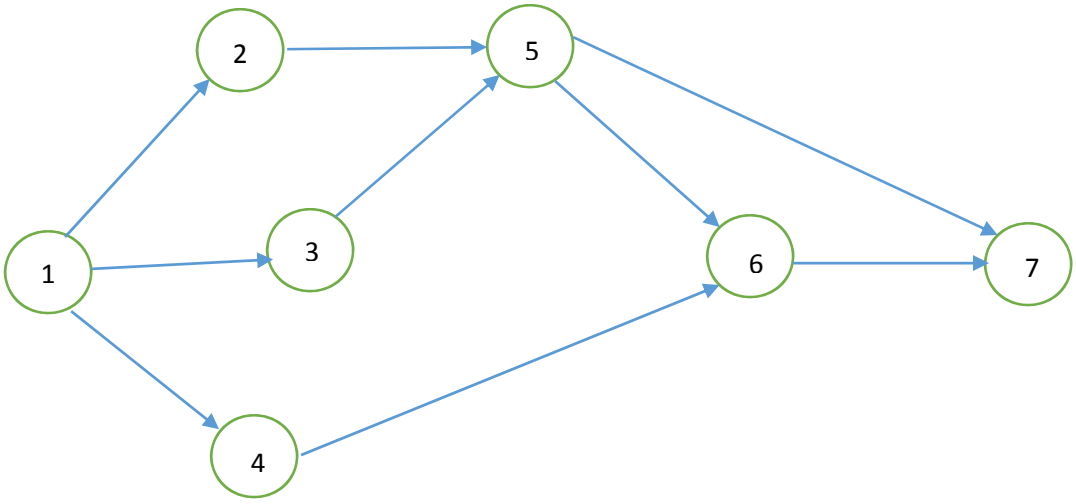
A certain project is composed of nine activities whose time estimates are given below:

Activity	Expected Duration (week)		
	Optimistic	Most likely	Pessimistic
1-2	1	1	7
1-3	3	5	7
1-4	2	2	8
2-5	1	1	1
3-5	3	6	9
4-6	2	5	8
5-6	4	6	14
5-7	3	7	11
6-7	6	8	10

a) Draw the project network and trace all the possible paths from it.
b) What is the expected project length?
c) Calculate the variance and standard deviation of the project length.

Solution

a)



***Possible Paths*:**

1 – 2 – 5 – 7,

1 – 2 – 5 – 6 – 7,

1 – 3 – 5 – 7,

1 – 3 – 5 – 6 – 7,

1 – 4 – 6 – 7.

b)

Activity	Time Estimates			Expected Time	Variance
	t_o	t_m	t_p		
1-2	1	1	7	2	1
1-3	3	5	7	5	0.45
1-4	2	2	8	3	1
2-5	1	1	1	1	0
3-5	3	6	9	6	1
4-6	2	5	8	5	1
5-6	4	6	14	7	2.79
5-7	3	7	11	7	1.77
6-7	6	8	10	8	0.45

The critical path is 1-3-5-6-7.

Expected Project length =5+6+7+8= 26.

c) For the project Length
Variance=Sum of the variances of the critical activities

=0.45+1+2.79+0.45=4.69

Standard $Deviation = \sqrt{Variance}$

$=\sqrt{4.69}$

=2.17

Question 5:

Activity	Expected Duration (week)		
	Optimistic	**Most likely**	**Pessimistic**
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

If the critical path for the above project is 1-3-5-6, then find the variance and standard deviation for the critical path.

Solution

As the critical path is 1 – 3 – 5 – 6

Activity	Expected Duration	Variance
1-2	2	1
1-3	4	1
1-4	3	1
2-5	1	0
3-5	6	4
4-6	5	1
5-6	7	4

Project Length=4+6+7=17

Variance=sum of the variance of the critical activities

=1+4+4=9

Standard Deviation=square root (variance) =3

Question 6:

A project schedule has the following characteristics:

Activity	Expected Duration (week)		
	Optimistic	Most likely	Pessimistic
1-2	3	3	3
2-3	3	6	9
2-4	2	4	6
3-5	4	6	8
4-6	4	6	8
5-6	0	0	0
5-7	3	4	5
6-7	2	5	8

a) Determine the critical path and the expected project time.
b) What is the probability that the project will be completed in 20 weeks?
c) What is the probability that the project will be completed in 23 weeks?

Solution

a) Critical path: 1-2-3-5-6-7
Expected project Time=20
Standard deviation of the critical path=2.6666

b)

$$z = \frac{T - T_e}{S.D} = \frac{20 - 20}{2.666} = 0$$

$$P(x < 20) = P\ (z < 0)$$

$$= 0.5$$

$$= 50\%$$

c)

$$z = \frac{T - T_e}{S.D} = \frac{23 - 20}{2.666} = 1.125$$

$$P(x < 23) = P(z < 1.125)$$

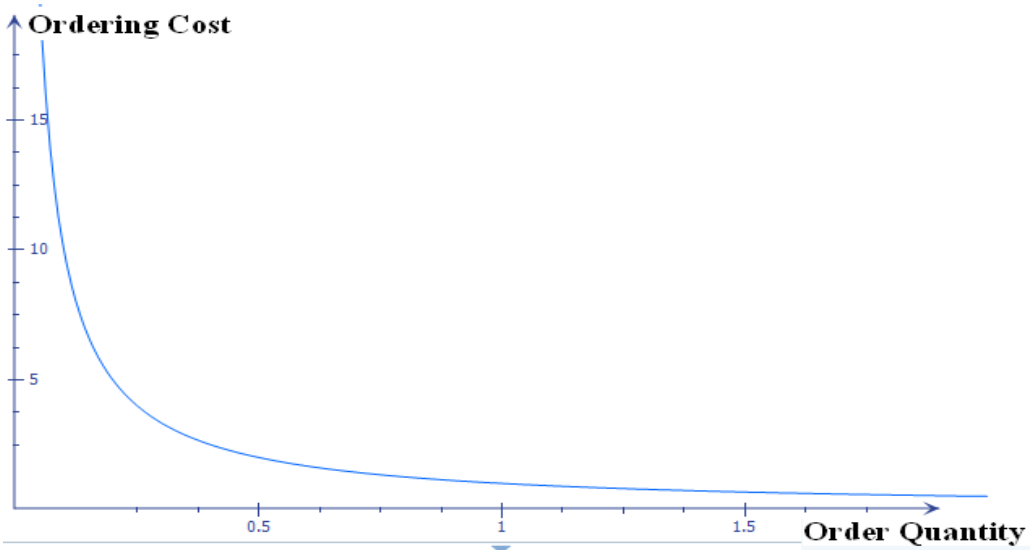
$$= 0.5 + P(0 < z < 1.125)$$

$$= 0.5 + 0.3686$$

$$= 0.8686$$

Question No 1:

Discuss the proportionality relation b/w ordering cost and order quantity.



When does the ordering cost per unit item would be maximum?

Question No 2:

Demand rate for a product is 600 units per year .Shortages are not allowed and the holding cost is Rs 0.60 per unit per year. Ordering cost is Rs 80.Find the economic order quantity.

Question No 3:

The storage cost of an item is Rs 1 per month and the setup cost is Rs 25 per run. If the production is instantaneous and the demand is 200 units per month, find the optimal size of the batch and the best time for the replacement of inventory.

Question No 4:

Consider the inventory system with the following data:

Demand Rate=15000 units /year

Item cost=Rs 4/item

Ordering Cost =Rs 100 per /order

Holding Cost=0.60 /item/month

Determine the following

a) Economic order quantity
b) The time between orders
c) Number of orders per year

Question No 5:

Assume you have a product with the following parameters:

Demand=360

Holding cost per year = $1.00 per unit

Order cost:= $100 per order

Determine the following

a) EOQ

b) Number of orders per year

Solution 1:

Curve shows the *inverse relation* b/w the Order Quantity and Ordering cost.
Since the ordering cost per unit item is inversely proportional to the quantity ordered, so it would be maximum whenever the ordered quantity will be minimum.

Solution No 2:

$D = 600 units$

$C_4 = 0$

$C_3 = 0.60$

$C_2 = 80$

$Economic\ Order\ Quantity:$

$$EOQ = \sqrt{\frac{2C_2D}{C_3}} = \sqrt{\frac{2*80*600}{0.60}}$$

$$= \sqrt{16000} = 400 units$$

Solution No 3:

$D = 200 units$

$C_4 = 0$

$C_3 = 1$

$C_2 = 25$

$Q = ?$

$t = ?$

$Economic\ Order\ Quantity:$

$$EOQ = Q^* = \sqrt{\frac{2C_2D}{C_3}} = \sqrt{\frac{2*25*200}{1}}$$

$$= \sqrt{10000} = 100$$

$$t = \frac{Q^*}{D} = \frac{100}{200} = 0.5$$

$Thus\ he\ should\ produce\ 100\ units\ of\ his\ product\ at\ \text{int}erval\ of\ 15\ days\ .$

Solution No 4:

$D = 15000$

$C_1 = Rs4 / Item$

$C_2 = Rs100 / order$

$C_3 = 0.60 / Item / month$

Economic Order Quantity:

$$EOQ = Q^* = \sqrt{\frac{2C_2D}{C_3}} = \sqrt{\frac{2*100*15000}{7.2}}$$

$$= \sqrt{416666.66} = 645.49 units$$

Time between order:

$$t = \frac{Q^*}{D} = \frac{645.49}{15000} = 0.5164 \text{ per month}$$

Number of orders per year:

$$N = \frac{D}{Q} = \frac{15000}{645.49} = 23.23$$

Solution No 5:

$D = 360$

$C_2 = 100$

$C_3 = 1$

Economic Order Quantity:

$$EOQ = Q^* = \sqrt{\frac{2C_2D}{C_3}} = \sqrt{\frac{2*100*360}{1}}$$

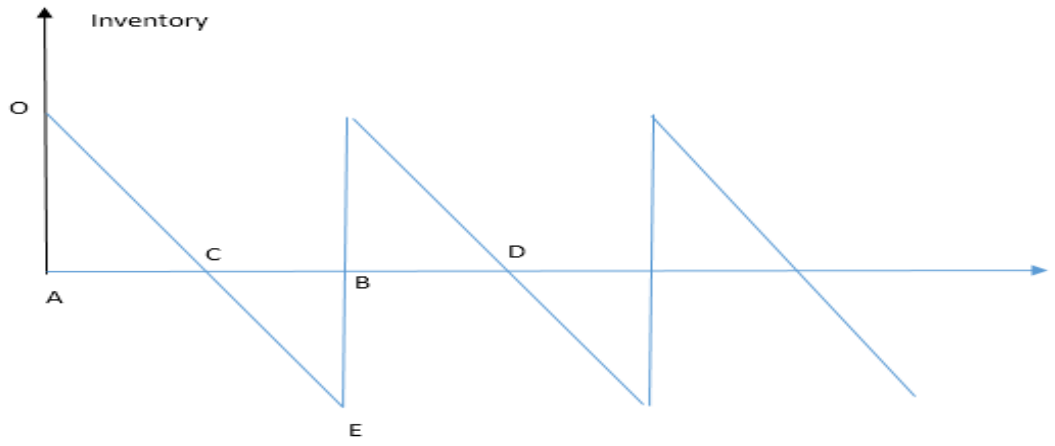
$$= \sqrt{72000} = 268.32 \text{ units}$$

Number of orders per year:

$$N = \frac{D}{Q} = \frac{360}{268.32} = 1.34$$

Question No 1:

Consider the following purchasing model with shortages, which of the area represent the availability and shortage of items in the interval AB



Question No 2:

The daily demand of a commodity is 100 units. The holding and ordering costs are Rs 0.02 and Rs 100.Shortage cost is Rs 5.Determine the economic order quantity.

Question No 3:

The demand for an item is 15,000 units/year. The cost of one purchase is Rs. 400. The holding cost is Rs. 1.2 per unit per year. The item cost is Rs. 2 per item. The shortage cost is Rs. 10 per unit per year. Determine:

a) The optimum order quantity.
b) The time between orders.
c) The number of orders per year.
d) The optimum shortages.
e) The maximum inventory.
f) Total cost per order.

Question No 4:

The cost function in two variables of an inventory model is as follows

$$C(S,Q) = C_1 D + \frac{C_2 D}{Q} + \frac{C_3 (Q-S)^2}{2Q} + \frac{C_4 S^2}{2Q}$$

Differentiate the function partially w.r.t Q and S.

Question No 5:

The cost function in two variables of an inventory model is as follows

$$C(S,Q) = C_1 D + \frac{C_2 D}{Q} + \frac{C_3 (Q-S)^2}{2Q} + \frac{C_4 S^2}{2Q}$$

Find the optimal value of S and Q.

Solution No 1:-

In the time subinterval AC, the area of triangle AOC above the x- axis represent the availability of items

In the time subinterval BC, the area of triangle CEB below the x- axis represent the shortage of items

Solution No 2:-

$$D = 100$$
$$C_3 = 0.02$$
$$C_2 = 100$$
$$C_4 = 5$$
$$EOQ = ?$$
$$EOQ = \sqrt{\frac{2C_2D}{C_3}} * \sqrt{\frac{C_3 + C_4}{C_4}}$$
$$EOQ = \sqrt{\frac{2*100*100}{0.02}} * \sqrt{\frac{0.02+5}{5}}$$
$$EOQ = \sqrt{\frac{20000*5.02}{0.02*5}}$$
$$EOQ = \sqrt{\frac{20000*5.02}{0.02*5}}$$
$$EOQ = \sqrt{1004000}$$
$$EOQ = 1001.998$$

Solution No 3:-

1) $$EOQ = \sqrt{\frac{2C_2D}{C_3}} * \sqrt{\frac{C_3 + C_4}{C_4}}$$
$$EOQ = \sqrt{\frac{2*400*15000}{1.2}} * \sqrt{\frac{1.2+10}{10}}$$
$$EOQ = \sqrt{\frac{800*15000*11.2}{1.2*10}}$$
$$EOQ = \sqrt{11200000}$$
$$EOQ = 3346.640$$
$$Q^* = 3346.640$$

2) $t = \frac{Q^*}{D}$

$t = \frac{3346.640}{15000} = 0.223109 Years$

$t = 2.67 Months$

3) $N = \frac{12}{t}$

$N = \frac{12}{2.67} = 4.5$

4) $S^* = \sqrt{\frac{2C_2D}{C_4}} * \sqrt{\frac{C_3}{C_3 + C_4}}$

$S^* = \sqrt{\frac{2*400*15000}{10}} * \sqrt{\frac{1.2}{1.2+10}}$

$S^* = \sqrt{80*15000} * \sqrt{\frac{1.2}{11.2}}$

$S^* = \sqrt{128571.4286}$

$S^* = 358.568$

5) $I = Q^* - S^*$

$I = 3346.640 - 358.568$

$I = 2988.072$

6) Total Cost per period:

$= C_1 Q^* + C_2 + \frac{C_3 (Q^* - S^*)^2}{2D} + \frac{C_4 S^{*2}}{2D}$

$= (2*3346.64) + 400 + \frac{1.2(2988.071)^2}{30,000} + \frac{10(358.568)^2}{30,000}$

$= 6693.28 + 400 + 357.1429 + 42.857 = 7493.0099$

Solution No 4:-

$$C(\mathrm{S},\mathrm{Q}) = \mathrm{C}_1 D + \frac{\mathrm{C}_2 D}{Q} + \frac{\mathrm{C}_3(\mathrm{Q}-\mathrm{S})^2}{2Q} + \frac{\mathrm{C}_4 S^2}{2Q}$$

$$\frac{\partial C}{\partial Q} = 0 - \frac{\mathrm{C}_2 D}{Q^2} + \frac{\mathrm{C}_3}{2}\left(\frac{2Q(\mathrm{Q}-\mathrm{S}) - (\mathrm{Q}^2 - 2\,\mathrm{QS} + S^2)}{Q^2}\right) - \frac{\mathrm{C}_4 S^2}{2Q^2}$$

$$= -\frac{\mathrm{C}_2 D}{Q^2} + \frac{\mathrm{C}_3}{2}\left(\frac{2Q^2 - 2Q\,\mathrm{S} - \mathrm{Q}^2 + 2\,\mathrm{QS} - S^2)}{Q^2}\right) - \frac{\mathrm{C}_4 S^2}{2Q^2}$$

$$= -\frac{\mathrm{C}_2 D}{Q^2} + \frac{\mathrm{C}_3}{2}\left(\frac{\mathrm{Q}^2 - S^2)}{Q^2}\right) - \frac{\mathrm{C}_4 S^2}{2Q^2}$$

$$\frac{\partial C}{\partial S} = 0 + 0 - C_3\left(\frac{(\mathrm{Q}-\mathrm{S})}{Q}\right) + \frac{\mathrm{C}_4 S}{Q}$$

$$= -C_3\left(\frac{\mathrm{Q}-\mathrm{S}}{Q}\right) + \frac{\mathrm{C}_4 S}{Q}$$

Solution No 5:

$$\frac{\partial C}{\partial Q} = -\frac{C_2 D}{Q^2} + \frac{C_3}{2}\left(\frac{Q^2 - S^2)}{Q^2}\right) - \frac{C_4 S^2}{2Q^2}$$

For optimal value putting $\frac{\partial C}{\partial Q}$ and $\frac{\partial C}{\partial S}$ *equals* to zero.

$$-2C_2 D + C_3(Q^2 - S^2) - C_4 S^2 = 0$$

$$C_3 Q^2 = 2C_2 D + C_3 S^2 + C_4 S^2$$

$$C_3 Q^2 = 2C_2 D + S^2(C_3 + C_4)$$

$$Q^2 = \frac{2C_2 D}{C_3} + S^2\left(\frac{C_3 + C_4}{C_3}\right) \qquad \text{(A)}$$

$$\frac{\partial C}{\partial S} = 0 + 0 - C_3\left(\frac{(Q - S)}{Q}\right) + \frac{C_4 S}{Q}$$

$$\frac{\partial C}{\partial S} = -C_3\left(\frac{Q - S}{Q}\right) + \frac{C_4 S}{Q}$$

$$C_3 Q = C_3 S + C_4 S$$

$$S = \frac{C_3 Q}{C_3 + C_4} \qquad \text{(B)}$$

Putting the value of S *in* (A)

$$Q^2 = \frac{2C_2 D}{C_3} + \left(\frac{C_3 Q}{C_3 + C_4}\right)^2\left(\frac{C_3 + C_4}{C_3}\right)$$

$$= \frac{2C_2 D}{C_3} + \left(\frac{C_3 Q^2}{C_3 + C_4}\right)$$

$$Q^2\left(1 - \frac{C_3}{C_3 + C_4}\right) = \frac{2C_2 D}{C_3}$$

$$Q = \sqrt{\frac{2C_2 D}{C_3}}\sqrt{\frac{C_3 + C_4}{C_4}}$$

Putting the value of Q in (B).

$$S = \frac{C_3}{C_3 + C_4}\sqrt{\frac{2C_2 D}{C_3}}\sqrt{\frac{C_3 + C_4}{C_4}}$$

$$S = \sqrt{\frac{2C_2 D}{C_4}}\sqrt{\frac{C_3}{C_3 + C_4}}$$

Question No 1:

In a Manufacturing Model with no shortages, Q* = 8490 units, ***D*** = 3000 units / month, R = 6000 units per month**.**
Determine the following:

a) The Time between orders.
b) The Number of orders per year.
c) The Time of manufacturing.

Question No 2:

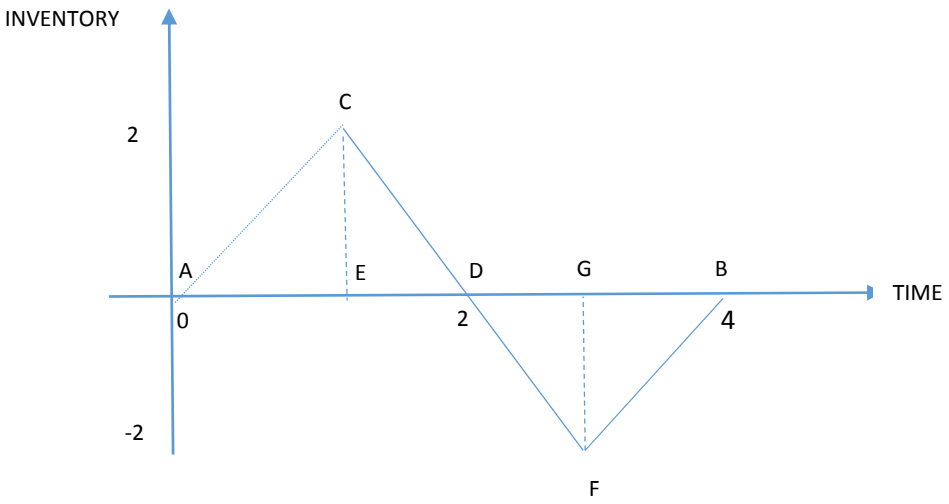
Find the optimum manufacturing quantity for the following data of the Manufacturing model with no shortages
Demand=2500 units/month
Production rate=5000/month
Holding cost=Rs 1/month
Ordering cost=Rs 400 per order.

Question No 3:

The cost of parameters and other factors for a production inventory system of an automobile pistons are given below
Demand per year =6000 units
Unit cost= Rs 40
Setup cost = Rs 500
Holding cost per year = Rs 8
Production rate per year=36000
Shortage cost per unit per year= Rs 20.Find
a) Optimum lot size
b) Number of shortages
c) Manufacturing time and time between orders

Question No 4:

Consider the following Manufacturing Model with Shortage, determine the Holding Cost(C_3) and Shortage Cost(C_4) in the interval ***AB*** such that the C_3 and C_4 are Rs.2 and Rs. 4 per item per unit time respectively.



Question No 5:

Write short notes on Economic order quantity and Dynamic order quantity.

Solution No.1:-

1) $t = \frac{Q^*}{D}$

$t = \frac{8490}{3000} = 2.83$

2) $N = \frac{12}{t}$

$t = \frac{12}{2.83} = 4.240$

3) $The\ time\ of\ manufacturing$

$t_1 = \frac{Q^*}{R}$

$t_1 = \frac{8490}{6000} = 1.415$

Solution No.2:-

$$Optimum\ Manufacturing\ Quantity = Q^* = \sqrt{\frac{2C_2D}{C_3}} * \sqrt{\frac{R}{R-D}}$$

$$Q^* = \sqrt{\frac{2*400*2500}{1}} * \sqrt{\frac{5000}{5000-2500}}$$

$$Q^* = \sqrt{\frac{2*400*2500*5000}{2500}}$$

$$Q^* = \sqrt{800*5000}$$

$$Q^* = 2000$$

Solution No.3:-

(a) $Optimum\ Manufacturing\ Quantity = Q^* = \sqrt{\frac{2C_2D}{C_3} * \frac{R}{R-D}} * \sqrt{\frac{C_3+C_4}{C_4}}$

$$Q^* = \sqrt{\frac{2*500*6000}{8} * \frac{36000}{36000-6000}} * \sqrt{\frac{8+20}{20}}$$

$$Q^* = \sqrt{\frac{1000*6000}{8} * \frac{36000}{30000}} * \sqrt{\frac{28}{20}}$$

$$Q^* = \sqrt{1,260,000}$$

$Optimum\ Manufacturing\ Quantity = Q^* = Approx.1123 units$

(b) $S = \sqrt{\frac{2C_2D}{C_4} * \frac{R-D}{R}} * \sqrt{\frac{C_3}{C_3+C_4}}$

$$S = \sqrt{\frac{2*500*6000}{20} * \frac{36000-6000}{36000}} * \sqrt{\frac{8}{8+20}}$$

$$S = \sqrt{\frac{2*500*6000}{20} * \frac{30000}{36000}} * \sqrt{\frac{8}{28}}$$

$$S = \sqrt{\frac{50*5000}{1} * \frac{2}{7}}$$

$$S = \sqrt{\frac{500,000}{7}}$$

$$S = approx.267$$

(c) *Time between orders*

$$t = \frac{Q^*}{D}$$

$$t = \frac{1123}{6000} = 0.19 Years$$

Manufacturing Time :

$$t^* = \frac{Q^*}{R}$$

$$t^* = \frac{1123}{36000} = 0.03$$

Solution No.4:-

$$\text{Holding Cost} = Rs.2 \text{ per item per unit time}$$

$$\text{Holding Cost for period } AD = C_3 * [\text{Area of } \Delta ACD]$$

$$= C_3 * [\frac{1}{2} * CE * AD]$$

$$= 2 * [\frac{1}{2} * 2 * 2]$$

$$\text{Holding Cost for period } AD = Rs.4$$

$$\text{Shortage Cost} = Rs.4 \text{ per item per unit time}$$

$$\text{Shortage Cost for period } DB = C_4 * [\text{Area of } \Delta DFB]$$

$$= 4 * [\frac{1}{2} * GF * DB]$$

$$= 4 * [\frac{1}{2} * 2 * 2]$$

$$= Rs.8$$

Question No 1:

Group the items given below into an ABC analysis.

Item No	Annual Usage
3	75000
4	37000
2	14000
1	36000
7	32000
8	8000
6	16000
5	11000
9	95000
10	4000

Question No 2:

The following information is known about a group of items. Classify the material into ABC analysis.

Model No	Annual Consumption	Unit Price(piasa)
101	30000	10
102	280000	15
103	3000	10
104	110000	5
105	4000	5
106	220000	10
107	15000	5
108	80000	5
109	60000	15
110	800	10

Question No 3:

Find % of cumulative Annual Usage by using ABC analysis.

Model No	Annual Consumption	Unit Price
1	7000	5
2	24000	3
3	15000	10
4	600	22
5	38000	1.5
6	40000	0.5
7	60000	0.2
8	3000	3.5

Question No 4:

Assign rank to each item using ABC analysis.

Product	Units	Unit Cost
1	40	350
2	30	30
3	50	200
4	20	70
5	8	100
6	20	20
7	10	100
8	40	20
9	10	60
10	5	20

Solution No 1:

Item No	Annual Usage	Descending Order	Rank
3	75000	95000	9
4	37000	75000	3
2	14000	37000	4
1	36000	36000	1
7	32000	32000	7
8	8000	16000	6
6	16000	14000	2
5	11000	11000	5
9	95000	8000	8
10	4000	4000	10

Cumulative Annual usage	%Cumulative Annual usage	Cumulative % of items	ABC Category
95000	28.96	10	A
170000	51.82	20	A
207000	63.1	30	B
243000	74.08	40	B
275000	83.83	50	B
291000	88.71	60	C
305000	92.98	70	C
316000	96.34	80	C
324000	98.78	90	C
328000	100	100	C

Solution No 2:

Model No	Annual consumption	Unit Price(Rs)	Usage Value	Descending Order
101	30000	0.1	3000	42000
102	280000	0.15	42000	22000
103	3000	0.1	300	9000
104	110000	0.05	5500	5500
105	4000	0.05	200	4000
106	220000	0.1	22000	3000
107	15000	0.05	750	800
108	80000	0.05	4000	750
109	60000	0.15	9000	300
110	8000	0.1	800	200

Cumulative Annual usage	%Cumulative Annual usage	Cumulative % of items	ABC Category
42000	48	10	A
64000	73	20	A
73000	83	30	B
78500	90	40	B
82500	94	50	B
85500	98	60	C
86300	98.6	70	C
87050	99.4	80	C
87350	99.6	90	C
87550	100	100	C

Solution No 3:

Item No	Units	Unit Cost	Usage Value	Descending Order
1	7000	5	35000	72000
2	24000	3	72000	57000
3	15000	10	15000	35000
4	600	22	13200	20000
5	38000	1.5	57000	15000
6	40000	0.5	20000	13200
7	60000	0.2	12000	12000
8	3000	3.5	10500	10500

Cumulative Usage Value	% Cumulative Usage Value
72000	30.67
129000	54.96
164000	69.87
184000	78.39
199000	84.78
212200	90.41
224000	95.44
234700	100

Solution No 4:

Product	Unit	Unit Cost	Usage value	Descending Order
1	40	350	14000	14000
2	30	30	900	10000
3	50	200	10000	1400
4	20	70	1400	1000
5	8	100	800	900
6	20	20	400	800
7	10	100	1000	800
8	40	20	800	600
9	10	60	600	400
10	5	20	100	100

Cumulative Usage Value	%Cumulative Annual usage	ABC Category
14000	46.7	A
24000	80	A
25400	84.7	B
26400	88	B
27300	91	C
28100	93.7	C
28900	96.3	C
29500	98.3	C
29900	99.7	C
30000	100	C

Question No 1:

A resourceful home decorator manufactures two types of lamps say A and B. Both lamps go through technicians first a cutter, second a finisher.

Lamp A requires 2 hours of the cutters time and 1 hour of finisher's time. Lamp B requires 1 hours of the cutters time and 2 hour of finisher's time. The cutter has 104 hours and finisher has 76 hours of available time each month. Profit on one lamp is Rs 6.00 and on one B lamp is Rs 11.00

Formulate the above linear programming model.

Question No 2:

An animal feed manufactures has to produce 200 kg of a feed mixture consisting of two ingredients X_1,X_2 . X_1 costs Rs 6 per kg while X_2 Rs 16 per kg. Not more than 80 kg of X_1 and not less than 60 kg of X_2 can be used .Formulate the above linear programming model.

Question No 3:

A company has two manufacturing plants ,ech of which produces three products from a common raw material .The proportion in which the products are produced are different in each plant and so are the plants operating cost per hour .Data on production and cost together with the currents orders on hand for each product are given in the following table:

Plants	**Products**			**Operating Cost**
	Prod 1	**Prod 2**	**Prod 3**	
Plant A	5	6	7	Rs 250/hr
Plant B	7	9	5	Rs 400/hr
Orders on hand	56	32	64	

Formulate the above linear programming problem.

Question No 4:

An electronic company manufactures integrated circuits for radios, televisions and videos. For a particular month it has available at least 1250 units of materials and not more than 850 man hours.

The demand and selling price for each of the above products are given in the following table:

Product	**Units of Materials**	**Man Hour**	**Selling Price**
Radio	3	2	2500
Television	11	9	14,500
Video	16	7	12,000

Formulate the problem as an LP model to determine a production schedule for maximizing income.

Question No 5:

Reliable Butchers shop is trying to determine the correct mix of two types of beefs feed 1 and feed 2, which cost Rs 60 and Rs 84 per kilograms respectively. Five essential ingredients are contained in the feed and are given in the following table:

Ingredients	**% Per Kilo Of feed**		**Minimum requirnment**
Kg	**Feed 1**	**Feed 2**	**(Kg)Daily**
1	25	35	30
2	50	20	58
3	10	15	32
4	0	40	70
5	24	32	48

Formulate the LP model.

Solution No 1:

$$\begin{aligned} &\textit{Maximize} && Z = 6x_1 + 11x_2 \\ &\textit{Subject to} && 2x_1 + x_2 \le 104 \\ & && x_1 + 2x_2 \le 76 \\ & && x_1, x_2 \ge 0 \end{aligned}$$

Solution No 2:

$$\begin{aligned} &\textit{Minimize} && Z = 6x_1 + 16x_2 \\ &\textit{Subject to} && x_1 + x_2 = 200 \\ & && x_1 \le 80 \\ & && x_2 \ge 60 \\ & && x_1, x_2 \ge 0 \end{aligned}$$

Solution No 3:

$$\begin{aligned} &\textit{Minimize} && Z = 250x_1 + 400x_2 \\ &\textit{Subject to} && 5x_1 + 7x_2 \ge 56 \\ & && 6x_1 + 9x_2 \ge 32 \\ & && 7x_1 + 5x_2 \ge 64 \\ & && x_1, x_2 \ge 0 \end{aligned}$$

Solution No 4:

$$\begin{aligned} &\textit{Maximize} && Z = 2500x_1 + 14500x_2 + 12000x_3 \\ &\textit{Subject to} && 3x_1 + 11x_2 + 16x_3 \ge 1250 \\ & && 2x_1 + 9x_2 + 7x_3 \le 850 \\ & && x_1, x, x_3 \ge 0 \end{aligned}$$

Solution No 5:

$$\begin{aligned} &\textit{Minimize} && Z = 60x_1 + 84x_2 \\ &\textit{Subject to} && 25x_1 + 35x_2 \ge 30 \\ & && 50x_1 + 20x_2 \ge 58 \\ & && 10x_1 + 15x_2 \ge 32 \\ & && 40x_2 \ge 70 \\ & && 24x_1 + 32x_2 \ge 48 \\ & && x_1, x_2 \ge 0 \end{aligned}$$

Question No.01:

Find the minimum value of "x" for the constraint $-4x + 2y \leq 11$ if the point (x, 4) satisfy the above constraint.

Question No.02:

Find the maximum value of "y" for the constraint $4x - 2y \leq 11$ if the point (2, -y) satisfy the above constraint.

Question No.03:

Find the corner points of the feasible region for the following constraints:

a) $4x + 2y \leq 12$

$x, y \geq 0$

b) $x + 2y \leq 16$

$x + y \leq 9$

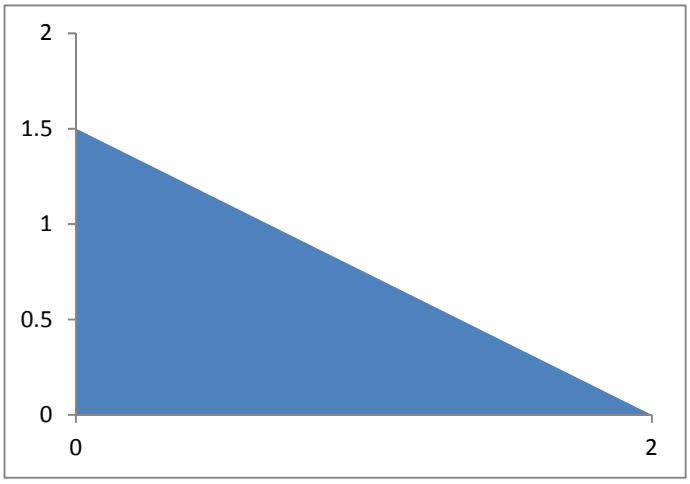
$x, y \geq 0$

c) $3x + 5y \leq 15$

$x, y \geq 0$

Question No.04:

Find the constraint expression for the below shaded region:



Question No.05:

Find the optimal values of "x" and "y" for the below given constraints:

a) $4x + 9y \leq 36$

$x, y \geq 0$

b) $4x + 9y \geq 36$

$x, y \geq 0$

Question No 06:

Maximize $Z = 4x + 3y$

Subject to the following constraints

$x + 4y \leq 20$

$2x + 3y \leq 30$

$x, y \geq 0$

Question No.07:

Minimize $Z = 4x + 3y$

Subject to the following constraints

$x + y \geq 6$

$3x + 8y \geq 24$

$x, y \geq 0$

Question No.08:

Find the corner points of feasible region for the given constraints:

$2x + 5y \leq 10$

$4x + 5y \geq 20$

$x, y \geq 0$

Solution No.01:

The given constraint is

$$-4x+2y \le 11$$

$$As\ the\ point\ (x,4)\ satisfy\ the\ given\ constraint$$

$$-4(x)+2(4) \le 11$$

$$-4x+8 \le 11$$

$$-4x \le 11-8$$

$$-4x \le 3$$

$$x \ge -\frac{3}{4}$$

Hence

$$\mathbf{X_{min} = -3/4}$$

Solution No.02:

The given constraint is

$$4x-2y \le 11$$

$$As\ the\ point\ (2,-y)\ satisfy\ the\ given\ constraint$$

$$4(2)-2(-y) \le 11$$

$$8+2y \le 11$$

$$2y \le 11-8$$

$$2y \le 3$$

$$y \le \frac{3}{2}$$

Hence

$$\mathbf{y_{max} = 3/2}$$

Solution No.03:

Part-a

The associated equation of the given constraint is:

4x + 2y = 12

As the feasible region of the given constraints comes only in the 1st quadrant because x and y are positive only in 1st quadrant. (x, y ≥ 0)

On Y-Axis, **x = 0;**

$\Longrightarrow$ 4(0) + 2y = 12

2y = 12

y = 6

1st corner point is A(0,6)

At intersection of line with X-Axis, **y = 0;**

$\Longrightarrow$ 4x + 2(0) = 12

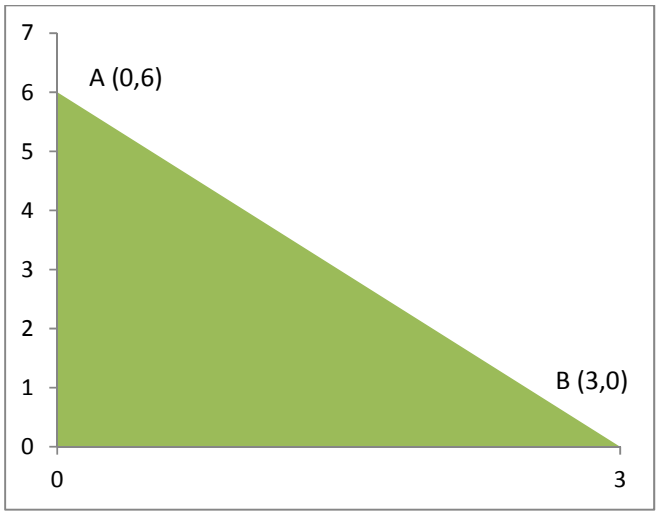
4x = 12

x = 3

2nd corner point is B(3,0)

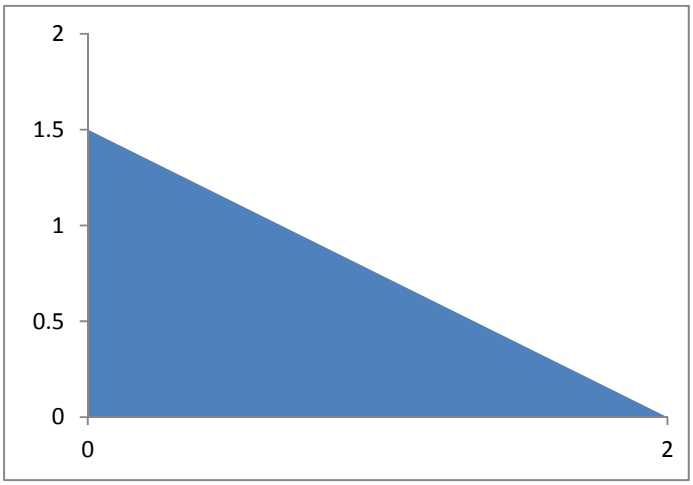
3rd corner point will be origin C(0,0)

The feasible region with corner points is shown below



Do **Part-b&c** by yourself (Similar to part-a)

Solution No.04:



From the graph, it is clear that

X-intercept = a = 2

Y-intercept = b = 1.5

So the associated equation of constraint can be written as:

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{2} + \frac{y}{1.5} = 1$$

$$\frac{x}{2} + \frac{10\,y}{15} = 1$$

$$\frac{x}{2} + \frac{2\,y}{3} = 1$$

$$3x + 4y = 6$$

So the set of constraints that will represent the above shaded region will be:

$3x + 4y \leq 6$

$x, y \geq 0$

Solution No.05:

Part-a

The optimal value of "x" will be on X-Axis where **y = 0;**

$\Longrightarrow$ $4x + 9(0) \leq 36$

$4x \leq 36$

$x \leq 9$

$\mathbf{x_{max} = 9}$

The optimal value of "y" will be on Y-Axis where **x = 0;**

$\Longrightarrow$ $4(0) + 9y \leq 36$

$9y \leq 36$

$y \leq 4$

$\mathbf{y_{max} = 4}$

As **"x"** and **"y"** cannot be negative as given by 2nd constraint equation i.e $x, y \geq 0$

So optimal values of "x" and "y" are:

$\mathbf{x_{max} = 9}$ $\mathbf{y_{max} = 4}$

$\mathbf{x_{min} = 0}$ $\mathbf{y_{min} = 0}$

Part-b

The optimal value of "x" will be on X-Axis where **y = 0;**

$\Longrightarrow$ $4x + 9(0) \geq 36$

$4x \geq 36$

$x \geq 9$

$\mathbf{x_{min} = 9}$

The optimal value of "y" will be on Y-Axis where **x = 0;**

$\Longrightarrow$ $4(0) + 9y \geq 36$

$9y \geq 36$

$y \geq 4$

$\mathbf{y_{min} = 4}$

So optimal values of "x" and "y" are:

$\mathbf{x_{max} = \infty}$ $\mathbf{y_{max} = \infty}$

$\mathbf{x_{min} = 9}$ $\mathbf{y_{min} = 4}$

Solution No.06:

First, it is needed to find the corner points of the feasible region for the given constraint equations:

The associated equations of the given constraints are:

x + 4y = 20 ……(1)

2x + 3y = 30 ……..(2)

For equation (1)

Putting x=0; we get

(0) + 4y = 20 $\Longrightarrow$ y = 5

A (0, 5)

Putting y=0; we get

x + 4(0) = 20 $\Longrightarrow$ x = 20

B(20, 0)

So the two points of equation (1) are **A (0, 5) and B(20, 0)**

For equation (2)

Putting x=0; we get

2(0) + 3y = 30 $\Longrightarrow$ y = 10

C (0, 10)

Putting y=0; we get

2x + 3(0) = 30 $\Longrightarrow$ x = 15

D(15, 0)

So the two points of equation (2) are **A (0, 10) and B(15, 0)**

The corner points of the feasible region are A(0, 5), D(15, 0), O(0, 0) and E(a, b).

Point "E(a, b)" is the intersection of given two lines.

Multiplying equation (1) by "2" and subtracting from equation (2)

$$\begin{array}{r} 2x+3y= \ 30 \\ \pm 2x \pm 8y = \pm 40 \\ \hline \end{array}$$

-5y = -10 $\Longrightarrow$ y = 2

Putting value of "y" in equation (1), we get

x + 4(2) = 20

x = 20-8

x = 12

So point E is E(12, 2)

The value of function Z at point "A(0, 5)"

$Z = 3(0) + 2(5) \quad \Longrightarrow \quad \mathbf{Z = 10}$

The value of function Z at point "D(15, 0)"

$Z = 3(15) + 2(0) \quad \Longrightarrow \quad \mathbf{Z_{max} = 45}$

The value of function Z at point "E(12, 2)"

$Z = 3(12) + 2(2) \quad \Longrightarrow \quad \mathbf{Z = 40}$

So the objective function has maximum value at point D(15, 0).

Question No.07:

First, it is needed to find the corner points of the feasible region for the given constraint equations:

The associated equations of the given constraints are:

x + y = 6 ……(1)

3x + 8y = 24 ……..(2)

For equation (1)

Putting x=0; we get

$(0) + y = 6 \quad \Longrightarrow \quad y = 6$

A (0, 6)

Putting y=0; we get

$x + (0) = 6 \quad \Longrightarrow \quad x = 6$

B(6, 0)

So the two points of equation (1) are **A (0, 6) and B(6, 0)**

For equation (2)

Putting x=0; we get

$3(0) + 8y = 24 \quad \Longrightarrow \quad y = 3$

C (0, 3)

Putting y=0; we get

$3x + 8(0) = 24 \quad \Longrightarrow \quad x = 8$

D(8, 0)

So the two points of equation (2) are **A (0, 3) and B(8, 0)**

The corner points of the feasible region are B(6, 0), C(0, 3), O(0, 0) and E(a, b).

Point "E(a, b)" is the intersection of given two lines.

Multiplying equation (1) by "3" and subtracting from equation (2)

$$3x + 8y = 24$$
$$\pm 3x \pm 3y = \pm 18$$

5y = 6 $\Longrightarrow$ y = 6/5

Putting value of "y" in equation (1), we get

x + 6/5 = 6

x = 6 – 6/5

x = 24/5

So point E is E(24/5, 6/5)

The value of function Z at point "B(6, 0)"

Z = 4(6) + 3(0) $\Longrightarrow$ **Z = 24**

The value of function Z at point "C(0, 3)"

Z = 4(0) + 3(3) $\Longrightarrow$ $\mathbf{Z_{min} = 9}$

The value of function Z at point "E(24/5, 6/5)"

$$Z = 4 * \frac{24}{5} + 3 * \frac{6}{5}$$
$$Z = \frac{96}{5} + \frac{18}{5}$$
$$Z = \frac{96 + 18}{5}$$
$$Z = \frac{114}{5}$$

So the objective function has minimum value at point C(0, 3).

Question No.08:

Try yourself