Date:

Mth-631

Q.1

Check the following series functions

$$\sum \frac{1}{x^2+n^2} \quad \text{and} \quad \sum \frac{\sin nx}{n^2}$$

are uniform convergence or not?

Ans:-

by dominated series we have

$$\frac{1}{x^2+n^2} \leq \frac{1}{n^2} \quad , \quad \left|\frac{\sin nx}{n^2}\right| \leq \frac{1}{n^2}$$

Taking $M_n = \frac{1}{n^2}$ and recall it

$$\sum M_n = \sum \frac{1}{n^2} < \infty$$ **M.M**

by weierstrass M-test, we can conclude

$$\sum \frac{1}{x^2+n^2} \quad \text{and} \quad \sum \frac{\sin nx}{n^2}$$

converge uniformly on $(-\infty, \infty)$.

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Q.2

Check the uniform convergence of the series

$$\sum f_n(x) = \sum \left(\frac{x}{1+x}\right)^n$$

Soln:-

The series is uniformly convergence on set S such that

$$\left|\frac{x}{1+x}\right| \leq r < 1 \quad , \quad x \in S \longrightarrow (i)$$

M-M

For set S we have $\|f_n\|_S \leq r^n$

by weierstrass M-test $\sum M_n = \sum r^n < \infty$

(i) will be

$$\frac{-r}{1+r} \leq x \leq \frac{r}{1-r} \quad , \quad x \in S$$

put $r=1$ then

$$\frac{-1}{2} \leq x \leq \infty$$

This mean the series converge uniformly on any compact subset of $(-\frac{1}{2}, \infty)$.

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Q.3

Define Absolute convergence?

Ans:-

A series of function Σf_n is said to be Absolute convergence on S if $\Sigma |f_n|$ converges pointwise on S and converges absolutely uniformly on S if $\Sigma |f_n|$ converges uniformly on S.

Q.4 M.M

Evaluate $\sum_{n=1}^{\infty} \int_0^1 x(e^x - 1)e^{-nx}\,dx$

Sol:-

Part-1

Given $\sum_{n=1}^{\infty} \int_0^1 x(e^x-1)e^{-nx}\,dx$

converge it to $F(x) = x$.

by the theorem of interchange of sum and integral sign, we have

$$\sum_{n=1}^{\infty} \int_0^1 x(e^x-1)e^{-nx}\,dx$$

$$= \int_0^1 \sum_{n=1}^{\infty} x(e^x-1)e^{-nx}\,dx$$

$$= \int_0^1 x\,dx$$

Part-2

$$= \left[\frac{x^2}{2}\right]_0^1$$

$$= \frac{1}{2} - 0$$

$$= \frac{1}{2}$$

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Q.45

If $\sum_{n=K}^{\infty} f_n$ converges uniformly to F on S and each f_n is continuous at a point x_0 in S, then so is F. Similar statements hold for continuity from the right and left?

Ans:-

Since it is proved in Q.2 that

$$F(x) = \sum_{n=0}^{\infty} \left(\frac{x}{1+x}\right)^n$$ M.M

is uniformly converge on every compact subset of $(\frac{-1}{2}, \infty)$.

As term of series is continuous on every such subset so it implies that F is also.

In fact we can state a stronger result, F is continuous on $(-1/2, \infty)$, since every point lies in $(-1/2, \infty)$ and it also lies in compact subinterval of $(-1/2, \infty)$.

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Q.6

The series $E(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ is converges uniformly on interval $[-r, r]$

soln:-

The series converges uniformly on every interval $[-r, r]$

by ~~[illegible]~~ weierstrass M-test

$$\frac{|x|^n}{n!} \leq \frac{r^n}{n!}$$ M.M

$$|x|^n \leq r^n \implies |x| \leq r$$

So, $$\sum \frac{r^n}{n!} \leq \infty$$

by Diff.

$$\sum_{n=1}^{\infty} \frac{x^n}{n!} = \sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)!}$$

Therefore, the differential series is also uniformly converges on $[-r, r]$.

Q.7

Power series:-

An infinite series of the form $\sum_{n=0}^{\infty} a_n (x - x_0)^n$ where x_0 and $a_0, a_1, \cdots a_n$ are constant is called a power series in $x - x_0$, if $x_0 = 0$ then power series will be

$$\sum_{n=0}^{\infty} a_n x^n.$$

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Q.8

Lemma: If $f(x)$ is a periodic function with period w, then $\int_c^{c+w} f(x)dx = \int_0^w f(x)dx$ whenever f is integrable on $[0, w]$?

Solu-

Given

$$\int_c^{c+w} f(x)dx = \int_0^w f(x)dx$$

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L.H.S

$$\int_c^{c+w} f(x)dx = \int_c^0 f(x)dx + \int_0^w f(x)dx + \int_w^{c+w} f(x)dx$$

$$= -\int_0^c f(x)dx + \int_0^w f(x)dx + \int_0^c f(x)dx$$

$$= \int_0^w f(x)dx$$

If show that the integral of a periodic functi with period w taken over an arbitrary interval of length w always has the same value.

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Q.9

Write the properties of fourier series of Even and odd function:-

Ans:-

i) The product of two even or odd functions is an even function.

ii) The sum of two even or odd functions is an even or odd function.

iii) The product of even and odd function is odd function.

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Q.11

The length of vectors is $X=(1,-2,6,5)$ and $Y=(3,-5,4,\frac{1}{2})$, Find the distance b/w X and Y and Find inner product.

Sol:-

Distance

$$|X-X| = \sqrt{(1-3)^2+(-2+5)^2+(6-4)^2+(5-\frac{1}{2})^2}$$

$$= \sqrt{4+9+4+\frac{81}{4}} = \frac{\sqrt{149}}{2}$$

inner product

$$X \cdot Y = x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4$$

$$= (1)(3)+(-2)(-5)+(6)(4)+(5)(\tfrac{1}{2})$$

$$= 3+10+24+\frac{5}{2} = \frac{79}{2} = 39\frac{1}{2}$$

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Q.10

Prove the function $f(x) = |\sin x|$ is defined for all x and has period π.

Sol:-

Clearly, f represent a continuous and even function of period π, therefore it is everywhere equal to its fourier series, containing cosine terms only.

here we have $c=0$ and $L=\frac{\pi}{2}$, then

$$a_k = \frac{2}{\pi}\int_0^{\pi} f(x)\cos(2kx)\,dx$$

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$$= \frac{2}{\pi}\int_0^{\pi} \sin x \cos(2kx)\,dx$$

$$= \frac{1}{\pi}\int_0^{\pi} [\sin(1+2k)x - \sin(2k-1)x]\,dx$$

$$= \frac{1}{\pi}\left(-\frac{\cos(1+2k)x}{2k+1} + \frac{\cos(2k-1)x}{2k-1}\right)\Bigg|_0^{\pi}$$

$$= -\frac{1}{\pi}\left(\frac{(-1)^{2k+1}-1}{2k+1} - \frac{(-1)^{2k-1}-1}{2k-1}\right)$$

$$= -\frac{4}{\pi(4k^2-1)}$$

Thus fourier series of $|\sin x|$ is

$$|\sin x| = \frac{2}{\pi} - \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\cos 2kx}{4k^2-1}, \quad x \in [-\pi, \pi]$$

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