Assignment # 01 MTH621 (Fall 2021)

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Math City Layyah with Muhammad Khalil

Maximum Marks: 20
Due Date: Dec 17, 2021

DON'T MISS THESE: Important instructions before attempting the solution of this assignment:

- **To solve this assignment, you should have good command over 1 - 10 lectures.**
- **Try to get the concepts, consolidate your concepts and ideas from these questions which you learn in the 1 to 10 lectures.**
- **Upload assignments properly through LMS, No assignment will be accepted through email.**
- **Write your ID on the top of your solution file.**
- **Don't use colorful back grounds in your solution files.**
- **Use Math Type or Equation Editor Etc. for mathematical symbols.**
- **You should remember that if we found the solution files of some students are same then we will reward zero marks to all those students.**
- **Try to make solution by yourself and protect your work from other students, otherwise you and the student who send same solution file as you will be given zero marks.**
- **Also remember that you are supposed to submit your assignment in Word format any other like scan images, etc. will not be accepted and we will give zero marks correspond to these assignments.**

Q1. If $a_1 = 2, a_2 = 0, a_3 = -14$, and $a_{n+1} = 9a_n - 23a_{n-1} + 15a_{n-2}, \quad n \geq 3.$

Show by induction that $a_n = 3^{n-1} - 5^{n-1} + 2, \quad n \geq 1.$

Solution

$$a_n = 3^{n-1} - 5^{n-1} + 2, n \geq 1$$

By Mathematically induction:

Step 1:

Put, **n=1** in given expression

$$a_1 = 3^{1-1} - 5^{1-1} + 2$$
$$a_1 = 3^0 - 5^0 + 2$$
$$a_1 = 1 - 1 + 2$$
$$a_1 = 2$$

Step 2:

We assume that the given expression is true for **n=k**

$$a_k = 3^{k-1} - 5^{k-1} + 2$$

Step 3:

Now we want to show that is true for **n=k+1**

$$a_{k+1} = 3^{k+1-1} - 5^{k+1-1} + 2$$
$$a_{k+1} = 3^k - 5^k + 2$$

Now we let

$$a_{k+1} = 9a_k - 23_{k-1} + 15_{k-2} \ldots\ldots\ldots (1)$$

Here

$$a_k = 3^{k-1} - 5^{k-1} + 2$$
$$a_{k-1} = 3^{k-2} - 5^{k-2} + 2$$
$$a_{k-2} = 3^{k-3} - 5^{k-3} + 2$$

Now equation (1) becomes

$$a_{k+1} = 9(3^{k-1} - 5^{k-1} + 2) - 23(3^{k-2} - 5^{k-2} + 2) + 15(3^{k-3} - 5^{k-3} + 2)$$

$$a_{k+1} = 9(\frac{3^k}{3} - \frac{5^k}{5} + 2) - 23(\frac{3^k}{9} - \frac{5^k}{25} + 2) + 15(\frac{3^k}{27} - \frac{5^k}{125} + 2)$$
$$a_{k+1} = \frac{9.3^k}{3} - \frac{9.5^k}{5} + 18 - \frac{23.3^k}{9} + \frac{23.5^k}{25} - 46 + \frac{15.3^k}{27} - \frac{15.5^k}{125} + 30$$
$$a_{k+1} = 3.3^k - \frac{23.3^k}{9} + \frac{5.3^k}{9} + \frac{23.5^k}{25} - \frac{9.5^k}{5} - \frac{3.5^k}{25} + 30 + 18 - 46$$
$$a_{k+1} = (3 - \frac{23}{9} + \frac{5}{9})3^k + (\frac{23}{25} - \frac{9}{5} - \frac{3}{25})5^k + 2$$
$$a_{k+1} = (\frac{27 - 23 + 5}{9})3^k + (\frac{23 - 45 - 3}{25})5^k + 2$$
$$a_{k+1} = (\frac{9}{9})3^k + (\frac{-25}{25})5^k + 2$$
$$a_{k+1} = 3^k - 5^k + 2$$

Hence proved

Q2. Show that the intersection of finitely many open sets is open. Also give an example showing that the intersection of infinitely many open sets may fail to be open.

Solution:

Let $A_1, A_2, A_3, ...A_n$ are open set

$A_1 \cap A_2 \cap A_3 \cap .....\cap A_n = X$

$\cap_{i=1}^{n} A_i = X$

We prove that

$X = \cap_{i=1}^{n} A_i$ is open set

Let

$x \in \cap_{i=1}^{n} A_i = X$

$\Rightarrow x \in A_1, x \in A_2, x \in A_3, ...x \in A_n$

Since each A_i is an open set therefore each A_i is neighborhood of X

$x \in (x - \delta_1 x + \delta_1) \subseteq A_1$

$x \in (x - \delta_2 x + \delta_2) \subseteq A_2$

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$x \in (x - \delta_n x + \delta_n) \subseteq A_n$

Let $\delta = \min\{\delta_1, \delta_2, ...\delta_n\}$

$\Rightarrow x \in (x - \delta, x + \delta) \subseteq A_i$ For all i

$\Rightarrow x \in (x - \delta, x + \delta) \subseteq \cap_{i=1}^{n} A_i = X$

$\Rightarrow x \in (x - \delta, x + \delta) \subseteq X$

Since X is the neighborhood of x, x is an arbitrary element

X is the neighborhood of each of its point, there x is an open set

$X = A_i^n = A_i$ Is an open set