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	MIDTERM EXAMINATION SEMESTER FALL 2004 MTH401- Differential Equations	Total Marks: 50 Duration: 60min

Instructions

1. Attempt all questions.
2. The Time allowed for this paper is 60 minutes.
3. This examination is closed book, closed notes, closed neighbors; any one found cheating will get zero grades in the course MTH401 Differential Equations.
4. You are not allowed to use any type of Table for Formulae of Differentiation and integration during your exam.
5. Each objective type question carries 2 marks and each Descriptive question carries 10 marks. So write every step of the solution of descriptive question to get maximum marks.
6. Do not ask any questions about the contents of this examination from anyone. If you think that there is something wrong with any of the questions, attempt it to the best of your understanding.

Question No: 1 **Marks: 2**

Order of the differential equation is the highest order derivative in a differential equation.

- **T**
- F

→ **02**

Question No: 2 **Marks: 2**

e^{-x} Is integrating factor of the differential equation $y(x+y+1)dx+(x+2y)dy=0$.

- T
- **F**

→ **02**

Question No: 3 **Marks: 2**

If $\Im_1: \ G(x,y,c_1)=0$ orthogonal intersect all curves of another family $\Im_2: \ H(x,y,c_2)=0$ then each curve of the families is said to be orthogonal trajectories of the other.

- **T**
- F

→ **02**

Question No: 4 **Marks:2**

The given functions $f_1(x)=1+x,\ f_2(x)=x,\ f_2(x)=x^2$ are linearly independent.

- **T**
- F → **02**

Question No: 5 **Marks: 2**

A set of functions whose wronskian is zero guarantees that set of functions is linearly dependent.

- T
- **F** → **02**

Question No: 6 **Marks: 10**

(a) Define separable form. Just separate the variables of the given differential equation.

$$(r\theta-4r+\theta-4)dr-\left(r^2\theta+20r^2-\theta-20\right)d\theta=0$$

Solution:

The differential equation of the form d y / d x = f (x, y) is called **separable** if it can be written in the form

d y / d x = h(x)g(y)

$$(r\theta-4r+\theta-4)dr-\left(r^2\theta+20r^2-\theta-20\right)d\theta=0$$

$$(r\theta-4r+\theta-4)dr=\left(r^2\theta+20r^2-\theta-20\right)d\theta$$

$$\left[r(\theta-4)+1(\theta-4)\right]dr=\left[r^2(\theta+20)-1(\theta+20)\right]d\theta$$

$$(r+1)(\theta-4)\,dr=\left(r^2-1\right)(\theta+20)\,d\theta$$

$$\frac{(r+1)}{\left(r^2-1\right)}dr=\frac{(\theta+20)}{(\theta-4)}d\theta$$

$$\frac{1}{r-1}dr=\frac{(\theta+20)}{(\theta-4)}d\theta$$

(b) Check whether the given differential equation is exact or not if not then make it exact also show that it is exact **(Just make the equation exact do not solve it further).**

$$\left(2y^2+3x\right)dx+2xydy=0$$

Solution

$$\left(2y^2+3x\right)dx+2xydy=0$$

$$M\left(x,y\right)=2y^2+3x,\quad N\left(x,y\right)=2xy$$

$$M_y=4y,\quad N_x=2y$$

$$M_y\neq N_x$$

Thus it is not exact now we apply techniques to make it exact

$$\left(2y^2+3x\right)dx+2xydy=0$$

$$\frac{M_y-N_x}{N}=\frac{4y-2y}{2xy}=\frac{2y}{2xy}=\frac{1}{x}$$

$$I.F=e^{\int\frac{1}{x}dx}=e^{\ln x}=x$$

$$\left(2xy^2+3x^2\right)dx+2x^2ydy=0$$

$$M\left(x,y\right)=2xy^2+3x^2,\quad N\left(x,y\right)=2x^2y$$

$$M_y=4xy, N_x=4xy$$

$$M_y=N_x$$

Which shows that now equation is exact

Question No: 7 **Marks:10**

(a) Solve given Bernoulli equation $xy-\frac{dy}{dx}=y^3e^{-x^2}$ **(Just make the given equation linear in v, do not integrate)**

Solution

$$xy-\frac{dy}{dx}=y^3e^{-x^2}$$

$$xy^{-2}-\frac{dy}{dx}y^{-3}=e^{-x^2}$$

$$put\ y^{-2}=v$$

$$-2\frac{dy}{dx}y^{-3}=\frac{dv}{dx}$$

$$-\frac{dy}{dx}y^{-3}=\frac{1}{2}\frac{dv}{dx}$$

Then

$$\frac{1}{2}\frac{dv}{dx}+vx=e^{-x^2}$$

$$\frac{dv}{dx}+2vx=2e^{-x^2}$$

Thus it is linear in “v”.

(b) The population of a town grows at rate proportional to the population at any time. Its initial population of 100 decreased by 20% in 20 years what will be the population in 30 years? **(Just make the model of the population dynamics as well as just describe the given conditions do not solve further)**

Solution

Suppose that P_0 is the initial population of the town, as given P_0 is 100 and P(t) the population at any time t then population growth by the differential equation

$$\frac{dP}{dt} \alpha P$$

$$\frac{dP}{dt} = kP$$

$$\frac{dP}{P} = kdt$$

Integrate both sides

$$\ln P = kt + c$$

$$P = e^{kt+c}$$

$$P = e^{kt}e^{c}$$

$$P = P_0 e^{kt} \qquad say\ P_0 = e^{c}$$

Where P_0 is the initial population of the town

P_0 =100 = P(0)

$$\text{P(20)=100-}\frac{20}{100}(100)$$

P(20)=80

P (30) =?

Question No: 8 **Marks: 10**

(a) Find a second solution of following differential equations where the first solution is given **(also write the formulae)**.

$$x^2y'' - 5xy' + 9y = 0; \quad y_1 = x^3 \ln x$$

Solution

$x^2y" - 5xy' + 9y = 0; y_1 = x^3 \ln x$

differential equation can be written as

$$x^2y" - \frac{5}{x}xy' + \frac{9}{x^2} = 0$$

the 2nd solution is given by

$$y_2 = y_1 \int \frac{e^{-\int pdx}}{y_1^{\,2}} dx$$

$$y_2 = x^3 \ln x \int \frac{e^{-\int -5/x\, dx}}{\left(x^3 \ln x\right)^2} dx$$

$$y_2 = x^3 \ln x \int \frac{e^{\int 5/x\, dx}}{\left(x^3 \ln x\right)^2} dx$$

$$y_2 = x^3 \ln x \int \frac{e^{5\ln x}}{\left(x^3 \ln x\right)^2} dx$$

$$y_2 = x^3 \ln x \int \frac{e^{\ln x^5}}{\left(x^3 \ln x\right)^2} dx$$

$$y_2 = x^3 \ln x \int \frac{x^5}{x^6 \left(\ln x\right)^2} dx$$

$$y_2 = x^3 \ln x \int \frac{(\ln x)^{-2}}{x} dx$$

$$y_2 = x^3 \ln x \frac{(\ln x)^{-2+1}}{-2+1}$$

$$y_2 = -x^3 \ln x \frac{1}{\ln x}$$

$$y_2 = -x^3$$

(b) Solve the differential equation by the undetermined coefficient **(superposition approach)**

$$y'' - 2y' - 3y = 2\cos\theta$$

If complimentary solution is given below

$$y_c = c_1 e^{3x} + c_2 e^{-x}$$

Then just find particular solution.

Solution

We find a particular solution of non-homogeneous differential equation.

Suppose input function

$$y_p = A\cos\theta + B\sin\theta$$

Then

$$y'_p = -A\sin\theta + B\cos\theta$$

$$y''_p = -A\cos\theta - B\sin\theta$$

Substituting in the given differential equation

$$-A\cos\theta - B\sin\theta - 2\left(-A\sin\theta + B\cos\theta\right) - 3\left(A\cos\theta + B\sin\theta\right) = 2\cos\theta$$

$$\left(-A - 2B - 3A\right)\cos\theta + \left(-B + 2A - 3B\right)\sin\theta = 2\cos\theta$$

From the resulting equations

$-A-2B-3A=2;$ $\quad -B+2A-3B=0$

$-4A-2B=2;$ $\quad 2A-4B=0$

$2A+B=-1;$ $\quad A-2B=0 \rightarrow A=2B$

$\rightarrow 2(2B)+B=-1$

$\rightarrow 5B=-1$

$\rightarrow B=\frac{-1}{5}$

$\rightarrow A=2\left(\frac{-1}{5}\right)=\frac{-2}{5}$

$y_p=\frac{-2}{5}\cos\theta+\frac{-1}{5}\sin\theta$

Question No: 9 **Marks:10**

(a) Solve the differential equation by the undetermined coefficient **(annihilator operator).**

$y''-4y'=x^3-2x+1$

If complimentary solution is given below

$y_c=c_1+c_2e^{4x}$

Then find general solution.

Solution

In this case of input function is

g(x)= x^3-2x+1

further

$D^4(g(x))=D^3(x^3-2x+1)=0$

Therefore the differential operator D^4 annihilates the function g. operating on both sides

$D^4(D^2-4D)y=D^4(x^3-2x+1)$

$D^4(D^2-4D)y=0$

This is the homogeneous equation of order 5. Next we solve this higher order equation.

Thus auxiliary equation is

$m^4(m^2-4m)=0$

$m^5(m-4)=0$

$m=0,0,0,0,0,4$

Thus its general solution of the differential equation must be

$y=c_1+c_2x+c_3x^2+c_4x^3+c_5x^4+c_6e^{4x}$

(b) Solve the differential equation by the variation of parameters

$y''-2y'-3y=x^3$

If complimentary solution is given below

$y_c = c_1 e^{3x} + c_2 e^{-x}$

Then just find particular solution **(do not integrate)**.

Solution……

$y'' - 2y' - 3y = x^3$

This equation is already in standard form

y’’+ P(x)y’ +Q(x)y = f(x)

Therefore, we identify the function f(x) as

f(x) = x^3

We construct the determinants

Since $y_1 = e^{3x}$, $y_2 = e^{-x}$ so

$$W(y_1, y_2) = \begin{vmatrix} e^{3x} & e^{-x} \\ 3e^{3x} & -e^{-x} \end{vmatrix} = -e^{3x-x} - 3e^{3x-x} = -4e^{2x}$$

$$W_1 = \begin{vmatrix} 0 & e^{-x} \\ x^3 & -e^{-x} \end{vmatrix} = -x^3 e^{-x}$$

$$W_2 = \begin{vmatrix} e^{3x} & 0 \\ 3e^{3x} & x^3 \end{vmatrix} = x^3 e^{3x}$$

We determine the derivatives of the function u_1 and u_2

$$u'_1 = \frac{W_1}{W} = \frac{-x^3 e^{-x}}{-4e^{2x}} \rightarrow u_1 = \int \frac{x^3}{4} e^{-3x} dx$$

$$u'_2 = \frac{W_2}{W} = \frac{x^3 e^{3x}}{-4e^{2x}} \rightarrow u_2 = \int \frac{x^3}{4} e^{x} dx$$

$$y_p = u_1 e^{3x} + u_2 e^{-x}$$

is required particular solution