Question # 3 of 10 (Start time: 03:41:30 PM, 15 June 2022

$If \sum a_n \quad ------, \ then$

Select the correct option

○	$\lim_{n\to\infty} a_n = -1$
○	$\lim_{n\to\infty} a_n = 0$ 
○	None of these
○	$\lim_{n\to\infty} a_n = 1$

Question # 7 of 10 (Start time: 02:36:27 PM, 15 June 2022) Total Marks: 1

Every Cauchy sequence has a

Select the correct option

- positive subsequence.
- decreasing subsequence.
- convergent subsequence.
- increasing subsequence.

Question # 8 of 10 (Start time: 02:37:01 PM, 15 June 2022)

Total Marks: 1

$$The\ series \sum_{n=k}^{\infty} a_n\ converges,\ if\ (-1)^n a_n > 0,\quad |a_{n+1}| < |a_n|,\ and \lim_{n\to\infty} a_n = ------.$$

Select the correct option

Reload Math Equations

○	-1
○	0
○	∞
○	1

MTH621:Quiz-1

Quiz Start Time: 02:31 PM, 15 June 2022

Question # 4 of 10 (Start time: 02:34:13 PM, 15 June 2022)

Total Marks: 1

Which of the following is a subsequenc of the sequence $\{\frac{1}{n}\}$?

Select the correct option

Reload Math Equations

- $\{\frac{1}{2n}\}$
- $\{\frac{2}{n}\}$
- none of these
- $\{\frac{n+1}{n}\}$

Question # 5 of 10 (Start time: 02:35:10 PM, 15 June 2022)

$For \ - - - - - - series \ \sum_{n=1}^{\infty} \frac{1}{n} = 0.$

Select the correct option

○	harmonic
○	geometric
○	None of these
○	arithmetic

Question # 1 of 10 (Start time: 02:31:46 PM, 15 June 2022) Total Marks:

Every sequence $\{s_n\}$ *of real numbers has a unique limit superior,* $\overline{s}$*, and a unique limit inferior,* $\underline{s}$*, in the extended reals, and* – – – –

Select the correct option

Reload Math Equations

- $\underline{s} \geq \overline{s}$
- $\underline{s} = \overline{s}$
- *None of these*
- $\underline{s} \leq \overline{s}$

Click to Save Answer & Move to Next Question

Question # 3 of 10 (Start time: 02:33:49 PM, 15 June 2022) Total Marks: 1

$$If\ \{s_n\}\ is\ a\ sequence\ of\ real\ numbers,\ then\ \lim_{n\to\infty} s_n = s\ iff\ \lim_{n\to\infty} \sup s_n ---- \lim_{n\to\infty} \inf s_n = s.$$

Select the correct option

Reload Math Equations

- $=$
- $>$
- $<$
- $None\ of\ these$

Click to Save Answer & Next Question

Question # 2 of 10 (Start time: 02:32:32 PM, 15 June 2022) Total Marks: 1

$A\ sequence\ \{s_n\}\ of\ real\ numbers\ is\ called\ a\ ________\ if\ for\ every\ \varepsilon > 0,\ there\ is\ an\ integer\ N\ such\ that\ |s_n - s_m| < \varepsilon if\ m, n \geq N.$

Select the correct option

Refresh Math Equations

- sequence
- bounded sequence
- cauchy sequence
- None of these



Click to Save Answer & Next Question

MTH621:Quiz-1

Quiz Start Time: 02:15 PM, 16 June 2022

Question # 9 of 10 (Start time: 02:23:48 PM, 16 June 2022)

Total Marks: 1

The set of terms of $\{a_{n_k}\}$ is contained in the set of terms of $\{a_n\}$ implies

Select the correct option

Reload Math Equations

- $\{a_{n_k}\}$ is convergent.
- $\sup\{a_n\} \geq \sup\{a_{n_k}\}$.
- $\sup\{a_n\} = \sup\{a_{n_k}\}$.
- $\sup\{a_n\} \leq \sup\{a_{n_k}\}$.

Question # 3 of 10 (Start time: 02:22:19 PM, 15 June 2020)

The given series $\sum_{n=1}^{\infty} \frac{1}{n}$ is ____.

Select the correct option

- convergent
- divergent

Question # 8 of 10 (Start time: 02:23:05 PM, 15 June 2022)

Let $\sum_{n=1}^{\infty} a_n = A$ and $\sum_{n=1}^{\infty} b_n = B$, where A and B are finite. There $\sum_{n=1}^{\infty} ca_n =$ ------

Select the correct option

O	cA
O	$c - A$
O	$\frac{c}{A}$
O	$c + A$

Question # 6 of 10 (Start time: 02:21:39 PM, 15 June 2022)

Which statement(s) is(are) true about the following sequence $a_1 = 1$ and $a_n = 1 - e^{-a_{n-1}}$

Select the correct option

- $a_{n+1} - a_n = -(e^{-a_n} - e^{-a_{n-1}})$
- $0 < a_n \leq 1$ for all n.
- All of the above. ✓
- Sequence is convergent.

If $\{x_n\}$ is ——————, then $\{x_n\}$ has a convergent subsequence.



bounded

unbounded

MTH621:Quiz-1

Question # 5 of 10 (Start time: 02:20:49 PM, 15 June 2022)

$\limsup_{n \to \infty} a_n = \langle a_n \rangle$ if

Select the correct option

- $\{a_n\}$ is not bounded above.
- $\{a_n\}$ has limit infimum
- $\{a_n\}$ has unique limit.
- $\{a_n\}$ is convergent.

Total Marks: 1

If $\{a_n\}$ is ———— then $\lim_{n\to\infty} a_n = \sup\{a_n\}$.

Select the correct option.

Reload Math Equations

- decreasing
- unincreasing
- increasing
- nondecreasing

MTH621:Quiz-1

Question # 7 of 10 (Start time: 02:10:11 PM, 15 June 2022)

$\liminf_{n\to\infty} s_n = -\infty$ if

Select the correct option

- $\{s_n\}$ is not bounded above.
- $\lim_{n\to\infty} s_n = 0.$
- $\{s_n\}$ is convergent .
- $\{s_n\}$ is not bounded below.

Question # 10 of 10 (Start time: 02:12:48 PM, 15 June 2022)

Every Cauchy sequence has a

Select the correct option

- decreasing subsequence.
- positive subsequence.
- increasing subsequence.
- convergent subsequence.

If $\{a_n\}_1^\infty$ is an infinite sequence of real numbers, the symbol $\sum_{n=1}^{\infty} a_n$ is ---------- and a_n is the n^{th} term of series

the correct option

finite series

infinite series

BC210407344: MADEEHA RUBAB

Time Left 87 sec(s)

MTH621:Quiz-1

Quiz Start Time: 01:56 PM, 15 June 2022

Question # 6 of 10 (Start time: 02:01:37 PM, 15 June 2022)

Total Marks: 1

If the sequence is convergent then

Select the correct option

- it is bounded.
- it is bounded below but may not be bounded above.
- it has two limits.
- it is bounded above but may not be bounded below.

Click to Save Answer & Move to Next Question

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3

IC200402420: HIRA RAB NAWAZ

MTH621:Quiz-1

Question # 6 of 10 (Start time: 10:34:09 AM, 04 June 2021)

If $f(x) = \sqrt{4-x^2}$ and $g(x) = \sqrt{x-1}$, then domain of the quotient function f/g is

and

$(fg)(x) = (\sqrt{4-x^2})(\sqrt{x-1}) = \sqrt{(4-x^2)(x-1)}$.

quotient f/g defined on $(1,2]$ by

$\left(\frac{f}{g}\right)(x) = \sqrt{\frac{4-x^2}{x-1}}$.

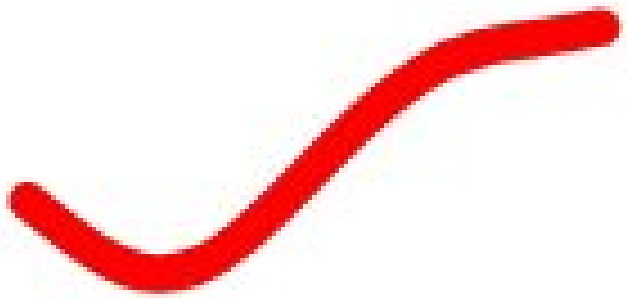
Select the correct option

- $[7,8]$.
- $[-3,3]$.
- $[1,2]$.
- $(1,2]$.

The sum of a convergent series is ———.

n

unique



different

MTH621:Quiz-1

Question # 8 of 10 (Start time: 01:40:59 PM, 15 June 2022)

For harmonic series $\sum_{n=1}^{\infty} \frac{1}{n} = - - - - - - -$.

Select the correct option

O	-1
O	1
O	0
O	None of these

MTH621:Quiz-1

Question # 6 of 10 (Start time: 01:39:52 PM, 15 June 2022)

A point x is a limit point of a set iff there is a sequence $\{x_n\}$ of point in S such that

$$x_n \neq x \text{ for } n \geq 1, \text{ and } - - - - - -$$

Select the correct option

○	$\lim_{n \to \infty} x_n = \bar{x}$
○	None of these
○	$\lim_{n \to \infty} x_n > \bar{x}$
○	$\lim_{n \to \infty} x_n \neq \bar{x}$

3G H+ 1:21 70

10:51 4G 671 B/s

Quiz
quiz.vu.edu.pk

MTH621:Quiz-1

Question # 9 of 10 (Start time: 10:51:36 AM, 04 June 2021)

If $f(x) = \sqrt{4 - x^2}$ and $g(x) = \sqrt{x - 1}$, then domain of the quotient functio

repeat

Select the correct option

- $(1, 2]$.
- $[7, 8]$.
- $[1, 2]$.
- $[-3, 3]$.

MTH621:Quiz-1

Question # 5 of 10 (Start time: 01:39:14 PM, 15 June 2022)

The limit of the seqJence

$$s_n = \frac{1}{n} + \frac{2(1+3/n)}{1+1/n}.$$

Select the correct option

- 4
- 2
- 3
- 1

MTH621:Quiz-1

Question # 2 of 10 (Start time: 01:36:56 PM, 15 June 2022)

The given series $\sum_{n=1}^{\infty} \frac{1}{n}$ is ------.

Select the correct option

- convergent
- divergent ✓

Question # 4 of 10 (Start time: 01:38:10 PM, 15 June 2022)

$\liminf_{n\to\infty} s_n = -\infty$ if

Select the correct option

- $\{s_n\}$ is not bounded above.
- $\lim_{n\to\infty} s_n = 0.$
- $\{s_n\}$ is not bounded below. ✓
- $\{s_n\}$ is convergent .

so that no constant M is greater than or equal to every partial sum of the harmonic series.

Example: Consider the geometric series $\sum r^n$.
If $|r| < 1$ and $m \geq n$, then

$$\begin{aligned} |A_m - A_n| &= |r^{n+1} + r^{n+2} + \cdots + r^m| \\ &\leq |r|^{n+1}(1 + |r| + \cdots + |r|^{m-n-1}) \\ &= |r|^{n+1}\frac{1 - |r|^{m-n}}{1 - |r|} < \frac{|r|^{n+1}}{1 - |r|}. \end{aligned}$$

If $\varepsilon > 0$, choose N so that

$$\frac{|r|^{N+1}}{1 - |r|} < \varepsilon.$$

Then Cauchy's convergence criterion implies that

$$|A_m - A_n| < \varepsilon \quad \text{if} \quad m \geq n \geq N.$$

Now Theorem implies that $\sum r^n$ converges if $|r| < 1$, as in Example ??.

Corollary: If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$

Proof: If $\sum a_n$ converges, then for each $\varepsilon > 0$ there is an integer K such that

$$\left|\sum_{n=k}^{\infty} a_n\right| < \varepsilon \quad \text{if} \quad k \geq K;$$

that is,

$$\lim_{k\to\infty} \sum_{n=k}^{\infty} a_n = 0.$$

Be careful: $\lim_{n\to\infty} a_n = 0$ Necessary condition.

Not sufficient.

Example: For the harmonic series $\lim_{n\to\infty} \frac{1}{n} = 0$.

But we have proved that

$$\sum_{n=1}^{\infty} \frac{1}{n} = 0$$

is divergent.

Corollary: If $\sum a_n$ converges, then for each $\varepsilon > 0$ there is an integer K such that

$$\left|\sum_{n=k}^{\infty} a_n\right| < \varepsilon \quad \text{if} \quad k \geq K;$$

Q: if $\sum a_n$ ——— ,
then $\lim_{n\to\infty} a_n = 0$

(a) converges
(b) diverges

If $\{A_n\}_k^\infty$ diverges, we say that $\sum_{n=k}^\infty a_n$ *diverges.*

In particular, if $\lim_{n\to\infty} A_n = \infty$ or $-\infty$, we say that $\sum_{n=k}^\infty a_n$ *diverges to* ∞ *or* $-\infty$, and write

$$\sum_{n=k}^{\infty} a_n = \infty \quad \text{or} \quad \sum_{n=k}^{\infty} a_n = -\infty.$$

2.8.2 Oscillatory Series

A divergent infinite series that does not diverge to $\pm\infty$ is said to *oscillate*, or *be oscillatory.*

Example: Consider the series.

$$\sum_{n=0}^{\infty} r^n, \quad -1 < r < 1.$$

Here $a_n = r^n$, $(n \geq 0)$. The nth term of sequence of partial sum is

$$A_n = 1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r},$$

If $r = -1$, then $A_{2m+1} = 0$ and $A_{2m} = 1$ for $m \geq 0$, while if $r =$ [illegible] $A_n = n + 1$; in both cases the series diverges.

Theorem: The sum of a convergent series is unique.
Proof: See Lecture.

Theorem: Let $\sum_{n=k}^{\infty} a_n = A$ and $\sum_{n=k}^{\infty} b_n = B$, where A and B are finite. Then

$$\sum_{n=k}^{\infty} (ca_n) = cA$$

if c is a constant,

$$\sum_{n=k}^{\infty} (a_n + b_n) = A + B,$$

and

$$\sum_{n=k}^{\infty} (a_n - b_n) = A - B.$$

These relations also hold [illegible] one or both of A and B is infinite, provided that the right sides are not indeterminate.

2.9 Dropping Finitely Many Terms

For example, suppose that we drop the first k terms of a series $\sum_{n=0}^{\infty} a_n$, and consider the new series $\sum_{n=k}^{\infty} a_n$. sums of the two series by

$$\begin{aligned} A_n &= a_0 + a_1 + \cdots + a_n, \quad n \geq 0, \\ A'_n &= a_k + a_{k+1} + \cdots + a_n, \quad n \geq k. \end{aligned}$$

$$A_n = (a_0 + a_1 + \cdots + a_{k-1}) + A'_n, \quad n \geq k,$$

it follows that $A = \lim_{n\to\infty} A_n$ exists (in the extended reals) if and only if $A' = \lim_{n\to\infty} A'_n$ does, and in this case

$$A = (a_0 + a_1 + \cdots + a_{k-1}) + A'.$$

An important principle follows from this.

Lemma: Suppose that for n sufficiently large (that is, for $n \geq$ [illegible]) the terms of $\sum_{n=k}^{\infty} a_n$ satisfy some condition that implies convergence of an infinite series. Then $\sum_{n=k}^{\infty} a_n$ converges.

Similarly, suppose that for n sufficiently large the terms $\sum_{n=k}^{\infty} a_n$ satisfy some condition that implies divergence of an infinite series. Then $\sum_{n=k}^{\infty} a_n$ diverges.

Example: *The series* $\sum_{k}^{\infty} a_n$ converges *if* $(-1)^n a_n > 0$, $|a_{n+1}| < |a_n|$, *and* $\lim_{n\to\infty} a_n = 0$

The terms of

$$\sum_{n=1}^{\infty} \frac{16+(-2)^n}{n2^n}$$

do not satisfy these conditions for all $n \geq 1$, but they do satisfy them for sufficiently large n.

Theorem: A series $\sum a_n$ converges if and only if for every $\varepsilon > 0$ there is an integer N such that

$$|a_n + a_{n+1} + \cdots + a_m| < \varepsilon \quad \text{if} \quad m \geq n \geq N.$$

Proof: In terms of the partial sums $\{A_n\}$ of $\sum a_n$,

$$a_n + a_{n+1} + \cdots + a_m = A_m - A_{n-1}.$$

Therefore, (??) can be written as

$$|A_m - A_{n-1}| < \varepsilon \quad \text{if} \quad m \geq n \geq N.$$

Since $\sum a_n$ converges if and only if $\{A_n\}$ converges, Theorem implies the conclusion.

Theorem: Show that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

Proof: Consider the sequence of partial sums

$$s_2 = 1 + \frac{1}{2}, s_3 = 1 + \frac{1}{2} + \frac{1}{3}, s_4 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}, \cdots$$

$n=k$ $n=k$

2.8.2 Oscillatory Series

A divergent infinite series that does not diverge to $\pm\infty$ is said to *oscillate* or *be oscillatory*.

Example: Consider the series

$$\sum_{n=0}^{\infty} r^n, \quad -1 < r < 1.$$

Here $a_n = r^n$ $(n \geq 0)$. The nth term of sequence of partial sum is

$$A_n = 1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r},$$

which converges to $1/(1-r)$ as $n \to \infty$. Thus, we write

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}, \quad -1 < r < 1.$$

Example: The nth term of sequence of partial sum is

$$\frac{1}{} \quad ^{n+1}$$

MCQs:

$$\liminf_{n\to\infty} [1+(-1)^n]\, n^2 \;=\; 0.$$

Theorem: If $\{s_n\}$ is a sequence of real numbers, then

$$\lim_{n\to\infty} s_n = s \tag{2.9}$$

if and only if

Amt

$$\limsup_{n\to\infty} s_n = \liminf_{n\to\infty} s_n = s. \tag{2.10}$$

Proof: If $s = \pm\infty$, the equivalence of (2.9) and (2.10) follows immediately from their definitions. If $\lim_{n\to\infty} s_n = s$ (finite), then definition of a convergent sequence implies that (2.4)–(2.7) hold with $\overline{s}$ and $\underline{s}$ replaced by s. Hence, (2.10) follows from the uniqueness of $\overline{s}$ and $\underline{s}$. For the converse, suppose that $\overline{s} = \underline{s}$ and let s denote their common value. Then (2.4) and (2.6) imply that

$$s - \varepsilon < s_n < s + \varepsilon$$

for large n, and (2.9) follows from Definition and the uniqueness of $\lim_{n\to\infty} s_n$.

2.7 Cauchy Sequence

A sequence $\{s_n\}$ of real numbers is called a Cauchy sequence if for every $\varepsilon > 0$, there is an integer N such that

$$|s_n - s_m| < \varepsilon \quad \text{if} \quad m, n \geq N.$$

Theorem: If $\{s_n\}$ is a Cauchy sequence of real numbers, then $\{s_n\}$ is bounded.

Proof: See Lecture

Theorem (Cauchy's convergence criterion): A sequence $\{s_n\}$ of real numbers converges **if and only if**, for every $\varepsilon > 0$, there is an integer N such that

$$|s_n - s_m| < \varepsilon \quad \text{if} \quad m, n \geq N.$$

Q:- For the convergent sequence $\{s_n\}$, which statement is true?

Amt $\lim_{n\to\infty} \sup s_n = \lim_{n\to\infty} \inf s_n = \lim_{n\to\infty} s_n$

Q:- Every Cauchy sequence has a ——— ?

Ans:- Convergent sequence.

Proof: Suppose that $\lim_{n\to\infty} s_n = s$ and $\varepsilon > 0$.
There is an integer N such that

$$|s_r - s| < \frac{\varepsilon}{2} \quad \text{if} \quad r \geq N.$$

Therefore,

$$|s_n - s_m| = |(s_n - s) + (s - s_m)| \leq |s_n - s| + |s - s_m| < \varepsilon,$$

whenever $n, m \geq N$. Therefore, the stated condition is necessary for convergence of $\{s_n\}$.

Recall a Cauchy sequence $\{s_n\}$ is bounded. So $\overline{s}$ and $\underline{s}$ are finite.
Now suppose that $\varepsilon > 0$ and N satisfies $|s_n - s_m| < \varepsilon$, $n, m \geq N$.
From definition of limit superior and limit inferior

$$|s_n - \overline{s}| < \varepsilon, \qquad |s_m - \underline{s}| < \varepsilon$$

for some integer $m, n > N$.
Since

$$\begin{aligned} |\overline{s} - \underline{s}| &= |(\overline{s} - s_n) + (s_n - s_m) + (s_m - \underline{s})| \\ &\leq |\overline{s} - s_n| + |s_n - s_m| + |s_m - \underline{s}|. \end{aligned}$$

We have

$$|\overline{s} - \underline{s}| < 3\varepsilon.$$

Since ε is an arbitrary positive number, this implies that $\overline{s} = \underline{s}$, so $\{s_n\}$ converges.

2.8 Series

If $\{a_n\}_k^\infty$ is an infinite sequence of real numbers, the symbol

Ans:- $\sum_{n=k}^{\infty} a_n$

is an *infinite series* and a_n is the *nth term* of the series.
We say that $\sum_{n=k}^{\infty} a_n$ *converges to the sum* A, and write

$$\sum_{n=k}^{\infty} a_n = A,$$

if the sequence $\{A_n\}_k^\infty$ defined by

$$A_n = a_k + a_{k+1} + \cdots + a_n, \quad n \geq k,$$

converges to A.

We also define

$$\limsup_{n\to\infty} s_n = \infty \quad \text{if } \{s_n\} \text{ is not bounded above,}$$

$$\limsup_{n\to\infty} s_n = -\infty \quad \text{if } \lim_{n\to\infty} s_n = -\infty,$$

$$\liminf_{n\to\infty} s_n = -\infty \quad \text{if } \{s_n\} \text{ is not bounded below,}$$

$$\liminf_{n\to\infty} s_n = \infty \quad \text{if } \lim_{n\to\infty} s_n = \infty.$$

Theorem: Every sequence $\{s_n\}$ of real numbers has a unique limit superior, $\overline{s}$, and a unique limit inferior, $\underline{s}$, in the extended reals, and

$$\underline{s} \leq \overline{s}.$$

Examples:

$$= \frac{3+0}{3+0}$$
$$= \frac{1}{6}.$$

2.5 Subsequence

A sequence $\{t_k\}$ is a *subsequence* of a sequence $\{s_n\}$ if

$$t_k = s_{n_k}, \quad k \geq 0,$$

where $\{n_k\}$ is an increasing infinite sequence of integers in the domain of $\{s_n\}$. We denote the subsequence $\{t_k\}$ by $\{s_{n_k}\}$.

Example: If

$$\{s_n\} = \left\{\frac{1}{n}\right\} = \left\{1, \frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{n}, \ldots\right\},$$

then letting $n_k = 2k$ yields the subsequence

$$\{s_{2k}\} = \left\{\frac{1}{2k}\right\} = \left\{\frac{1}{2}, \frac{1}{4}, \ldots, \frac{1}{2k}, \ldots\right\},$$

and letting $n_k = 2k+1$ yields the subsequence

$$\{s_{2k+1}\} = \left\{\frac{1}{2k+1}\right\} = \left\{1, \frac{1}{3}, \ldots, \frac{1}{2k+1}, \ldots\right\}.$$

Example: The sequence $\{s_n\}$ defined by

$$s_n = (-1)^n \left(1 + \frac{1}{n}\right)$$

does not converge, but $\{s_n\}$ has subsequences that do. For example,

$$\{s_{2k}\} = \left\{1 + \frac{1}{2k}\right\}$$

Since the set of terms of $\{s_{n_k}\}$ is contained in the set of terms of $\{s_n\}$, implies ?

Ans $\sup\{s_n\} \geq \sup\{s_{n_k}\}$.

Since $\{s_n\}$ is nondecreasing, there is for every n an integer n_k such that $s_n \leq s_{n_k}$. This implies that

$$\sup\{s_n\} \leq \sup\{s_{n_k}\}.$$

Theorem: A point $\overline{x}$ is a limit point of a set S **if and only if** there is a sequence $\{x_n\}$ of points in S such that $x_n \neq \overline{x}$ for $n \geq 1$, and

Ans $\lim_{n\to\infty} x_n = \overline{x}$.

Proof: By definition for each $\varepsilon > 0$, there is an integer N such that $0 < |x_n - \overline{x}| < \varepsilon$ if $n \geq N$.

Therefore, every ε-neighborhood of $\overline{x}$ contains infinitely many points of S. This means that $\overline{x}$ is a limit point of S.

For necessity, let $\overline{x}$ be a limit point of S. Then, for every integer $n \geq 1$, the interval $(\overline{x} - 1/n, \overline{x} + 1/n)$ contains a point x_n $(\neq \overline{x})$ in S.

Since $|x_m - \overline{x}| \leq 1/n$ if $m \geq n$, $\lim_{n\to\infty} x_n = \overline{x}$.

Theorem:

Ans

1. If $\{x_n\}$ is bounded then $\{x_n\}$ has a convergent subsequence.
2. If $\{x_n\}$ is unbounded above, then $\{x_n\}$ has a subsequence $\{x_{n_k}\}$ such that $\lim_{k\to\infty} x_{n_k} = \infty$. Ans
3. If $\{x_n\}$ is unbounded below, then $\{x_n\}$ has a subsequence $\{x_{n_k}\}$ such that $\lim_{k\to\infty} x_{n_k} = -\infty$. Ans

Proof: Let S be the set of distinct numbers that occur as terms of $\{x_n\}$.

(For example, if $\{x_n\} = \{(-1)^n\}$, $S = \{1, -1\}$; if $\{x_n\} = \{1, \frac{1}{2}, 1, \frac{1}{3}, \ldots, 1, 1/n, \ldots\}$, $S = \{1, \frac{1}{2}, \ldots, 1/n, \ldots\}$.)

If S contains only finitely many points, then some $\overline{x}$ in S occurs infinitely often in $\{x_n\}$; that is, $\{x_n\}$ has a subsequence $\{x_{n_k}\}$ such that $x_{n_k} = \overline{x}$ for all k. Then $\lim_{k\to\infty} x_{n_k} = \overline{x}$, and we are finished in this case.

If S is infinite, then, since S is bounded (by assumption), the Bolzano-Weierstrass theorem implies that S has a limit point $\overline{x}$.

There is a sequence of points $\{y_j\}$ in S, distinct from $\overline{x}$, such that

$$\lim_{j\to\infty} y_j = \overline{x}.$$

2.6 Limit Superior and Limit Inferior

Theorem:

- If $\{s_n\}$ is bounded above and does not diverge to $-\infty$, then there is a unique real number $\overline{s}$ such that, if $\varepsilon > 0$,

$$s_n < \overline{s} + \varepsilon \quad \text{for large } n \tag{2.4}$$

and

$$s_n > \overline{s} - \varepsilon \quad \text{for infinitely many } n. \tag{2.5}$$

- If $\{s_n\}$ is bounded below and does not diverge to ∞, then there is a unique real number $\underline{s}$ such that, if $\varepsilon > 0$,

$$s_n > \underline{s} - \varepsilon \quad \text{for large } n \tag{2.6}$$

and

$$s_n < \underline{s} + \varepsilon \quad \text{for infinitely many } n. \tag{2.7}$$

Proof: Proof of the first part. Since $\{s_n\}$ is bounded above, there is a number β such that $s_n < \beta$ for all n. Since $\{s_n\}$ does not diverge to $-\infty$, there is a number

so that

$$0 < x_n \leq \frac{p-1}{n}.$$

Theorem: Show that $\lim_{n\to\infty} \sqrt[n]{n} = 1.$

Proof: Take $x_n = \sqrt[n]{n} - 1$.
Then $x_n \geq 0$, and, by the binomial theorem

$$n = (1+x_n)^n \geq \frac{n(n-1)}{2} x_n^2.$$

Hence

$$0 \leq x_n \leq \sqrt{\frac{2}{n-1}}, \qquad n \geq 2.$$

Theorem: If $p > 0$ and α is real, then $\lim_{n\to\infty} \frac{n^\alpha}{(1+p)^n} = 0.$

Proof: Let k be an integer such that $k > \alpha$, $k > 0$.
For $n > 2k$,

$$(1+p)^n > \frac{n(n-1)\ldots(n-k+1)}{k!} p^k > \frac{n^k p^k}{2^k k!}.$$

Hence

$$0 < \frac{n^\alpha}{(1+p)^n} < \frac{2^k k!}{p^k} n^{\alpha-k}, \qquad n > 2k.$$

Since $\alpha - k < 0$, $n^{\alpha-k} \to 0$ by

Theorem: If $|x| < 1$, then $\lim_{n\to\infty} x^n = 0.$

Theorem: Let $\lim_{n\to\infty} s_n = s$ and $\lim_{n\to\infty} t_n = t$, where s and t are finite. Then

$$\lim_{n\to\infty} (cs_n) = cs \tag{2.2}$$

if c is a constant;

$$\lim_{n\to\infty} (s_n + t_n) = s + t,$$

$$\lim_{n\to\infty} (s_n - t_n) = s - t,$$

$$\lim_{n\to\infty} (s_n t_n) = st,$$

$$\lim_{n\to\infty} \frac{s_n}{t_n} = \frac{s}{t}$$

Q. if $\{s_n\}$ converges to -1 and $\{t_n\}$ converges to 5, then the sequence $\{s_n + t_n\}$ ——?
Ans: will be convergent and its limit is 4.

This implies that $|s_n - \beta| < \varepsilon$ if $n \geq N$, so $\lim_{n\to\infty} s_n = \beta$, by definition.

If $\beta = \infty$ and b is any real number, then $s_N > b$ for some integer N. Then $s_n > b$ for $n \geq N$, so $\lim_{n\to\infty} s_n = \infty$.

For the proof of the second part try yourself.

options ①

Example: If $s_0 = 1$ and $s_n = 1 - e^{-s_{n-1}}$, then $0 < s_n \leq 1$ for all n, by induction.

② Since $s_{n+1} - s_n = -(e^{-s_n} + e^{-s_{n-1}})$ if $n \geq 1$.

The mean value theorem implies that

$$s_{n+1} - s_n = e^{-t_n}(s_n - s_{n-1}) \quad \text{if} \quad n \geq 1, \tag{2.1}$$

where t_n is between s_{n-1} and s_n. Since $s_1 - s_0 = -1/e < 0$, it follows by induction from (2.1) that $s_{n+1} - s_n < 0$ for all n.

Hence, $\{s_n\}$ is bounded and decreasing, and therefore ③ convergent. Sequence

Remark: Let $\{x_n\}$ and $\{s_n\}$ be two sequences. If $0 \leq x_n \leq s_n$ for $n \geq N$, where N is some fixed number, and if

$$s_n \to 0,$$

option ④ :- all of them.

then

$$x_n \to 0.$$

2.4 Some Special Sequences

Theorem: If $p > 0$, then

$$\lim_{n\to\infty} \frac{1}{n^p} = 0.$$

Proof: Before presenting the proof let us recall the following:

Recall the binomial theorem:

$$(1+x)^n = 1 + nx + x(n-1)\frac{x^2}{2!} + \ldots.$$

We have the following inequality

$$1 + nx \leq (1+x)^n, x > 0.$$

Theorem: If $p > 0$, then

$$\lim_{n\to\infty} \sqrt[n]{p} = 1.$$

Proof: We will discuss the three cases when $p = 1$, when $p > 1$ and when $0 < p < 1$.

If $p > 1$, put $x_n = \sqrt[n]{p} - 1$. Then $x_n > 0$, and, by the binomial theorem

$$1 + nx_n \leq (1 + x_n)^n = p,$$

Q- Which statement(s) is(are) true about the following sequence: $s_0 = 1$ and $s_n = 1 - e^{-s_{n-1}}$

Ans:- All of the above

$|t_n \quad t| \qquad |t_n||t| \qquad |t|$

hence, $\lim_{n\to\infty} 1/t_n = 1/t$. Now we obtain (2.3) in the general case from (2.3) with $\{t_n\}$ replaced by $\{1/t_n\}$.

Example: Find the limit of the sequence

$$s_n = \frac{1}{n}\sin\frac{n\pi}{4} + \frac{2(1+3/n)}{1+1/n}.$$

$= 2$

Ans

that is,

$$\lim_{k\to\infty}\sum_{n=k}^{\infty} a_n = 0.$$

Example: If $|r| < 1$, then

$$\left|\sum_{n=k}^{\infty} r^n\right| = \left|r^k \sum_{n=k}^{\infty} r^{n-k}\right| = \left|r^k \sum_{n=0}^{\infty} r^n\right| = \frac{|r|^k}{1-r}.$$

Therefore, if

$$\frac{|r|^K}{1-r} < \varepsilon,$$

then

$$\left|\sum_{n=k}^{\infty} r^n\right| < \varepsilon \quad \text{if} \quad k \geq K,$$

which implies that $\lim_{k\to\infty}\sum_{n=k}^{\infty} r^n = 0$.

2.10 Series of Nonnegative Terms

The series $\sum a_n$ is said to be series of nonnegative terms if $a_n \geq 0$ for $n \geq k$. The theory of series $\sum a_n$ with terms that are nonnegative for sufficiently large n is simpler than the general theory, since such

The series of nonnegative terms either converges to a finite limit or diverges to ∞.

Recall the following theorem:

- If $\{s_n\}$ is nondecreasing, then $\lim_{n\to\infty} s_n = \sup\{s_n\}$.
- If $\{s_n\}$ is nonincreasing, then $\lim_{n\to\infty} s_n = \inf\{s_n\}$.

Theorem: If $a_n \geq 0$ for $n \geq k$, then $\sum a_n$ converges if its partial sums are bounded, or diverges to ∞ if they are not. These are the only possibilities and, in either case,

$$\sum_{n=k}^{\infty} a_n = \sup\{A_n : n \geq k\}$$

where

$$A_n = a_k + a_{k+1} + \cdots + a_n, \quad n \geq k.$$

Proof: Since $A_n = A_{n-1} + a_n$ and $a_n \geq 0$ $(n \geq k)$, the sequence $\{A_n\}$ is nondecreasing, so the conclusion follows from Theorem and definition of convergence of a series.

Recall the theorem: If $a_n \geq 0$ for $n \geq k$, then $\sum a_n$ converges if its partial sums are bounded, or diverges to ∞ if they are not. These are the only possibilities and, in either case,

$$\sum_{n=k}^{\infty} a_n = \sup\{A_n : n \geq k\}$$

that is,

$$\lim_{k\to\infty}\sum_{n=k}^{\infty} a_n = 0.$$

Example: If $|r| < 1$, then

$$\left|\sum_{n=k}^{\infty} r^n\right| = \left|r^k \sum_{n=k}^{\infty} r^{n-k}\right| = \left|r^k \sum_{n=0}^{\infty} r^n\right| = \frac{|r|^k}{1-r}.$$

Therefore, if

$$\frac{|r|^K}{1-r} < \varepsilon,$$

then

$$\left|\sum_{n=k}^{\infty} r^n\right| < \varepsilon \quad \text{if} \quad k \geq K,$$

which implies that $\lim_{k\to\infty}\sum_{n=k}^{\infty} r^n = 0$.

2.10 Series of Nonnegative Terms

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Theorem: If $a_n \geq 0$ for $n \geq k$, then $\sum a_n$ converges if its partial sums are bounded, or diverges to ∞ if they are not. These are the only possibilities and, in either case,

$$\sum_{n=k}^{\infty} a_n = \sup\{A_n : n \geq k\}$$

where

$$A_n = a_k + a_{k+1} + \cdots + a_n, \quad n \geq k.$$

Proof: Since $A_n = A_{n-1} + a_n$ and $a_n \geq 0$ $(n \geq k)$, the sequence $\{A_n\}$ is nondecreasing, so the conclusion follows from Theorem and definition of convergence of a series.

Recall the theorem: If $a_n \geq 0$ for $n \geq k$, then $\sum a_n$ converges if its partial sums are bounded, or diverges to ∞ if they are not. These are the only possibilities and, in either case,

$$\sum_{n=k}^{\infty} a_n = \sup\{A_n : n \geq k\}$$

2.8.1 Sequence of Partial Sums

The finite sum A_n is the *nth partial sum of* $\sum_{n=k}^{\infty} a_n$.

If $\{A_n\}_k^{\infty}$ diverges, we say that $\sum_{n=k}^{\infty} a_n$ *diverges*.

In particular, if $\lim_{n\to\infty} A_n = \infty$ or $-\infty$, we say that $\sum_{n=k}^{\infty} a_n$ *diverges to* ∞ *or* $-\infty$, and write

$$\sum_{n=k}^{\infty} a_n = \infty \quad \text{or} \quad \sum_{n=k}^{\infty} a_n = -\infty.$$

2.8.2 Oscillatory Series

A divergent infinite series that does not diverge to $\pm\infty$ is said to *oscillate* or *be oscillatory*.

Example: Consider the series

$$\sum_{n=0}^{\infty} r^n, \quad -1 < r < 1.$$

Here $a_n = r^n$ $(n \geq 0)$. The nth term of sequence of partial sum is

$$A_n = 1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r},$$

which converges to $1/(1-r)$ as $n \to \infty$. Thus, we write

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}, \quad -1 < r < 1.$$

Example: The nth term of sequence of partial sum is

$$A_n = 1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r},$$

If $|r| > 1$, then $\sum_{n=0}^{\infty} r^n$ diverges; if $r > 1$, then

$$\sum_{n=0}^{\infty} r^n = \infty,$$

Example: The nth term of sequence of partial sum is

$$A_n = 1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r},$$

If $r < -1$, $\sum_{n=0}^{\infty} r^n$ oscillates, since its partial sums alternate in sign and their magnitudes become arbitrarily large for large n.

The terms of $$\sum_{n=1}^{\infty} \frac{16+(-2)^n}{n2^n}$$

do not satisfy these conditions for all $n \geq 1$, but they do satisfy them for su
large n.

Theorem: A series $\sum a_n$ converges if and only if for every $\varepsilon > 0$ there is a
N such that

$$|a_n + a_{n+1} + \cdots + a_m| < \varepsilon \quad \text{if} \quad m \geq n \geq N.$$

Proof: In terms of the partial sums $\{A_n\}$ of $\sum a_n$,

$$a_n + a_{n+1} + \cdots + a_m = A_m - A_{n-1}.$$

Therefore, (??) can be written as

$$|A_m - A_{n-1}| < \varepsilon \quad \text{if} \quad m \geq n \geq N.$$

Since $\sum a_n$ conv ges if and only if $\{A_n\}$ converges, Theorem implie s the co

Theorem Sho that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

Proof: C nsider the sequence of partial sums

where

$$A_n = a_k + a_{k+1} + \cdots + a_n, \quad n \geq k.$$

* Quiz: 1 End.

2.11 The Comparison Test

Theorem: Suppose that

$$0 \leq a_n \leq b_n, \quad n \geq k.$$

Then ?

1. $\sum a_n < \infty$ if $\sum b_n < \infty$.
2. $\sum b_n = \infty$ if $\sum a_n = \infty$.

Proof: If

$$A_n = a_k + a_{k+1} + \cdots + a_n \quad \text{and} \quad B_n = b_k + b_{k+1} + \cdots + b_n, \quad n \geq k,$$

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MTH621:Quiz-1

Question # 1 of 10 (Start time: 10:30:24 AM, 04 June 2021)

The set of terms of $\{s_{n_k}\}$ is contained in the set of terms of $\{s_n\}$ implies

Since the set of terms of $\{s_{n_k}\}$ is contained in the set of terms of $\{s_n\}$,

$\sup\{s_n\} \geq \sup\{s_{n_k}\}$.

Select the correct option

- $\{s_n\}$ is convergent.
- $\sup\{s_n\} = \sup\{s_{n_k}\}$.
- $\sup\{s_n\} \geqslant \sup\{s_{n_k}\}$.
- $\sup\{s_n\} \leqslant \sup\{s_{n_k}\}$.

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Since the set of terms of $\{s_{n_k}\}$ is contained in the set of terms of $\{s_n\}$, implies

$$\sup\{s_n\} \geq \sup\{s_{n_k}\}.$$

Since $\{s_n\}$ is nondecreasing, there is for every n an integer n_k such that $s_n <$ s
his implies that

$$\sup\{\quad\} \leq \quad \{s_{n_k}\}.$$

Theorem: A point $\overline{x}$ is a limit point of a set S **if and only if** there is a sequence $\{x_n\}$ of points in S such that $x_n \neq \overline{x}$ for $n \geq 1$, and

$$\lim_{n\to\infty} x_n = \overline{x}.$$

Proof: By definition for each $\varepsilon > 0$, there is an integer N such that $0 < |x_n - \overline{x}| < \varepsilon$ if $n \geq N$.

Therefore, every ε-neighborhood of $\overline{x}$ contains infinitely many points of S. This means that $\overline{x}$ is a limit point of S.

For necessity, let $\overline{x}$ be a limit point of S. Then, for every integer $n \geq 1$, the interval $(\overline{x} - 1/n, \overline{x} + 1/n)$ contains a point x_n $(\neq \overline{x})$ in S.

Since $|x_m - \overline{x}| \leq 1/n$ if $m \geq n$, $\lim_{n\to\infty} x_n = \overline{x}$.

Theorem:

- If $\{x_n\}$ is bounded, then $\{x_n\}$ has a convergent subsequence.
- If $\{x_n\}$ is unbounded above, then $\{x_n\}$ has a subsequence $\{x_{n_k}\}$ such that

$$\lim_{k\to\infty} x_{n_k} = \infty.$$

- If $\{x_n\}$ is unbounded below, then $\{x_n\}$ has a subsequence $\{x_{n_k}\}$ such that

$$\lim \quad = -\infty$$

Total Marks: 1

Question # 9 of 10 (Start time: 12:01:53 PM, 15 June 2022)

Every sequence $\{s_n\}$ *of real numbers has a unique limit superior,* $\overline{s}$*, and a unique limit inferior,* $\underline{s}$*, in the extended reals, and* – – – –

Reload Math Equations

Select the correct option

- $\underline{s} \geqslant \overline{s}$
- $\underline{s} = \overline{s}$
- $\underline{s} \leqslant \overline{s}$
- *None of these*

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9 of 10 (Start time: 10:42:10 AM, 04 June 2021)

sequence $\{s_n\}$ of real numbers has a unique limit superior, $\overline{s}$, and a unique limit inferior, $\underline{s}$, in th

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correct option

$\underline{s} \leq \overline{s}$

$\underline{s} = \overline{s}$

None of these

$\underline{s} \geq \overline{s}$

Theorem: Every sequence $\{s_n\}$ of real numbers has a unique limit superior, $\overline{s}$, and a unique limit inferior, $\underline{s}$, in the extended reals, and

$$\underline{s} \leq \overline{s}.$$

Examples:

The set of terms of $\{s_{n_k}\}$ is contained in the set of terms of $\{s_n\}$ implie

---

Since the set of terms of $\{s_{n_k}\}$ is contained in the set of terms of $\{s_n\}$,

$$\sup\{s_n\} \geq \sup\{s_{n_k}\}.$$

elect the correct option

- $\sup\{s_n\} \geqslant \sup\{s_{n_k}\}.$
- $\sup\{s_n\} = \sup\{s_{n_k}\}.$
- $\sup\{s_n\} \leqslant \sup\{s_{n_k}\}.$
- $\{s_n\}$ is convergent.

MTH621:Quiz-1

Total Marks: 1

Question # 6 of 10 (Start time: 11:57:26 AM, 15 June 2022)

If $\{s_n\}$ is bounded above and does not diverge to $-\infty$, then there is a unique real number $\overline{s}$ such that

$$,if\ \varepsilon > 0,\ \ s_n > \ \overline{s}\ - \varepsilon\ \ - - - - -$$

Reload Math Equations

Select the correct option

- for finitely n
- for \infinitely many n

Click to Save Answer & Move to Next Question

If $\{x_n\}$ is ----, then $\{x_n\}$ has a convergent subsequence.

lect the correct option

Relo

○	*unbounded*
○	*bounded*



Click to Save Answer & Move to N

The series $\sum_{n=0}^{\infty} r^n, \quad -1 < r < 1$, is

Select the correct option

○	convergent and converges to $\frac{1}{1+r^2}$.
○	convergent and converges to 5.
○	divergent.
○	convergent and converges to $\frac{1}{1-r}$.



Question # 9 of 10 (Start time: 11:48:25 AM, 15 June 2022)

The given series $\sum_{n=1}^{\infty} \frac{1}{n}$ *is* ------.

Select the correct option

○	divergent
○	convergent

Let $\sum_{n=k}^{\infty} a_n = A$ and $\sum_{n=k}^{\infty} b_n = B$, where A and B are finite.

Then $\sum_{n=k}^{\infty} (a_n + b_n) = ------$

Select the correct option

- $A + B$
- AB
- $A - B$
- $\frac{A}{B}$

Click to Save Answer & M

Question # 7 of 10 (Start time: 11:45:50 AM, 15 June 2022)

Every Cauchy sequence has a

Select the correct option

- convergent subsequence.
- increasing subsequence.
- decreasing subsequence.
- positive subsequence.

Question # 10 of 10 (Start time: 08:01:33 AM, 15 June 2022)

$For \ \ - - - - - - - series \ \ \sum_{n=1}^{\infty} \frac{1}{n} = 0.$

Select the correct option

O	harmonic
O	arithmetic
O	None of these
O	geometric

Question # 2 of 10 (Start time: 11:39:29 AM, 15 June 2022)

$If \sum a_n$ ------, $then \lim_{n \to \infty} a_n = 0$

Select the correct option

○	converges 
○	diverges

Question # 8 of 10 (Start time: 08:00:19 AM, 15 June 2022)

If the sequence is convergent then

Select the correct option

- it is bounded.
- it has two limits.
- it is bounded above but may not be bounded below.
- it is bounded below but may not be bounded above.

Question # 6 of 10 (Start time: 07:59:08 AM, 15 June 2022)

Total Marks: 1

$$\text{The series } \sum_{n=k}^{\infty} a_n \text{ converges, if } (-1)^n a_n > 0, \quad |a_{n+1}| < |a_n|, \text{ and } \lim_{n\to\infty} a_n = \text{-----}.$$

Select the correct option

Reload Math Equations

○	1
○	-1
○	0
○	∞

Question # 7 of 10 (Start time: 07:59:50 AM, 15 June 2022)

$For\ harmonic\ series\ \sum_{n=1}^{\infty} \frac{1}{n} = - - - - - -.$

Select the correct option

○	-1
○	1
○	None of these
○	0

Total Marks: 1

Question # 4 of 10 (Start time: 07:57:54 AM, 15 June 2022)

$A\ sequence\ \{s_n\}\ of\ real\ numbers\ is\ called\ a\ \cdots\cdots\cdots\ if\ for\ every\ \varepsilon > 0,\ there\ is\ an\ integer\ N\ such\ that\ |s_n - s_m| < \varepsilon if\ m, n \geq N.$

Select the correct option

Reload Math Equations

- cauchy sequence
- bounded sequence
- sequence
- None of these

Click to Save Answer & Move to Next Question

MTH621:Quiz-1

Question # 5 of 10 (Start time: 07:58:33 AM, 15 June 2022)

$Let \sum_{n=k}^{\infty} a_n = A \ and \ \sum_{n=k}^{\infty} b_n = B, \ where \ A \ and \ B \ are \ finite. Then \ \sum_{n=k}^{\infty} ca_n = - - - -$

Select the correct option

O	cA
O	$c + A$
O	$c - A$
O	$\frac{c}{A}$

Question # 2 of 10 (Start time: 07:56:43 AM, 15 June 2022)

If $\{s_n\}$ converges to -1 and $\{t_n\}$ converges to 5, then the sequence $\{s_n + t_n\}$

Select the correct option

○	may be convergent and may be divergent, it depends on the particular given sequences $\{s_n\}$ and $\{t_n\}$.
○	will be divergent.
○	will be convergent and its limit is 4.
○	will be convergent and its limit is 5.

20 AM, 15 June 2022)

$$If\ \{s_n\}\ is\ \text{-----},\ then\ \lim_{n\to\infty} s_n = \inf\{s_n\}.$$

increasing

nondecreasing

nonincreasing



decreasing

Click to Save Answer

Question # 1 of 10 (Start time: 07:55:50 AM, 15 June 2022)

If $f(x) = \sqrt{4 - x^2}$ and $g(x) = \sqrt{x - 1}$, then domain of the quotient function f/g is

Select the correct option

○	$[-3, 3]$.
○	$[1, 2]$.
○	$[7, 8]$.
○	$(1, 2]$.

$$\limsup_{n\to\infty} [1+(-1)^n]\, n^2 = \infty,$$
$$\liminf_{n\to\infty} [1+(-1)^n]\, n^2 = 0.$$

Theorem: If $\{s_n\}$ is a sequence of real numbers, then

$$\lim_{n\to\infty} s_n = s \tag{2.9}$$

if and only if

$$\limsup_{n\to\infty} s_n = \liminf_{n\to\infty} s_n = s. \tag{2.10}$$

Proof: If $s = \pm\infty$, the equivalence of (2.9) and (2.10) follows immediately from their definitions. If $\lim_{n\to\infty} s_n = s$ (finite), then definition of a convergent sequence implies that (2.4)–(2.7) hold with $\overline{s}$ and $\underline{s}$ replaced by s. Hence, (2.10) follows from

Quiz Start Time: 05:02 PM

Start time: 05:13:30 PM, 14 June 2022)

$$\text{The series } \sum_{n=k}^{\infty} a_n \text{ converges, if } (-1)^n a_n > 0, \quad |a_{n+1}| < |a_n|, \text{ and } \lim_{n\to\infty} a_n = \text{------}.$$

on

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1

∞

0

-1

MTH621:Quiz-1

Quiz Start Time: 05:02 PM, 14 June 2022

Question # 8 of 10 (Start time: 05:11:29 PM, 14 June 2022)

Total Marks: 1

A divergent infinite series that does not diverge to ---------- is said to be oscillate

Select the correct option

Reload Math Equations

- ○ ± 1
- ○ ± 2
- ○ $\pm \infty$
- ○ None of these

MTH621:Quiz-1

Question # 6 of 10 (Start time: 05:08:48 PM, 14 June 2022)

$\liminf_{n\to\infty} s_n = -\infty$ if

Select the correct option

- $\lim_{n\to+\infty} s_n = 0.$
- $\{s_n\}$ is convergent .
- $\{s_n\}$ is not bounded below.
- $\{s_n\}$ is not bounded above.

MTH621:Quiz-1

Question # 5 of 10 (Start time: 05:07:12 PM, 14 June 2022)

Which of the following is a subsequenc of the sequence $\{\frac{1}{n}\}$?

Select the correct option

○	$\{\frac{2}{n}\}$
○	$\{\frac{n+1}{n}\}$
○	$\{\frac{1}{2n}\}$
○	none of these

MTH621:Quiz-1

Quiz Start Time: 04:47 PM, 14 June 2022

Question # 8 of 10 (Start time: 04:52:51 PM, 14 June 2022)

Total Marks: 1

If $\{a_n\}_k^\infty$ is an infinite sequence of real numbers, the symbol $_____$ is an infinite series and is the n^{th} term of series.

Select the correct option

Reload Math Equations

- $\sum_{n=k}^{\infty} a_n$
- None of these
- a_n
- $\prod_{k}^{\infty} a_n$

Click to Save Answer & Move to Next Question

Question # 3 of 10 (Start time: 04:47:58 PM, 14 June 2022)

$A\ point\ \bar{x}\ is\ a\ limit\ point\ of\ a\ set\ iff\ there\ is\ a\ sequence\ \{x_n\}\ of\ point\ in\ S\ such\ that\ x_n \neq x\ for\ n \geqslant 1,\ and\ ------$

Select the correct option

O	None of these
O	$\lim_{n\to\infty} x_n = \bar{x}$
O	$\lim_{n\to\infty} x_n \neq \bar{x}$
O	$\lim_{n\to\infty} x_n > \bar{x}$

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Question # 7 of 10 (Start time: 10:40:28 AM, 04 June 2021)

$\limsup_{n\to\infty} s_n = \infty$ if

$\limsup_{n\to\infty} s_n = \infty$ if $\{s_n\}$ is not bounded above,
$\limsup_{n\to\infty} s_n = -\infty$ if $\lim_{n\to\infty} s_n = -\infty$,
$\liminf_{n\to\infty} s_n = -\infty$ if $\{s_n\}$ is not bounded below,
$\liminf_{n\to\infty} s_n = \infty$ if $\lim_{n\to\infty} s_n = \infty$.

Select the correct option

- $\{s_n\}$ has unique limit.
- $\{s_n\}$ has limit inferior.
- $\{s_n\}$ is convergent.
- $\{s_n\}$ is not bounded above.

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MC200405702: MISBAH NAZEER

MTH621:Quiz-1

Question # 7 of 10 (Start time: 10:40:28 AM, 04 June 2021)

$\limsup_{n\to\infty} s_n = \infty$ if

genne

$$\limsup_{n\to\infty} s_n = \infty \text{ if } \{s_n\} \text{ is not bounded above,}$$
$$\limsup_{n\to\infty} s_n = -\infty \text{ if } \lim_{n\to\infty} s_n = -\infty,$$
$$\liminf_{n\to\infty} s_n = -\infty \text{ if } \{s_n\} \text{ is not bounded below,}$$
$$\liminf_{n\to\infty} s_n = \infty \text{ if } \lim_{n\to\infty} s_n = \infty.$$

Select the correct option

- $\{s_n\}$ has unique limit.
- $\{s_n\}$ has limit inferior.
- $\{s_n\}$ is convergent.
- $\{s_n\}$ is not bounded above.

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Question # 9 of 10 (Start time: 12:58:15 PM, 14 June 2022) Total Marks: 1

$If\ \{s_n\}\ is\ a\ sequence\ of\ real\ numbers,\ then \lim_{n\to\infty} s_n = s\ iff \lim_{n\to\infty} \sup s_n ---- \lim_{n\to\infty} \inf s_n = s.$

Select the correct option

Reload Math Equations

- $<$
- $=$
- $None\ of\ these$
- $>$

Submit Your Answer & Move to Next Question

MC210201381: Sehrish Sarfraz

Quiz Start Time: 12:54 PM, 14 June 2022

MTH621:Quiz-1

Total Marks: 1

Question # 7 of 10 (Start time: 12:57:13 PM, 14 June 2022)

A sequence $\{s_n\}$ of real numbers is called a Cauchy sequence if for every $\varepsilon > 0$, there is an integer N such that $|s_n - s_m| < \varepsilon$ if $m, n \geq N$.

Select the correct option

Reload Math Equations

- $>$
- $\geq$
- $<$
- $\leq$

Click Next to Submit & Move to Next Question

Question # 6 of 10 (Start time: 12:56:48 PM, 14 June 2022) Total Marks: 1

If $\{s_n\}$ is bounded above and does not diverge to $-\infty$, then there is a unique real number $\bar{s}$ such that
, if $\varepsilon > 0$, $s_n < \bar{s} + \varepsilon$ ------

Select the correct option

Reload Math Equations

- for large n
- for small n

Question # 5 of 10 (Start time: 12:56:20 PM, 14 June 2022) Total Marks: 1

If $f(x) = \sqrt{4 - x^2}$ and $g(x) = \sqrt{x - 1}$, then domain of the quotient function f/g is

Select the correct option

Reload Math Equations

- $[-3, 3]$.
- $[1, 2]$.
- $(1, 2]$.
- $[7, 8]$.

Question # 1 of 10 (Start time: 12:54:08 PM, 14 June 2022)

The given series $\sum_{n=1}^{\infty} \frac{1}{n}$ is ——————.

Select the correct option

○	convergent
○	divergent

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Question # 4 of 10 (Start time: 12:30:36 PM, 14 June 2022)

If $\sum a_n$ ——————, then $\lim_{n \to \infty} a_n = 0$

Select the correct option

- converges
- diverges

Total Marks: 1

Question # 1 of 10 (Start time: 12:29:01 pm, 14 June 2022)

A divergent infinite series that does not diverge to ---------- is said to be oscillate

Reload Math Equations

Select the correct option

- ○ ± 1
- ○ None of these
- ○ ± 2
- ○ $\pm \infty$

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Quiz Start Time: 10:51 AM, 14 June 2022

Question # 2 of 10 (Start time: 10:52:39 AM, 14 June 2022)

Total Marks: 1

$If\ \{x_n\}\ unbounded\ above,\ then\ \{x_n\}\ has\ a\ subsequence\ \{x_n\}\ such\ that\ \lim_{k\to\infty} x_{n_k} = ----$

Select the correct option

Reload Math Equations

- 1
- ∞
- $-\infty$
- 0

The sum of a convergent series is _____.

tion

different

unique

itart time: 10:57:20 AM, 14 June 2022)

A divergent infinite series that does not diverge to $\pm\infty$ *is said to be* _____.

ion

convergent

oscillate

None of these

bounded

$$If\ \{s_n\}\ is\ ------,\ then\ \lim_{n\to\infty} s_n = \sup\{s_n\}.$$

on

increasing

nonincreasing

nondecreasing



decreasing

MTH621:Quiz-1

Question # 7 of 10 (Start time: 10:56:04 AM, 14 June 2022)

The series $\sum_{n=0}^{\infty} r^n, \quad -1 < r < 1$, is

Select the correct option

- convergent and converges to $\frac{1}{1-r}$.
- divergent.
- convergent and converges to 5.
- convergent and converges to $\frac{1}{1+r^2}$.

MTH621:Quiz-1

Quiz Start Time: 10:51 AM, 14 June 2022

Question # 8 of 10 (Start time: 10:56:36 AM, 14 June 2022)

Total Marks: 1

$$If\ \{s_n\}\ is\ a\ sequence\ of\ real\ numbers,\ then \lim_{n\to\infty} s_n = s\ iff\ \lim_{n\to\infty} \sup s_n - - - - \lim_{n\to\infty} \inf s_n = s.$$

Select the correct option

Reload Math Equations

- $>$
- $=$
- $<$
- $None\ of\ these$

10:54

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MC220204041: IQRA ZAINAB

Time Left 87 sec(s)

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Quiz Start Time: 10:51 AM, 14 June 2022

Question # 5 of 10 (Start time: 10:54:23 AM, 14 June 2022)

Total Marks: 1

The given series $\sum_{n=1}^{\infty} \frac{1}{n}$ *is* $-----$.

Select the correct option

Reload Math Equations

- convergent
- divergent

Click to Save Answer & Move to Next Question