SPECIAL THEORY OF RELATIVITY.

INERTIAL FRAME.

A frame of reference which is at rest or moving with constant velocity and has no acceleration is called inertial frame. Newton's law hold in an inertial frame.

Frames which may move relative to one another but in which all observers find the same value for acceleration of a given moving particle are called inertial framesof reference.

NON-INERTIAL FRAME.

A frame of reference which is moving with changing velocity and possesses acceleration relative to an inertial frame is called non-inertial frame. Newton's laws do not hold in non-inertial frame.

SPECIAL THEORY OF RELATIVITY.

Theory of relativity which involves the reference frames moving with constant velocity relative to one another is called special theory of relativity.

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POSTULATES OF SPECIAL THEORY OF RELATIVITY.

(1). THE PRINCIPLE OF RELATIVITY.

The laws of physics are the same in all inertial reference frames. It means that laws of physics are absolute, universal and same for all inertial observers.

Laws that hold for one inertial observer can not be violated for any inertial observer.

(2). THE PRINCIPLE OF CONSTANCY OF SPEED OF LIGHT.

The speed of light in free space has same value C in all inertial frames. It means ultimate speed in nature is C. It is same in all directions and in all inertial reference frames.

Light and all electro-magnetic radiations and massless particles such as Neutrinos travel with speed $C = 3 \times 10^8 \, m s^{-1}$.

EXAMPLE:- Neutral pions π° were accelerated to speed of 0.99975 C. Every Pi-on deay into two γ-ray photons. It was seen that speed of every γ-ray photon is C and not 1.9997 C as may be expected by classical theory.

$$\pi^\circ \longrightarrow \gamma + \gamma$$

It proves second postulate.

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CONSEQUENCE:- The two postulates taken togather have another consequence. It is not possible to accelerate a particle to a speed greater than C, no matter how much K·E we give it.
When an electron is accelerated by applying 15×10^6 volts, it approaches speed of light C, but never atlains speed C or exceeds it, by what amount voltage is increase.
However its K·E increases due to increase in mass.

PROPER EVENT. Any event which is stationary to observer is called proper event.

CONSEQUENCES OF EINSTEIN'S POSTULATES.

THE RELATIVITY OF TIME OR TIME DILATION.

Observer S is at rest on ground and observer S′ is moving with speed V relative to S

FOR OBSERVER S′.



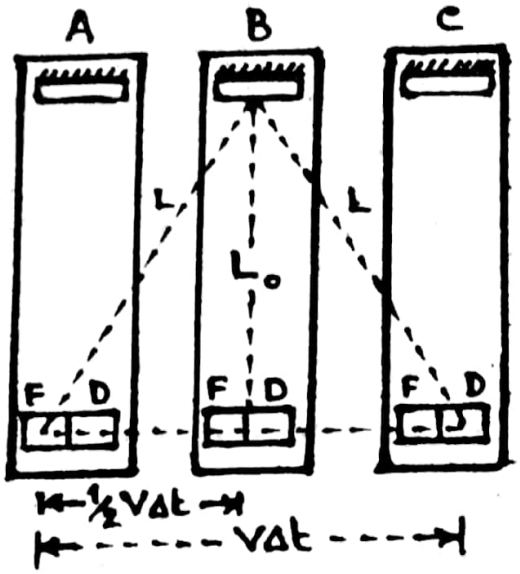
In his own frame, observer S′ measures the time Δt_0, for light flash to move from F to mirror M and back to detecter D.
If distance between F and M is L_0

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then, Time interval $\Delta t_o = \frac{2L_o}{c}$ ⟶ ①

Time interval Δt_o is proper time where as clock is at rest relative to S′.

FOR OBSERVER S,— Now consider the measurement of this time interval as measured by stationary observer S on the clock fixed in S′ which is moving with speed V relative to S. According to S, Light is emitted from flash F when S′ is in position A, this light flash reaches mirron M, when S′ is in position B and after reflection to detector D, when S is in position C. So, he sees the motion of light along diagonal path. If S measures this time interval Δt, then distance moved towards right by S′ or clock during this time Δt is $V\Delta t$. So from figure.



$$L = \sqrt{L_o^2 + \left(\frac{1}{2}V\Delta t\right)^2} \longrightarrow ②$$

But by equ. ①

$$L_o = \frac{c\Delta t_o}{2}$$

Put value of L_o in equ. ②

$$L = \sqrt{\left(\frac{1}{2}c\Delta t_o\right)^2 + \left(\frac{1}{2}V\Delta t\right)^2}$$

Time interval measured by S $= \Delta t = \frac{2L}{c} = \frac{2}{c}L$

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$$\Delta t = \frac{2}{c}\sqrt{\left(\frac{1}{2}c\Delta t_o\right)^2 + \left(\frac{1}{2}v\Delta t\right)^2}$$

$$\Delta t^2 = \frac{4}{c^2}\left[\frac{c^2\Delta t_o^2}{4} + \frac{v^2\Delta t^2}{4}\right] = \Delta t_o^2 + \frac{v^2}{c^2}\Delta t^2$$

$$\Delta t^2 - \frac{v^2}{c^2}\Delta t^2 = \Delta t_o^2 \implies \left(1 - \frac{v^2}{c^2}\right)\Delta t^2 = \Delta t_o^2$$

$$\boxed{\Delta t = \frac{\Delta t_o}{\sqrt{1 - v^2/c^2}}} \longrightarrow (3)$$

As $\sqrt{1 - v^2/c^2} \leq 1$, so $\Delta t > \Delta t_o$

DEF. OF TIME DILATION. If a clock is in motion relative to an observer then he measures greater time interval of an event than the time interval measured of this event by an observer relative to which clock is at rest.

ORDINARY SPEED.

At ordinary speed $v << c$, $v^2/c^2 \simeq 0$ so equ. (3) gives

$$\Delta t = \frac{\Delta t_o}{\sqrt{1-0}} = \Delta t_o$$

Equ. (3) is valid for any direction of the relative motion of S and S′. It is valid for any type of clock used. Biological clocks such as human aging are expected to be affected by time dilation.

RELATIVITY OF LENGTH OR LENGTH CONTRACTION.

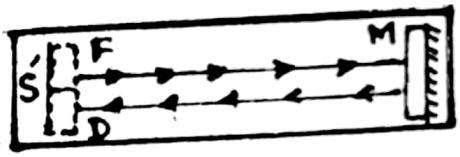
DEFINITION.— The length of an object

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appears to be shortend or contracted to an observer relative to which the body moves than to an observer relative to which object is rest or who moves along with object. It is called length contraction. Length contraction is noted or felt only along the direction of motion. Length measurments transverse to the direction of motion are unaffected.

EXPLANATION. Observer S′ is moving with constant velocity V to-wards right relative to stationary observer S.

OBSERVATIONS OF S′.



In his own frame S′ notes the flash of light to move from F to mirror M and back to detector D. He notes proper length L_o and proper time Δt_o, for round trip motion. Thus

$$\Delta t_o = 2L_o/c \longrightarrow (1)$$

OBSERVATIONS OF S. Flash of light is emitted at position A, and reaches the mirror M (position B) in time Δt_1. In this time flash of light moves distance $c\Delta t_1$ This distance is sum of two distances or lengths.

1). Distance or length L between F and M as measured by observer S.

2). Distance or length of path $V\Delta t_1$

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moved by S' relative to S. So

$$C\Delta t_1 = L + V\Delta t_1$$

$$(C - V)\Delta t_1 = L$$

$$\Delta t_1 = \frac{L}{C-V}$$

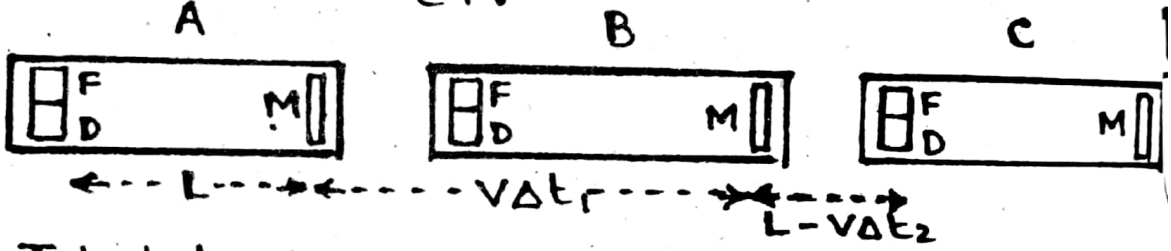
During return trip, flash takes time Δt_2 to move from mirror M to detector D as measured by (stationary observer) S'. Light travels distance $C\Delta t_2$ from M toward F and F or D itself travels distance $V\Delta t_2$ toward M. So

$$C\Delta t_2 = L - V\Delta t_2$$

$$C\Delta t_2 + V\Delta t_2 = L$$

$$(C + V)\Delta t_2 = L$$

$$\Delta t_2 = \frac{L}{C+V}$$



Total time measured by S = $\Delta t = \Delta t_1 + \Delta t_2$

$$\Delta t = \frac{L}{C-V} + \frac{L}{C+V} = L\left[\frac{C+V+C-V}{C^2-V^2}\right]$$

$$\Delta t = \frac{2LC}{C^2\left[1 - \frac{V^2}{C^2}\right]}$$

$$\Delta t = \frac{2L}{C}\,\frac{1}{\left[1 - V^2/C^2\right]} \longrightarrow (2)$$

Also $$\Delta t = \frac{\Delta t_0}{\sqrt{1 - V^2/C^2}} \longrightarrow (3)$$

Put value of $\Delta t_0 = \frac{2L_0}{C}$ from equ. (1) in equ. (3)

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$$\Delta t = \frac{2L_o}{C} \frac{1}{\sqrt{1 - V^2/C^2}}$$

Put value of Δt from equ. ②

$$\frac{2L}{C} \frac{1}{(1 - V^2/C^2)} = \frac{2L_o}{C} \frac{1}{\sqrt{1 - V^2/C^2}}$$

$$\boxed{L = L_o \sqrt{1 - V^2/C^2}}$$

ORDINARY SPEED.

At ordinary speed $V << C$ so $\frac{V^2}{C^2} \simeq 0$

Now $L = L_o \sqrt{1 - 0} = L_o$

So, at ordinary speed of objects, their Lengthsreal and appearent are same.

THE LORENTZ TRANSFORMATION.

An inertial frame S' is moving with speed V relative to frame S, in positive x-direction. An observer in S reports space time co-ordinates x, y, z, t for an event and an observer in S' reports x', y', z', t' for the same event y and z co-ordinates, which are perpendicular to the motion, are not affected by the motion.

So, $y = y'$ and $z = z'$

GALILEAN TRANSFORMATION EQUATIONS.

At very low speed i.e $V << C$, Galilean transformation equations are valid i.e

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Lecture # 39

(4) Inertial Frames. (Where there is no motion)
It is that frame where there is no acceleration.
Non-inertial frame: (acceleration)

(5) $$x' = x - vt$$

Differentiate b.s $\frac{dx'}{dt} = \frac{dx}{dt} - \frac{v\,dt}{dt}$

$$\frac{x'}{dt} = \frac{x}{dt} - v \longrightarrow (1)$$

As time is constant so, $t = t'$

Then $$\frac{x'}{dt} = \frac{x'}{dt'}$$

So eq. (1) becomes as.

$$\frac{x'}{dt'} = \frac{dx}{dt} - v \longrightarrow (2)$$

Similarly,

$$\frac{dy'}{dt'} = \frac{dy}{dt} \quad \& \quad \frac{dz'}{dt'} = \frac{dz}{dt}$$

Now

$$\frac{dx'}{dt} = u'_x \quad \& \quad \frac{dx}{dt} = u_x$$

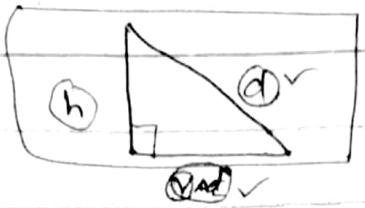
So, eq. (2) becomes as

$$u'_x = u_x - v, \quad u'_y = u_y$$

and $$u'_z = u_z.$$

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(10)



$d^2 = \sqrt{(h)^2 + (v\Delta t)^2}$

$d = \sqrt{h^2 + (v\Delta t)^2}$ ✓

$\Delta t = d/v \longrightarrow (1)$

$\Delta t = \dfrac{\sqrt{h^2 + (v\Delta t)^2}}{c}$ ✓

Now T.S.O.B.S.

$(\Delta t)^2 = \dfrac{\left(\sqrt{h^2 + (v\Delta t)^2}\right)^2}{(c)^2}$

$(c\Delta t)^2 = h^2 + (v\Delta t)^2$

$(c\Delta t)^2 - (v\Delta t)^2 = h^2$ ✓

$\Delta t^2 (c^2 - v^2) = h^2$

$\sqrt{\Delta t^2} = \sqrt{\dfrac{h^2}{c^2 - v^2}}$

$\Delta t = \dfrac{h}{\sqrt{c^2 - v^2}} \times \dfrac{c}{c}$

$= \dfrac{h}{\sqrt{c^2 - v^2}} \cdot \dfrac{1}{c}$

$= \dfrac{h}{c} \cdot \dfrac{1}{\sqrt{\dfrac{c^2 - v^2}{c^2}}}$

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$$\Delta t = \frac{h}{c} \cdot \frac{1}{\sqrt{\frac{c^2}{c^2} - \frac{v^2}{c^2}}}$$

Δl

$$= \frac{h}{c} \cdot \boxed{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} \Rightarrow (\gamma)$$

$$= \frac{h}{c} \gamma \rightarrow \Delta t'$$

$$\boxed{\Delta t = \Delta t' \gamma}$$

(12)

$$\Delta t_1 = \frac{\Delta x + v\Delta t_1}{c} \rightarrow (1)$$

$$c\Delta t_1 = \Delta x + v\Delta t_1$$

$$c\Delta t_1 - v\Delta t_1 = \Delta x$$

$$\Delta t_1 (c - v) = \Delta x$$

$$\Delta t_1 = \frac{\Delta x}{c - v} \checkmark$$

$$\Delta t_2 = \frac{\Delta x - v\Delta t_2}{c} \rightarrow (2)$$

Solve (2) as above and we get

$$\boxed{\Delta t_2 = \frac{\Delta x}{c + v}}$$

Total Time $= \Delta t_1 + \Delta t_2 = \Delta t$

$$\Delta t = \frac{\Delta x}{c - v} + \frac{\Delta x}{c + v} = \frac{\Delta x(c + v) + \Delta x(c - v)}{(c - v)(c + v)}$$

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$$= \frac{\Delta x\left[(C+V)+(C-V)\right]}{C^2-V^2}$$

$$\Delta t = \frac{\Delta x(C+V+C-V)}{C^2-V^2} = \frac{\Delta x\, 2C}{C^2-V^2}$$

$$\Delta t = \frac{2\Delta x C}{\frac{C^2}{C^2}\cdot\frac{V^2}{C^2}} = \frac{2\Delta x C}{C^2\left(1-\frac{V^2}{C^2}\right)}$$

$$\Delta t = \frac{2\Delta x}{C}\cdot\frac{1}{\left(1-\frac{V^2}{C^2}\right)}$$

$$\Delta t = \frac{2\Delta x}{C}\cdot\frac{1}{\left(1-\frac{V^2}{C^2}\right)}$$

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