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	MIDTERM EXAMINATION SEMESTER FALL 2004 MTH401- Differential Equations	Total Marks: 50 Duration: 60min

Instructions

1. Attempt all questions.
2. The Time allowed for this paper is 60 minutes.
3. This examination is closed book, closed notes, closed neighbors; any one found cheating will get zero grades in the course MTH401 Differential Equations.
4. You are not allowed to use any type of Table for Formulae of Differentiation and integration during your exam.
5. Each objective type question carries 2 marks and each Descriptive question carries 10 marks. So write every step of the solution of descriptive question to get maximum marks.
6. Do not ask any questions about the contents of this examination from anyone. If you think that there is something wrong with any of the questions, attempt it to the best of your understanding.

Question No: 1 **Marks: 2**

The differential equation $\frac{dy}{dx} - y = e^x y^2$ is not Bernoulli equation.

- T
- **F**

Question No: 2 **Marks: 2**

$f(x,y) = \frac{x}{x^2 + y^2}$ Is homogeneous.

- T
- **F**

Question No: 3 **Marks: 2**

Population dynamics are not practical application of the first order differential equations.

- T
- **F**

Question No: 4 **Marks: 2**

A set

$$\{y_1, y_2, ..., y_n\}$$

Of *n* linearly dependent solutions, on interval I, of the homogeneous linear *n*th-order differential equation

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = 0$$

Is said to be a fundamental set of solutions on the interval I.

- T
- **F**

Question No: 5 **Marks: 2**

The differential operator that annihilates $10x^3 - 2x$ is D^4.

- **T**
- F

Question No: 6 **Marks:10**

(a) Define separable form. Just separate the variables of the given differential equation.

$$(3r\theta - 3\theta + r - 1)\,dr - (2r\theta + 4\theta - r - 2)\,d\theta = 0$$

Solution

The differential equation of the form d y / d x = f (x, y) is called **separable** if it can be written in the form

d y / d x = h(x)g(y)

$$(3r\theta - 3\theta + r - 1)\,dr - (2r\theta + 4\theta - r - 2)\,d\theta = 0$$

$$(3r\theta - 3\theta + r - 1)\,dr = (2r\theta + 4\theta - r - 2)\,d\theta$$

$$[3\theta(r-1) + 1(r-1)]\,dr = [2\theta(r+2) - 1(r+2)]\,d\theta$$

$$(r-1)(3\theta+1)\,dr = (r+2)(2\theta-1)\,d\theta$$

$$\frac{(r-1)}{(r+2)}\,dr = \frac{(2\theta-1)}{(3\theta+1)}\,d\theta$$

(b) Check whether the given differential equation is exact or not if not then make it exact also show that it is exact **(Just make the equation exact do not solve it further).**

$$\left(\frac{3y^2 - x^2}{y^5}\right)\frac{dy}{dx} + \frac{x}{2y^5} = 0$$

Solution

It can also be written as

$$xdx + 2\left(3y^2 - x^2\right)dy = 0$$

$$M\left(x,y\right) = x, \quad N\left(x,y\right) = 6y^2 - 2x^2$$

$$M_y = 0, \quad N_x = -4x$$

$$M_y \neq N_x$$

Thus it is not exact now we apply techniques to make it exact

$$xdx + \left(6y^2 - 2x^2\right)dy = 0$$

$$\frac{N_x - M_y}{M} = \frac{-4x}{x} = -4 = g(y)$$

$$I.F = e^{\int -4dy} = e^{-4y}$$

$$xe^{-4y}dx + e^{-4y}\left(3y^2 - 2x^2\right)dy = 0$$

$$M\left(x,y\right) = xe^{-4y}, \quad N\left(x,y\right) = e^{-4y}\left(3y^2 - 2x^2\right)$$

$$M_y = -4xe^{-4y}, N_x = -4xe^{-4y}$$

$$M_y = N_x$$

Which shows that equation is exact

Question No: 7 **Marks:10**

(a) Solve the Bernoulli equation $x^3\frac{dy}{dx} + 2xy = y^5$ (**Just make the given equation linear in v, do not integrate**)

Solution

$$x^3\frac{dy}{dx} + 2xy = y^5$$

$$\frac{dy}{dx}y^{-5} + \frac{2}{x^2}y^{-4} = \frac{1}{x^3}$$

$$put\ y^{-4} = v$$

$$-4\frac{dy}{dx}y^{-5} = \frac{dv}{dx}$$

$$\frac{dy}{dx}y^{-5} = -\frac{1}{4}\frac{dv}{dx}$$

Then

$$-\frac{1}{4}\frac{dv}{dx} + \frac{2v}{x^2} = \frac{1}{x^3}$$

$$\frac{dv}{dx} - \frac{8v}{x^2} = \frac{-4}{x^3}$$

Thus it is linear in "v".

(b) Initially there were 200 milligrams of a radioactive substance present. After 8 hours the mass increased by 4%. If the rate of decay is proportional to the amount of the substance present at any time, determine half-life of the radioactive substance? **(Just make the model of the radioactive decay as well as describe the given conditions do not solve further)**

Solution

Suppose that A_0 is the initial amount, as given A_0 is 200 and A(t) be the amount present at time t then its governed by the differential equation

$$\frac{dA}{dt} \alpha A$$

$$\frac{dA}{dt} = kA$$

$$\frac{dA}{A} = kdt$$

Integrate both sides

$$\ln A = kt + c$$

$$A = e^{kt+c}$$

$$A = e^{kt}e^{c}$$

$$A = A_0 e^{kt} \qquad say\ A_0 = e^{c}$$

Where A_0 is the initial amount

A_0=200 = A(0)

$$A(8)=200+\frac{4}{100}(200)$$

A(8)=208

A (T) =100

Question No: 8 **Marks:10**

(a) Find a second solution of following differential equations where the first solution is given **(also write the formulae)**. $(1+2x)y''+2xy'-4=0 \quad , y_1 = e^{-2x}$

Solution

$(1+2x)y''+2xy'-4=0 \quad , y_1 = e^{-2x}$

differential equation can be written as

$$y''+\frac{2x}{1+2x}y'-\frac{4}{1+2x}=0$$

the 2nd solution is given by

$$y_2 = y_1 \int \frac{e^{-\int p dx}}{y_1^{\,2}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{-\int \frac{2x}{1+2x} dx}}{\left(e^{-2x}\right)^2} dx$$

$$y_2 = e^{-2x} \int \frac{e^{-\int \frac{1+2x-1}{1+2x} dx}}{e^{-4x}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{\int (\frac{1}{1+2x} - 1) dx}}{e^{-4x}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{-x} e^{\ln(1+2x)}}{e^{-4x}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{2\ln(1+2x)}}{e^{-3x}} dx$$

$$y_2 = e^{-2x} \int \frac{e^{\ln(1+2x)}}{e^{-3x}} dx$$

$$y_2 = e^{-2x} \int (1+2x) e^{3x} dx$$

$$y_2 = e^{-2x} \left[(1+2x) \frac{e^{3x}}{3} - \int (1+2x)' \frac{e^{3x}}{3} dx \right]$$

$$y_2 = e^{-2x} \left[(1+2x) \frac{e^{3x}}{3} - \int \frac{2e^{3x}}{3} dx \right]$$

$$y_2 = (1+2x) \frac{e^{x}}{3} - \frac{2e^{x}}{9}$$

(b) Solve the differential equation by the undetermined coefficient **(superposition approach)**

$$y'' - 2y' - 3y = e^{4x}$$

If complimentary solution is given below

$$y_c = c_1 e^{3x} + c_2 e^{-x}$$

Then just find particular solution.

Solution

We find a particular solution of non-homogeneous differential equation.

Suppose input function

$y_p = Ae^{4x}$

Then

$y'_p = 4Ae^{4x}$

$y''_p = 16Ae^{4x}$

Substituting in the given differential equation

$16Ae^{4x} - 2\left(4Ae^{4x}\right) - 3Ae^{4x} = e^{4x}$

$e^{4x}\left(16A - 8A - 3A\right) = e^{4x}$

From the resulting equations

$5A = 1$

$A = \frac{1}{5}$

$y_p = \frac{1}{5}e^{4x}$

Question No: 9 **Marks: 10**

(a) Solve differential equation by the undetermined coefficient **(annihilator operator).**

$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} = x\cos x$

If complimentary solution is given below

$y_c = c_1 + c_2e^{4x}$

Then just find general solution.

Solution

In this case of input function is

g(x)= $x\cos x$

further

$(D^2+1)^2(g(x)) = (D^2+1)^2(x\cos x) = 0$

Therefore the differential operator $(D^2+1)^2$ annihilates the function g. operating on both sides

$(D^2+1)^2(D^2-4D)y = (D^2+1)^2(x\cos x)$

$(D^2+1)^2(D^2-4D)y = 0$

This is the homogeneous equation of order 6. Next we solve this higher order equation.

Thus auxiliary equation is

$(m^2+1)^2(m^2-4m)y=0$

$m(m-4)(m^2+1)^2=0$

$m=0,4,i,i,-i,-i$

Thus its general solution of the differential equation must be

$y=c_1+c_2e^{4x}+(c_3+c_4x)\cos x+(c_5+c_6x)\sin x$

(b) Solve the differential equation by the variation of parameters

$y"-y=x^2$

Complimentary solution is given below If

$y_c=c_1e^x+c_2e^{-x}$

Then just find particular solution **(do not integrate).**

Solution

$y"-y=x^2$

This equation is already in standard form

y''+ P(x)y' +Q(x)y = f(x)

Therefore, we identify the function f(x) as

f(x) = x^2

We construct the determinants

Since $y_1=e^x$, $y_2=e^{-x}$ so

$$W(y_1,y_2)=\begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix}=-e^{x-x}-e^{x-x}=-2$$

$$W_1=\begin{vmatrix} 0 & e^{-x} \\ x^2 & -e^{-x} \end{vmatrix}=-x^2e^{-x}$$

$$W_2=\begin{vmatrix} e^x & 0 \\ e^x & x^2 \end{vmatrix}=x^2e^x$$

We determine the derivatives of the function u_1 and u_2

$$u'_1=\frac{W_1}{W}=\frac{-x^2e^{-x}}{2}\rightarrow u_1=\int\frac{x^2}{2}e^{-x}dx$$

$$u'_2=\frac{W_2}{W}=\frac{x^2e^{-x}}{2}\rightarrow u_2=-\int\frac{x^2}{2}e^{-x}dx$$

$y_p=u_1e^x+u_2e^{-x}$

is required particular solution