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MTH # 645 MID

"SOLVED PAPER"

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Q.NO;01 Show $x \Delta_4 x$?.

Sol: As we know that

$$x \Delta_4 y = \frac{xy}{x+y-xy} \quad -(1)$$

By using Eq (1) we write

$$x \Delta_4 x = \frac{xx}{x+x-xx}$$

$$= \frac{x^2}{2x-x^2}$$

$$= \frac{x^2}{x(2-x)}$$

$$x \Delta_4 x = \frac{x}{2-x} \quad \text{Ans.}$$

Q.NO;02 $A = \{10, 15, 20\}$ Find A^2 and show that Graphically ?.

Sol: p.t.o

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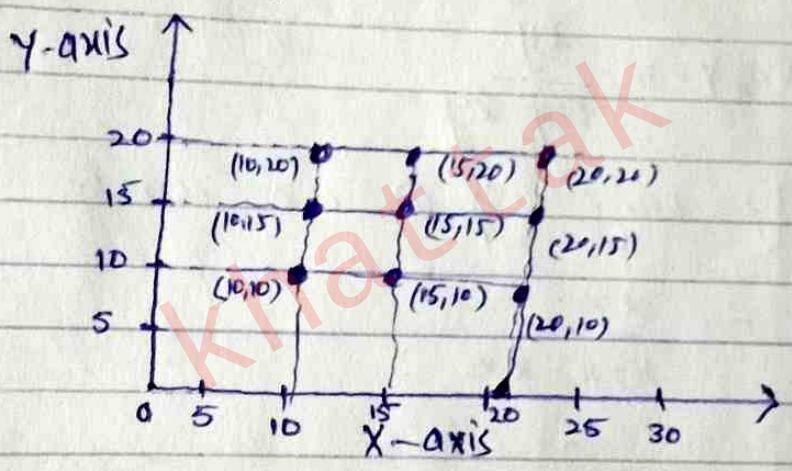
Since Given that $A = \{10, 15, 20\}$

Plan

$A^2 = A \times A = \{10, 15, 20\} \times \{10, 15, 20\}$

$A^2 = \{(10,10), (10,15), (10,20), (15,10), (15,15), (15,20), (20,10), (20,15), (20,20)\}$.

Graphically shown



QNO;03 write the relation for R?

Sol:

	x_1	x_2	x_3
a_1	0	1	0
a_2	1	0	0
a_3	0	1	0
a_4	0	0	1

Sol:

$R = \{(a_1, x_2), (a_2, x_1), (a_3, x_2), (a_4, x_3)\}$

Q NO; 04 Define the intersection of two interval.

Sol: Let Suppose I_1, I_2 be the two interval such that $I_1 = [a_1, b_1]$ and $I_2 = [a_2, b_2]$ than intersection of these two interval can be define As

$$I_1 \cap I_2 = [a_1, b_1] \cap [a_2, b_2]$$

$$= [\max\{a_1, a_2\}, \min\{b_1, b_2\}]$$

and $I_1 \cap I_2 = \phi$ if $a_2 > b_1$ and $a_1 > b_2$

Q NO; 05 Prove that $(\tilde{A} \cap \tilde{A}')(x) \leq 0.5$

Sol:

Proof: As we know that

$$\tilde{A}'(x) = 1 - \tilde{A}(x)$$

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$(\tilde{A}(x)$

now $(\tilde{A} \cap \tilde{A}')(x) = \min(\tilde{A}(x), \tilde{A}'(x))$

$= \min(\tilde{A}(x), 1 - \tilde{A}(x))$.

Here we discuse two cases.

Case 1:-

$\tilde{A}(x) \leq 0.5$

$\Rightarrow 1 - \tilde{A}(x) \geq 0.5$

$\Rightarrow \tilde{A}'(x) \geq 0.5$

But here is intersection we always take minimum

$\Rightarrow (\tilde{A} \cap \tilde{A}')(x) = \min(\tilde{A}(x), \tilde{A}'(x)) \leq 0.5$

$(\tilde{A} \cap \tilde{A}')(x) \leq 0.5$ — (1)

Case 2:- when $\tilde{A}(x) \geq 0.5$

$\Rightarrow 1 - \tilde{A}(x) \leq 0.5$

$\tilde{A}'(x) \leq 0.5$

Same satuation Here we take min.

$\Rightarrow (\tilde{A} \cap \tilde{A}')x = \min(\tilde{A}(x), \tilde{A}'(x)) \leq 0.5$

$(\tilde{A} \cap \tilde{A}')(x) \leq 0.5$ — (2)

Thus from (1) & (2) it clear that $\tilde{A} \cap \tilde{A}' \leq 0.5$ always.

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Q.NO:06 Define empty fuzzy set?

Sol: A fuzzy set is said to be an empty fuzzy set if all the element maps on zero "0" and membership value of all the element is zero.

Example: let $A = \{y_1, y_2, y_3\}$.

$$\mu_A(x) = \{(y_1, 0), (y_2, 0), (y_3, 0)\}.$$

Q.NO:07

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$= x + a$; $x \in X, a \in (Y+Z)$

$=$

Q no 10; 07 Prove that $x + (y+z) = (x+y) + z$

Proof; L.H.S

$x + (y+z) = x + a$; $x \in X, a \in (y+z)$

$= x + y + z$; $x \in X, y \in Y, z \in Z$

$= (x+y) + z$

$= b + z$; $b \in (x+y), z \in Z$

$= (x+y) + z$

$=$ R.H.S

Proved.

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Q NO; 08 FIND $X \Delta_1 X$: ?

Sol. As we know that

$$X \Delta_1 Y = 0 \vee (x+y-1) \quad -(1)$$

By using Eq (1) we write

$$X \Delta_1 X = 0 \vee (x+x-1)$$

$$= 0 \vee (2x-1)$$

$$X \Delta_1 X = 2x-1$$

Q NO; 09 Define Union of two fuzzy set.

Sol:- Let $\tilde{A}$ and $\tilde{B}$ be the two fuzzy set than their union can be define as

$$\mu_{\tilde{A} \cup \tilde{B}} = \mu_{\tilde{C}} = \max \{ \mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x) \}$$

$$= \{ \mu_A(x) \vee \mu_B(x) \} \quad \forall x \in X$$

Where $\tilde{C} = \tilde{A} \cup \tilde{B}$

Q NO; 10 Define degree of subsethood?

Sol:- degree of subsethood of two fuzzy set $\tilde{A}$ and $\tilde{B}$ can be define as

$$S(\tilde{A}, \tilde{B}) = \frac{|\tilde{A} \cap \tilde{B}|}{|\tilde{A}|}$$

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Q.NO;11 Define Normal fuzzy set with example ?

Sol:- A fuzzy set is said to be normal fuzzy set if there exist at least one element whose membership value is one

Example :- $X = \{x_1, x_2, x_3\}$.

$\tilde{A} = \{(x_1, 0.8), (x_2, 0), (x_3, 1)\}$

$\tilde{A}$ is normal fuzzy set.

Q.NO;12 Make the membership table for the fuzzy relation.?

$R = \{((a,c), 0.7), ((b,a), 0.9), ((c,b), 1), ((c,d), 0.2)\}$

Sol

The membership value of the given relation can be written as in matric form.

	a	b	c	d
a	0	0	0.7	0
b	0.9	0	0	0
c	0	1	0	0.2
d	0	0	0	0

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Q NO; 13 Define Support of a fuzzy set with Example. ?

Sol: Support of a fuzzy set define as the set whose all the element have membership value greater than "zero"

Support $\tilde{A} = \{ x : x \in X \text{ and } \tilde{A}(x) > 0 \}$.

Example $X = \{ x_1, x_2, x_3, x_4 \}$

$\tilde{A} = \{ (x_1, 10), (x_2, 0.1), (x_3, 0.5), (x_4, 1) \}$.

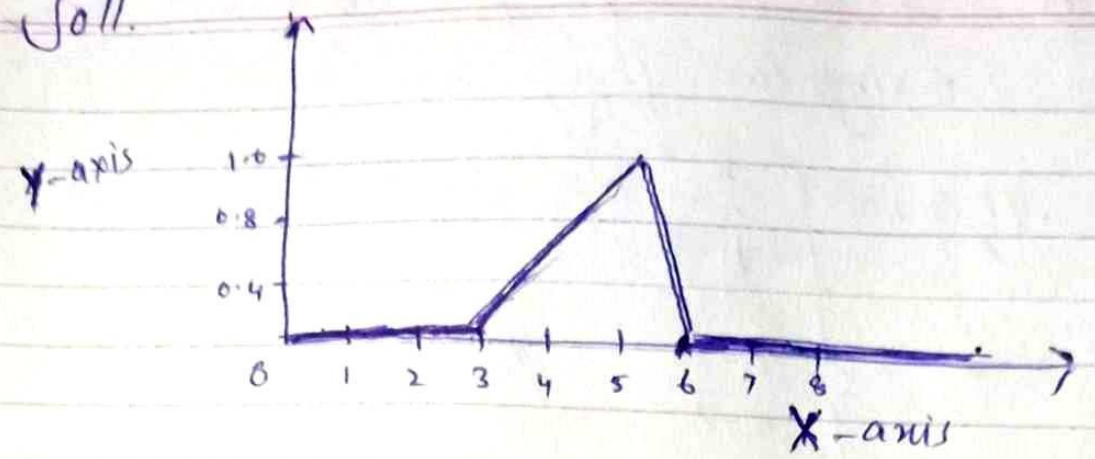
Support $(\tilde{A}) = \{ x_2, x_3, x_4 \}$

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Q NO; 14 Draw the graph of ~~trapezoidal~~ traingular fuzzy no whose membership function as.

$$\mu_A(x) \begin{cases} 0 & ; x < 3 \\ \frac{x-3}{5-3} & ; 3 \le x \le 5 \\ \frac{6-x}{6-5} & ; 5 \le x \le 6 \\ 0 & ; \text{otherwise} \end{cases}$$

Sol.



Q No; 15 If Δ_2 is T-norm and 0.5, 0.4 are two fuzzy number find the value of $0.5 \Delta_2 0.4$

Sol: As we know that Δ_2 T-norm can be defin as.

$$x \Delta_2 y = \frac{xy}{2-(x+y-xy)} \quad \text{—(1)}$$

Here $x = 0.5$, $y = 0.4$

by using Eq (1) we write

$$0.5 \Delta_2 0.4 = \frac{(0.5)(0.4)}{2-(0.5+0.4-(0.5)(0.4))}$$

$$= \frac{0.2}{2-(0.9-0.2)}$$

$$= \frac{0.2}{2-0.7} = \frac{0.2}{1.3}$$

$$0.5 \Delta_2 0.4 = 0.16$$

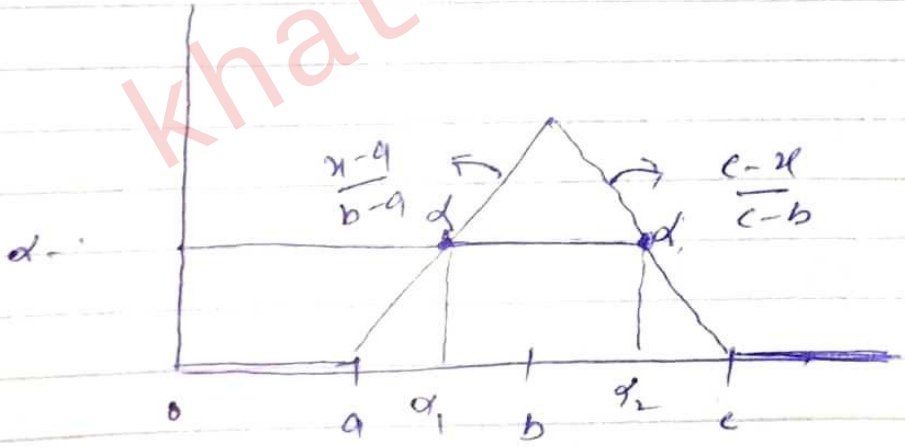
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Q No. 16 membership function of traingular fuzzy no $\tilde{A}$

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a} & a \le x \le b \\ \dfrac{c-x}{c-b} & b \le x \le c \\ 0 & \text{otherwise} \end{cases}$$

α-cut of fuzzy number is written as $A_\alpha = (a_1, a_2)$ find a_1, a_2 ?

Sol



This is the graph of traingular fuzzy no $\tilde{A}$.

let $\dfrac{x-a}{b-a} = \alpha$

$\alpha(b-a) = x-a$

$a + \alpha(b-a) = x = a_1$

$\Rightarrow a_1 = a + \alpha(b-a)$

$\dfrac{c-x}{c-b} = \alpha$

$c-x = \alpha(c-b)$

$c - \alpha(c-b) = x = a_2$

$a_2 = c - \alpha(c-b)$

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Q.NO; 17 Define Union of TWO fuzzy set.

Sol: Let $(\tilde{F}, \subseteq)$ be the collection of fuzzy set and $\tilde{A}, \tilde{B}$ belong to $\tilde{F}$ than $\tilde{A} \cup \tilde{B}$ can be define as.

$$\tilde{A} \cup \tilde{B} = \mu_{\tilde{A}}(x) \vee \mu_{\tilde{B}}(x)$$

$$= \max\left(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right).$$

Q.NO; 18 Define intersection of two fuzzy set.

Sol: Let $\tilde{F}$ be the collection of fuzzy set in "X" and $\tilde{A}, \tilde{B}$ be the subset of $\tilde{F}$ than their intersection $\tilde{A} \cap \tilde{B}$ can be define As.

$$\tilde{A} \cap \tilde{B} = \mu_{\tilde{A}}(x) \wedge \mu_{\tilde{B}}(x)$$

$$= \min\left(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right).$$

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QNO;19 $\tilde{A} = \{(x_1, 0.1), (x_2, 0.9), (x_3, 1), (x_4, 0), (x_5, 0.3), (x_6, 0.7)\}$

$\tilde{B} = \{(x_1, 0.3), (x_2, 0.8), (x_3, 0), (x_4, 1), (x_5, 0.1), (x_6, 0.9)\}$

Find the degree of subsethood?

Soln

As we know that

$$S(\tilde{A}, \tilde{B}) = \frac{|\tilde{A} \cap B|}{|\tilde{A}|} \quad -(1)$$

$\tilde{A} \cap \tilde{B} = \{(x_1, 0.1), (x_2, 0.8), (x_3, 0), (x_4, 0), (x_5, 0.1), (x_6, 0.7)\}$

$|\tilde{A} \cap \tilde{B}| = 0.1 + 0.8 + 0.1 + 0.7$

$|\tilde{A} \cap \tilde{B}| = 0.17$

$|\tilde{A}| = 0.1 + 0.9 + 1 + 0.3 + 0.7$

$|\tilde{A}| = 1.20$ put in (1)

$$S(\tilde{A}, \tilde{B}) = \frac{0.17}{1.20}$$

$S(\tilde{A}, \tilde{B}) = 0.14$

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Q.

Q.No: 20

M_R	a	b	c
1	0.1	0.7	0
2	0	1	0.9
3	0.5	0	0.2

S_R	a	b	c
1	1	0.8	0.1
2	0.3	0	0.5
3	0.9	1	0

Find MMRUS.

Sol:

MMRUS	a	b	c
1	1	0.8	0.1
2	0.3	1	0.9
3	0.9	1	0.2

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