Assignment#1

MTH622 (Fall 2021)

Like & Subscribe My YouTube Channel

Math City Layyah

With

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Total marks: 20

Modules#1-36

Due date: December 16, 2021

DON'T MISS these important instructions:

- Upload assignments properly through LMS.
- All students are directed to use the font and style of text as is used in this document.
- This is an individual assignment, not group assignment, so keep in mind that you are supposed to submit your own and self-made assignment even if you discuss the questions with your class fellows. All similar assignments (even with some meaningless modifications) will be awarded zero marks and no excuse will be accepted. This is your responsibility to keep your assignment safe from others.
- Solve the assignment on MS word document.

Question#1

Find the directional derivative of $\varphi = xy + yz + zx^2$ at (1,-1,1) in the direction of $\hat{i} + 2\hat{j} + 3\hat{k}$. MARKS 10

Solution

Given $\varphi = xy + yz + zx^2$

At (1,-1, 1) , direction $(\hat{i} + 2\hat{j} + 3\hat{k})$

Directional derivative= $\nabla\varphi.\hat{a}$

$$\nabla\varphi = (\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k})(xy + yz + zx^2)$$

$$\nabla\varphi = (y + 2xz)\hat{i} + (x + z)\hat{j} + (y + x^2)\hat{k}$$

$\nabla\varphi$ at (1,-1,1)

$$\nabla\varphi(1,-1,1) = (-1 + 2(1)(1))\hat{i} + (1+1)\hat{j} + (-1 + (1)^2)\hat{k}$$

$$\nabla\varphi(1,-1,1) = (-1+2)\hat{i} + (2)\hat{j} + (0)\hat{k}$$

$$\nabla\varphi(1,-1,1) = \hat{i} + 2\hat{j} + 0\hat{k}$$

The unit vector in the given direction can be calculated as:

$$\hat{a} = \frac{\vec{a}}{|a|}$$

$$\hat{a} = \frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{1+4+9}}$$

$$\hat{a} = \frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}}$$

$$\hat{a} = \frac{\hat{i}}{\sqrt{14}} + \frac{2\hat{j}}{\sqrt{14}} + \frac{3\hat{k}}{\sqrt{14}}$$

The required directional unit vector is

$$\nabla\varphi.\hat{a} = (\hat{i} + 2\hat{j} + 0\hat{k})(\frac{\hat{i}}{\sqrt{14}} + \frac{2\hat{j}}{\sqrt{14}} + \frac{3\hat{k}}{\sqrt{14}})$$

$$\nabla\varphi.\hat{a} = \frac{1}{\sqrt{14}} + \frac{4}{\sqrt{14}} + \frac{0}{\sqrt{14}}$$

$$\nabla\varphi.\hat{a} = \frac{1+4}{\sqrt{14}}$$

$$\nabla\varphi.\hat{a} = \frac{5}{\sqrt{14}}$$

Since this is positive, φ is increasing in this direction

Question#2

Determine the constant 'a' so that the vector

$\vec{V} = (2x+5y)\hat{i} + (5x-4y)\hat{j} + (ax+4z)\hat{k}$ is solenoidal.

MARKS 10

Solution

$$\vec{V} = (2x+5y)\hat{i} + (5x-4y)\hat{j} + (ax+4z)\hat{k}$$

$$\nabla.\vec{V} = (\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k})(2x+5y)\hat{i} + (5x-4y)\hat{j} + (ax+4z)\hat{k}$$

$$\nabla.\vec{V} = \frac{\partial}{\partial x}(2x+5y) + \frac{\partial}{\partial y}(5x-4y) + \frac{\partial}{\partial z}(ax+4z)$$

$$\nabla.\vec{V} = 2 + (-4) + 4$$

$$\nabla.\vec{V} = 2 - 4 + 4$$

$$\nabla.\vec{V} = 2$$

Hence we substitute **a=2** is given vector field then $\vec{V}$ will become solenoidal.