PHYSICS 101

AN INTRODUCTION TO PHYSICS

This course of 45 video lectures, as well as accompanying notes, have been developed and presented by Dr. Pervez Amirali Hoodbhoy, professor of physics at Quaid-e-Azam University, Islamabad, for the Virtual University of Pakistan, Lahore.

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GENERAL INFORMATION

Purpose: This course aims at providing the student a good understanding of physics at the elementary level. Physics is essential for understanding the modern world, and is a definite part of its culture.

Background: It will be assumed that the student has taken physics and mathematics at the F.Sc level, i.e. the 12^{th} year of schooling. However, B.Sc students are also likely to find the course useful. Calculus is not assumed and some essential concepts will be developed as the course progresses. Algebra and trigonometry are essential. However, for physics, the more mathematics one knows the better.

Scope and Duration: The course has 45 lectures, each of somewhat less than one hour duration. All main fields of physics will be covered, together with several applications in each.

Language: For ease of communication, all lectures are in Urdu. However, English or Latin technical terms have been used where necessary. The student must remember that further study and research in science is possible only if he or she has an adequate grasp of English.

Textbook: There is no prescribed textbook. However, you are strongly recommended to read a book at the level of "College Physics" by Halliday and Resnick (any edition). There are many other such books too, such as "University Physics" by Young and Freedman. Study any book that you are comfortable with, preferably by a well-established foreign author. Avoid local authors because they usually copy. After listening to a lecture, go read the relevant chapter. Please remember that these notes cover only some things that you should know and are not meant to be complete.

Assignments: There will be total Eight Assignment in this course and its schedules will be announced from time to time. The book you choose to consult will have many more. Those students who are seriously interested in the subject are advised to work out several of the questions posed there. In physics you cannot hope to gain mastery of the subject without extensive problem solving.

Examinations: Their schedules will be announced from time to time.

Tutors: Their duty is to help you, and they will respond to all genuine questions. However, please do not overload them as they have to deal with a large number of students. Happy studying!

Acknowledgements: I thank the Virtual University team and administration for excellent cooperation, as well as Mansoor Noori and Naeem Shahid, for valuable help.

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Summary of Lecture 1 – INTRODUCTION TO PHYSICS

1. Physics is a science. Science works according to the *scientific method*. The *scientific method* accepts only reason, logic, and experimental evidence to tell between what is scientifically correct and what is not. Scientists do not simply believe – they test, and keep testing until satisfied. Just because some "big scientist" says something is right, that thing does not become a fact of science. Unless a discovery is repeatedly established in different laboratories at different times by different people, or the same theoretical result is derived by clear use of established rules, we do not accept it as a scientific discovery. The real strength of science lies in the fact that it continually keeps challenging itself.

2. It is thought that the laws of physics do not change from place to place. This is why experiments carried out in different countries by different scientists – of any religion or race – have always led to the same results if the experiments have been done honestly and correctly. We also think that the laws of physics today are the same as they were in the past. Evidence, contained in the light that left distant stars billions of years ago, strongly indicates that the laws operating at that time were no different than those today. The spectra of different elements then and now are impossible to tell apart, even though physicists have looked very carefully.

3. This course will cover the following broad categories:
   a) *Classical Mechanics*, which deals with the motion of bodies under the action of forces. This is often called Newtonian mechanics as well.
   b) *Electromagnetism*, whose objective is to study how charges behave under the influence of electric and magnetic fields as well as understand how charges can create these fields.
   c) *Thermal Physics*, in which one studies the nature of heat and the changes that the addition of heat brings about in matter.
   d) *Quantum Mechanics*, which primarily deals with the physics of small objects such as atoms, nuclei, quarks, etc. However, Quantum Mechanics will be treated only briefly for lack of time.

4. Every physical quantity can be expressed in terms of three fundamental dimensions: Mass (M), Length (L), Time (T). Some examples:

Speed	LT^{-1}
Acceleration	LT^{-2}
Force	MLT^{-2}
Energy	$ML^{2}T^{-2}$
Pressure	$ML^{-1}T^{-2}$

You cannot add quantities that have different dimensions. So force can be added to force, but force can never be added to energy, etc. A formula is definitely wrong if the dimensions on the left and right sides of the equal sign are different.

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5. Remember that any function $f(x)$ takes as input a dimensionless *number x* and outputs a quantity *f* (which may, or may not have a dimension). Take, for example, the function $f(\theta) = \sin\theta$. You know the expansion: $\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$ If θ had a dimension then you would be adding up quantities of different dimensions, and that is not allowed.

6. Do not confuse units and dimensions. We can use different units to measure the same physical quantity. So, for example, you can measure the mass in units of kilograms, pounds, or even in *sair* and *chatak!* In this course we shall always use the MKS or **M**etre-**K**ilogram-**S**econd system. When you want to convert from one system to another, be methodical as in the example below:

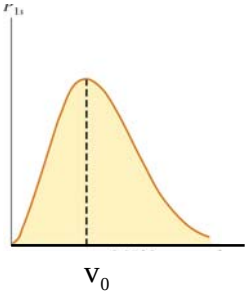
$$1\frac{mi}{hr} = 1\frac{mi}{hr} \times 5280\frac{ft}{mi} \times \frac{1}{3.28}\frac{m}{ft} \times \frac{1}{3600}\frac{hr}{s} = 0.447\frac{m}{s}$$

When you write it out in this manner, note that various quantities cancel out cleanly in the numerator and denominator. So you never make a mistake!

7. A good scientist first thinks of the larger picture and then of the finer details. So, estimating *orders of magnitude* is extremely important. Students often make the mistake of trying to get the decimal points right instead of the first digit – which obviously matters the most! So if you are asked to calculate the height of some building using some data and you come up with 0.301219 metres or 4.01219×10^6 metres, then the answer is plain nonsense even though you may have miraculously got the last six digits right. Physics is commonsense first, so use your intelligence before submitting any answer.

8. Always check your equations to see if they have the same dimensions on the left side as on the right. So, for example, from this principle we can see the equation $v^2 = u^2 + 2at$ is clearly wrong, whereas $v^2 = u^2 + 13a^2t^2$ could possibly be a correct relation. (Here v and *u* are velocities, *a* is acceleration, and *t* is time.) Note here that I use the word *possibly* because the dimensions on both sides match up in this case.

9. Whenever you derive an equation that is a little complicated, see if you can find a special limit where it becomes simple and transparent. So, sometimes it is helpful to imagine that some quantity in it is very large or very small. Where possible, make a "mental graph" so that you can picture an equation. So, for example, a formula for the distribution of molecular speeds in a gas could look like $f(v) = ve^{-(v-v_0)^2/a^2}$. Even without knowing the value of a you can immediately see that
   a) $f(v)$ goes to zero for large values of v, and $v = 0$.
   b) The maximum value of $f(v)$ occurs at v_0 and the function decreases on both side of this value.



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Summary of Lecture 2 – KINEMATICS I

1. *x(t)* is called displacement and it denotes the position of a body at time. If the displacement is positive then that body is to the right of the chosen origin and if negative, then it is to the left.

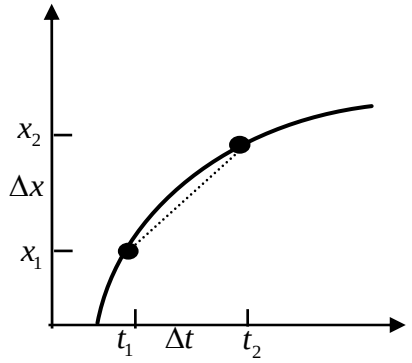
2. If a body is moving with average speed v then in time *t* it will cover a distance *d*=v*t*. But, in fact, the speed of a car changes from time to time and so one should limit the use of this formula to small time differences only. So, more accurately, one defines an average speed over the small time interval Δt as:

$$\text{average speed} = \frac{\text{distance travelled in time } \Delta t}{\Delta t}$$

3. We define *instantaneous velocity* at any time t as:

$$\text{v} = \frac{x(t_2) - x(t_1)}{t_2 - t_1} \equiv \frac{\Delta x}{\Delta t} \quad .$$

Here Δx and Δt are both very small quantities that tend to zero but their ratio v does not.



4. Just as we have defined velocity as the rate of change of distance, similarly we can define *instantaneous acceleration* at any time t as:

$$a = \frac{\text{v}(t_2) - \text{v}(t_1)}{t_2 - t_1} \equiv \frac{\Delta \text{v}}{\Delta t} \quad .$$

Here Δv and Δt are both very small quantities that tend to zero but their ratio *a* is not zero, in general. Negative acceleration is called deceleration. The speed of a decelerating body decreases with time.

5. Some students are puzzled by the fact that a body can have a very large acceleration but can be standing still at a given time. In fact, it can be moving in the opposite direction to its acceleration. There is actually nothing strange here because position,

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6. For constant acceleration and a body that starts from rest at $t = 0$, v increases linearly with time, $v \propto t$ (or $v = at$). If the body has speed v_0 at $t = 0$, then at time t, $v = at + v_0$.
7. We know in (6) above how far a body moving at constant speed moves in time t. But what if the body is changing its speed? If the speed is increasing linearly (i.e. constant acceleration), then the answer is particularly simple: just use the same formula as in (6) but use the average speed: $(v_0 + v_0 + at)/2$. So we get $x = x_0 + (v_0 + v_0 + at)t/2 = x_0 + v_0 t + \frac{1}{2}at^2$. This formula tells you how far a body moves in time t if it moves with constant acceleration a, and if started at position x_0 at $t=0$ with speed v_0.

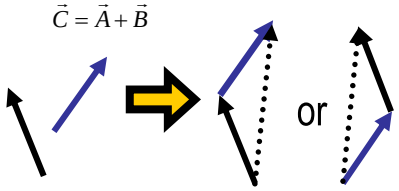
8. We can eliminate the time using (7), and arrive at another useful formula that tells us what the final speed will be after the body has traveled a distance equal to $x - x_0$ after time t, $v^2 = v_0^2 + 2a(x - x_0)$.

9. Vectors: a quantity that has a size as well as direction is called a *vector*. So, for example, the wind blows with some speed and in some direction. So the *wind velocity* is a vector.

10. If we choose axes, then a vector is fixed by its components along those axes. In one dimension, a vector has only one component (call it the x-component). In two dimensions, a vector has both x and y components. In three dimensions, the components are along the x,y,z axes.

11. If we denote a vector $\vec{r} = (x, y)$ then, $r_x = x = r\cos\theta$, and $r_y = y = r\sin\theta$. Note that $x^2 + y^2 = r^2$. Also, that $\tan\theta = y/x$.

13. Two vectors can be added together geometrically. We take any one vector, move it without changing its direction so that both vectors start from the same point, and then make a parallelogram. The diagonal of the parallelogram is the resultant.



14. Two vectors can also be added algebraically. In this case, we simply add the components of the two vectors along each axis separately. So, for example, Two vectors can be put together as $(1.5, 2.4) + (1, -1) = (2.5, 1.4)$.

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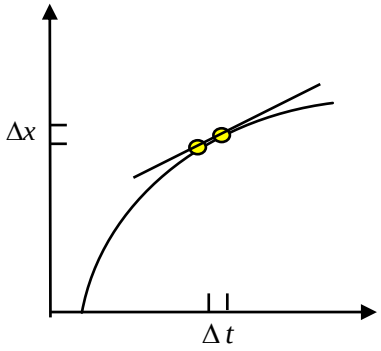
Summary of Lecture 3 – KINEMATICS II

1. The concept of the derivative of a function is exceedingly important. The derivative shows how fast a function changes when its argument is changed. (Remember that for $f(x)$ we say that f is a function that depends upon the argument x. You should think of f as a machine that gives you the value f when you input x.)

2. Functions do not always have to be written as $f(x)$. $x(t)$ is also a function. It tells us where a body is at different times t.

3. The derivative of $x(t)$ at time t is defined as:

$$\frac{dx}{dt} \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$
$$= \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}.$$



4. Let's see how to calculate the derivative of a simple function like $x(t) = t^2$. We must first calculate the difference in x at two slightly different values, t and $t + \Delta t,$ while remembering that we choose Δt to be extremely small:

$$\Delta x = (t + \Delta t)^2 - t^2$$
$$= t^2 + (\Delta t)^2 + 2t\Delta t - t^2$$
$$\frac{\Delta x}{\Delta t} = \Delta t + 2t \Rightarrow \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} = 2$$

5. In exactly the same way you can show that if $x(t) = t^n$ then:

$$\frac{dx}{dt} \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = nt^{n-1}$$

This is an extremely useful result.

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6. Let us apply the above to the function $x(t)$ which represents the distance moved by a body with constant acceleration (see lecture 2):

$$x(t) = x_0 + \mathrm{v}_0 t + \frac{1}{2}at^2$$

$$\frac{dx}{dt} = 0 + \mathrm{v}_0 + \frac{1}{2}a(2t) = \mathrm{v}_0 + a\,t$$

This clearly shows that $\frac{d\mathrm{v}}{dt} = 0 + a = a$ (acceleration is constant)

7. A stone dropped from rest increases its speed in the downward direction according to $\frac{d\mathrm{v}}{dt} = g \approx 9.8$ m/sec. This is true provided we are fairly close to the earth, otherwise the value of g decreases as we go further away from the earth. Also, note that if we measured distances from the ground up, then the acceleration would be negative.

8. A useful notation: write $\frac{d\mathrm{v}}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$. We call $\frac{d^2x}{dt^2}$ the second derivative of x with respect to t, or the rate of rate of change of x with respect to t.

9. It is easy to extend these ideas to a body moving in both the x and y directions. The position and velocity in 2 dimensions are:

$$\vec{r} = x(t)\hat{i} + y(t)\,\hat{j}$$

$$\vec{\mathrm{v}} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\,\hat{j}$$

$$= \mathrm{v}_x\hat{i} + \mathrm{v}_y\,\hat{j}$$

Here the unit vectors $\hat{i}$ and $\hat{j}$ are fixed, meaning that they do not depend upon time.

10. The scalar product of two vectors $\vec{A}$ and $\vec{B}$ is defined as:

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$

You can think of:

$$\vec{A}\cdot\vec{B} = (A)(B\cos\theta)$$

$$= (\text{length of } \vec{A}) \times (\text{projection of } \vec{B} \text{ on } \vec{A})$$

OR,

$$\vec{A}\cdot\vec{B} = (B)(A\cos\theta)$$

$$= (\text{length of } \vec{B}) \times (\text{projection of } \vec{A} \text{ on } \vec{B}).$$

Remember that for unit vectors $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = 1$ and $\hat{i}\cdot\hat{j} = 0$.

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Summary of Lecture 4 – FORCE AND NEWTON'S LAWS

1. Ancient view: objects tend to stop if they are in motion; force is required to keep something moving. This was a natural thing to believe in because we see objects stop moving after some time; frictionless motion is possible to see only in rather special circumstances.

2. Modern view: objects tend to remain in their initial state; force is required to *change* motion. Resistance to changes in motion is called *inertia*. More inertia means it is harder to make a body accelerate or decelerate.

3. Newton's First Law: An object will remain at rest or move with constant velocity unless acted upon by a net external force. (A non-accelerating reference frame is called an inertial frame; Newton's First Law holds only in inertial frames.)
4. More force leads to more acceleration: $\Rightarrow \quad a \propto F$

5. The greater the mass of a body, the harder it is to change its state of motion. More mass means more inertia. In other words, more mass leads to less acceleration:

$$\Rightarrow \quad a \propto \frac{1}{m}$$

Combine both the above observations to conclude that:

$$a \propto \frac{F}{m}$$

6. Newton's Second Law: $a = \frac{F}{m}$ (or, if you prefer, write as $F = ma$).

7. $F = ma$ is one relation between three independent quantities (m, a, F). For it to be useful, we must have separate ways of measuring mass, acceleration, and force. Acceleration is measured from observing the rate of change of velocity; mass is a measure of the amount of matter in a body (e.g. two identical cars have twice the mass of a single one). Forces (due to gravity, a stretched spring, repulsion of two like charges, etc) will be discussed later.

8. Force has dimensions of [mass] $\times$ [acceleration] $= M L T^{-2}$. In the MKS system the unit of force is the Newton. It has the symbol N where:
1 Newton = 1 kilogram.metre/second^2.

9. Forces can be internal or external. For example the mutual attraction of atoms within a block of wood are called internal forces. Something pushing the wood

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is an external force. In the application of $F = ma$, remember that F stands for the total external force upon the body.

10. Forces are vectors, and so they must be added vectorially:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$$

This means that the components in the $\hat{x}$ direction must be added separately, those in the $\hat{y}$ direction separately, etc.

11. Gravity acts directly on the mass of a body - this is a very important experimental observation due to Newton and does not follow from $F = ma$. So a body of mass m_1 experiences a force $F_1 = m_1 g$ while a body of mass m_2 experiences a force $F_2 = m_2 g$, where g is the acceleration with which any body (big or small) falls under the influence of gravity. (Galileo had established this important fact when he dropped different masses from the famous leaning tower of Pisa!)

12. The weight of a body W is the force which gravity exerts upon it, $W = mg$. Mass and weight are two completely different quantities. So, for example, if you used a spring balance to weigh a kilo of grapes on earth, the same grapes would weigh only 1/7 kilo on the moon.

13. Newton's Third Law: for every action there is an equal and opposite reaction. More precisely, $F_{AB} = -F_{BA}$, where F_{AB} is the force exerted by body B upon A whereas F_{BA} is the force exerted by body A upon B. Ask yourself what would happen if this was not true. In that case, a system of two bodies, even if it is completely isolated from the surroundings, would have a net force acting upon it because the net force acting upon both bodies would be $F_{AB} + F_{BA} \neq 0$.

14. If action and reaction are always equal, then why does a body accelerate at all? Students are often confused by this. The answer: in considering the acceleration of a body you must consider only the (net) force acting upon that body. So, for example, the earth pulls a stone towards it and causes it to accelerate because there is a net force acting upon the stone. On the other hand, by the Third Law, the stone also pulls the earth towards it and this causes the earth to accelerate towards the stone. However, because the mass of the earth is so large, we are only able to see the acceleration of the stone and not that of the earth.

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Summary of Lecture 5 – APPLICATIONS OF NEWTON'S LAWS – I

1. An obvious conclusion from $F = ma$ is that if $F = 0$ then $a = 0$! How simple, yet how powerful ! This says that for any body that is not accelerating the *sum of all the forces acting upon it* must vanish.

2. Examples of systems in equilibrium: a stone resting on the ground; a pencil balanced on your finger; a ladder placed against the wall, an aircraft flying at a constant speed and constant height.

3. Examples of systems out of equilibrium: a stone thrown upwards that is at its highest point; a plane diving downwards; a car at rest whose driver has just stepped on the car's accelerator.

4. If you know the acceleration of a body, it is easy to find the force that causes it to accelerate. Example: An aircraft of mass m has position vector,

$$\vec{r} = (at + bt^3)\hat{i} + (ct^2 + dt^4)\,\hat{j}$$

What force is acting upon it?

SOLUTION:
$$\vec{F} = m\frac{d^2x}{dt^2}\hat{i} + m\frac{d^2y}{dt^2}\,\hat{j}$$
$$= 6\,b\,m\,t\,\hat{i} + m(2c + 12d\,t^2)\,\hat{j}$$

5. The other way around is not so simple: suppose that you know F and you want to find x. For this you must solve the equation,

$$\frac{d^2x}{dt^2} = \frac{F}{m}$$

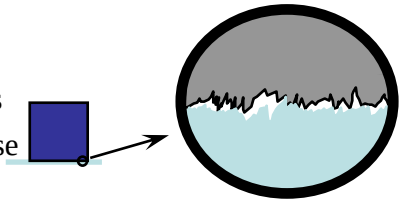
This may or may not be easy, depending upon F (which may depend upon both x as well as t if the force is not constant).

6. Ropes are useful because you can pull from a distance to change the direction of a force. The tension, often denoted by T, is the force you would feel if you cut the rope and grabbed the ends. For a massless rope (which may be a very good approximation in many situations) the tension is the same at every point along the rope. Why? Because if you take any small slice of the rope it weighs nothing (or very little). So if the force on one side of the slice was any different from the force force on the other side, it would be accelerating hugely. All this was for the "ideal rope" which has no mass and never breaks. But this idealization if often good enough.

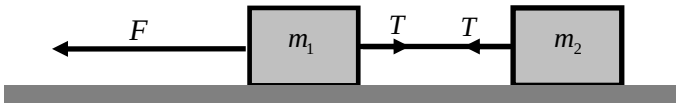
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7. We are all familiar with frictional force. When two bodies rub against each other, the frictional force acts upon each body separately opposite to its direction of motion (i.e it acts to slow down the motion). The harder you press two bodies against each other, the greater the friction. Mathematically, $\vec{F} = \mu\vec{N}$, where $\vec{N}$ is the force with which you press the two bodies against each other (normal force). The quantity μ is called the coefficient of friction (obviously!). It is large for rough surfaces, and small for smooth ones. Remember that $\vec{F} = \mu\vec{N}$ is an empirical relation and holds only approximately. This is obviously true: if you put a large enough mass on a table, the table will start to bend and will eventually break.

8. Friction is caused by roughness at a microscopic level - if you look at any surface with a powerful microscope you will see unevenness and jaggedness. If these big bumps are levelled somehow, friction will still not disappear because there will still be little bumps due to atoms. More precisely, atoms from the two bodies will interact each other because of the electrostatic interaction between their charges. Even if an atom is neutral, it can still exchange electrons and there will be a force because of surrounding atoms.



9. Consider the two blocks below on a frictionless surface:



We want to find the tension and acceleration: The total force on the first mass is $F - T$ and so $F - T = m_1 a$. The force on the second mass is simply T and so $T = m_2 a$. Solving the above, we get: $T = \dfrac{m_2 F}{m_1 + m_2}$ and $a = \dfrac{F}{m_1 + m_2}$.

10. There is a general principle by which you solve equilibrium problems. For equilibrium, the sum of forces in every direction must vanish. So $F_x = F_y = F_z = 0$. You may always choose the x, y, z directions according to your convenience. So, for example, as in the lecture problem dealing with a body sliding down an inclined plane, you can choose the directions to be along and perpendicular to the surface of the plane.

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Summary of Lecture 6 – APPLICATIONS OF NEWTON'S LAWS – II

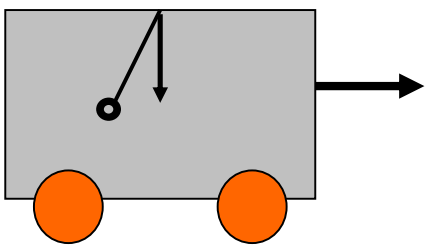
1. As a body moves through a body it displaces the fluid. it has to exert a force on the fluid to push it out of the way. By Newton's third law, the fluid pushes back on the body with an equal and opposite force. The direction of the fluid resistance force on a body is always opposite to the direction of the body's velocity relative to the fluid.

2. The magnitude of the fluid resistance force usually increases with the speed of the body through the fluid. Typically, $f = kv$ (an empirical law!). Imagine that you drop a ball bearing into a deep container filled with oil. After a while the ball bearing will approach its maximum (terminal) speed when the forces of gravity and friction balance each other: $mg = kv$ from which $v_{final} = mg/k$.

3. The above was a simple example of equilibrium under two forces. In general, while solving problems you should a)draw a diagram, b)define an origin for a system of coordinates, c)identify all forces (tension, normal, friction, weight, etc) and their x and y components, d)Apply Newton's law separately along the x and y axes. e) find the accelerations, then velocities, then displacements. This sounds very cook-book, and in fact it will occur to you naturally how to do this while solving actual problems.

4. Your weight in a lift: suppose you are in a lift that is at rest or moving at constant velocity. In either case $a=0$ and the normal force N and the force due to gravity are exactly equal, $N - Mg = 0 \Rightarrow N = Mg$. But if the lift is accelerating downwards then $Mg - N = Ma$ or $N = M(g - a)$. So now the normal force (i.e. the force with which the floor of the lift is pushing on you) is decreased. Note that if the lift is accelerating downwards with acceleration a (which it will if the cable breaks!) then $N=0$ and you will experience weightlessness just like astronauts in space do. Finally, if the lift is accelerating upwards then a is negative and you will feel heavier.

5. Imagine that you are in a railway wagon and want to know how much you are accelerating. You are not able to look out of the windows. A mass is hung from the roof. Find the acceleration of the car from the angle made by the mass.



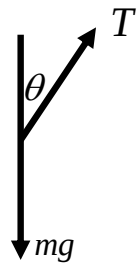
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We first balance the forces vertically: $T\cos\theta = mg$

And then horizontally: $T\sin\theta = ma$

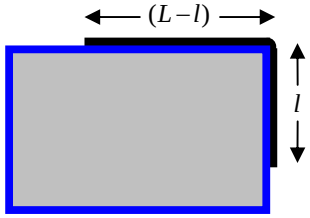
From these two equations we find that: $\tan\theta = \frac{a}{g}$

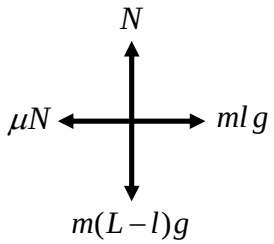
Note that the mass m doesn't matter - it cancels out!



6. Friction is a funny kind of force. It doe not make up its mind which way to act until some other force compels it to decide. Imagine a block lying on the floor. If you push it forward, friction will act backward. And if you push it to the left, friction will act to the right. In other words, the direction of the frictional force is always in the opposite direction to the applied force.

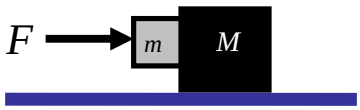
7. Let us solve the following problem: a rope of total length L and mass per unit length m is put on a table with a length l hanging from one edge. What should be l such that the rope just begins to slip?





To solve this, look at the balance of forces in the diagram below: in the vertical direction, the normal force balances the weight of that part of the rope that lies on the table: $N = m(L-l)g$. In the horizontal direction, the rope exerts a force mlg to the right, which is counteracted by the friction that acts to the left. Therefore $\mu N = mlg$. Substituting N from the first equation we find that $l = \frac{\mu L}{\mu + 1}$. Note that if μ isvery small then even a small piece of string that hangs over the edge will cause the entire string to slip down.

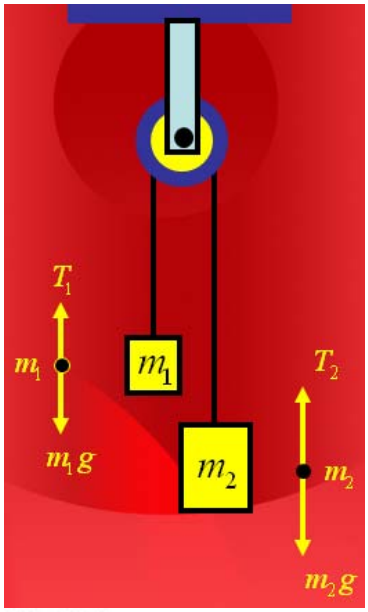
9. In this problem, we would like to calculate the minimum force F such that the small block does not slip downwards. Clearly, since the 2 bodies move together, $F = (m+M)a$. This gives $a = \frac{F}{(m+M)}$. We want the friction μN to be at least as large as the downwards force, mg.



So, we put $N = ma = m\left(\frac{F}{(m+M)}\right)$ from which the minimum horizontal force needed to prevent slippage is $F = \frac{(m+M)g}{\mu}$.

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There is an error in video lecture # 6 (video recording at 41:00) please consider the following problem with correct solution.
Consider two bodies of unequal masses m_1 and m_2 connected by the ends of a string, which passes over a frictionless pulley as shown in the diagram.



$m_2 > m_1$

$$T - m_1 g = m_1 a$$

$$m_2 g - T = m_2 a$$

$$m_2 g - m_1 g = (m_1 + m_2) a$$

$$a = \frac{(m_2 - m_1)}{(m_1 + m_2)} g$$

$m_2 > m_1$

$$T - m_1 g = m_1 a$$

$$a = \frac{T - m_1 g}{m_1} \quad \text{(i}$$

$$m_2 g - T = m_2 a$$

$$a = \frac{m_2 g - T}{m_2} \quad \text{(ii}$$

equating (i and (ii

$$\frac{T - m_1 g}{m_1} = \frac{m_2 g - T}{m_2}$$

$$m_2 T - m_1 m_2 g = m_1 m_2 g - m_1 T$$

$$m_2 T + m_1 T = m_1 m_2 g + m_1 m_2 g$$

$$T(m_2 + m_1) = 2 m_1 m_2 g$$

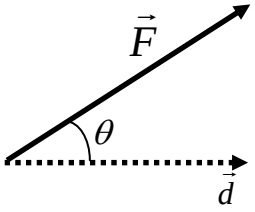
$$T = \frac{2 m_1 m_2 g}{(m_2 + m_1)}$$

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Summary of Lecture 7 – WORK AND ENERGY

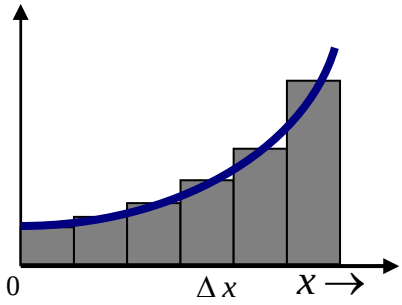
1. Definition of work: force applied in direction of displacement × displacement. This means that if the force F acts at an angle θ with respect to the direction of motion, then $W = \vec{F} \cdot \vec{d} = Fd\cos\theta$



2. a) Work is a scalar - it has magnitude but no direction.
   b) Work has dimensions: $M \times (L\ T^{-2}) \times L\ =\ M\ L^2T^{-2}$
   c) Work has units: 1 Newton × 1 Metre ≡ 1 Joule (J)

3. Suppose you lift a mass of 20 kg through a distance of 2 metres. Then the work you do is 20 kg × 9.8 Newtons × 2 metres = 392 Joules. On the other hand, the force of gravity is directed opposite to the force you exert and the work done by gravity is -392 Joules.

4. What if the force varies with distance (say, a spring pulls harder as it becomes longer). In that case, we should break up the distance over which the force acts into small pieces so that the force is approximately constant over each bit. As we make the pieces smaller and smaller, we will approach the exact result:



Now add up all the little pieces of work:

$$W = \Delta W_1 + \Delta W_2 + \cdots + \Delta W_N = F_1\Delta x + F_2\Delta x + \cdots + F_N\Delta x \equiv \sum_{n=1}^{N} F_n \Delta x$$

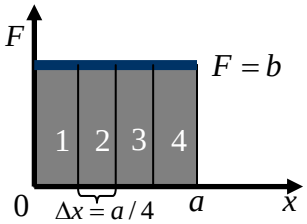
To get the exact result let $\Delta x \to 0$ and the number of intervals N $\to \infty$: $W = \lim_{\Delta x \to 0} \sum_{n=1}^{\infty} F_n \Delta x$

Definition: $W = \lim_{\Delta x \to 0} \sum_{n=1}^{\infty} F_n\ \Delta x \equiv \int_{x_i}^{x_f} F(x)dx$ is called the integral of F with respect to x from

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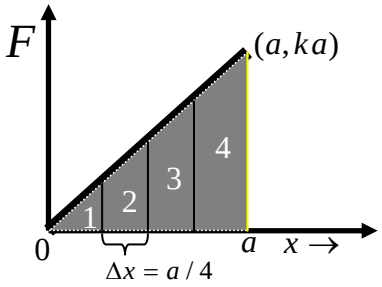
x_i to x_f. This quantity is the work done by a force, constant or non-constant. So if the force is known as a function of position, we can always find the work done by calculating the definite integral.

5. Just to check what our result looks like for a constant force, let us calculate W if $F = b$,



$$\frac{1}{4}a(b)+\frac{1}{4}a(b)+\frac{1}{4}a(b)+\frac{1}{4}a(b)=ab \qquad \therefore \int_0^a F dx = ab$$

6. Now for a less trivial case: suppose that $F=kx$, i.e. the force increases linearly with x.



Area of shaded region $= \frac{1}{2}(a)(ka) = k\frac{a^2}{2}$ $\qquad \therefore \quad \int_0^a F dx = k\frac{a^2}{2}$

7. Energy is the capacity of a physical system to do work:
   - it comes in many forms – mechanical, electrical, chemical, nuclear, etc
   - it can be stored
   - it can be converted into different forms
   - it can never be *created* or *destroyed*

8. Accepting the fact that energy is conserved, let us derive an expression for the kinetic energy of a body. Suppose a *constant force* accelerates a mass *m* from speed *0* to speed v over a *distance d*. What is the work done by the force? Obviously the answer is: $W = Fd$. But $F = ma$ and $v^2 = 2ad$. This gives $W = (ma)d = \frac{mv^2}{2d}d = \frac{1}{2}mv^2$. So, we conclude that the work done by the force has gone into creating kinetic energy. and that the amount of kinetic energy possessed by a body moving with speed v is $\frac{1}{2}mv^2$.
9. The work done by a force is just the force multiplied by the distance – it does not depend upon time. But suppose that the same amount of work is done in half the time. We then say that the *power* is twice as much.

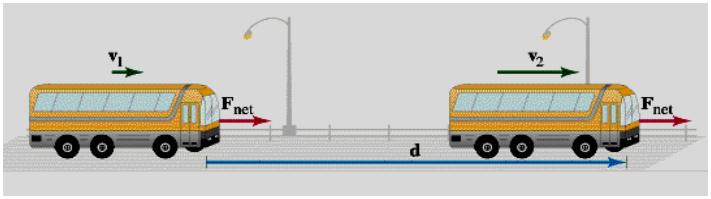
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We define:

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

If the force does not depend on time: $\frac{\text{Work}}{\text{Time}} = \frac{F\,\Delta x}{\Delta t} = F\,\text{v}$. Therefore, Power = F v.

10. Let's work out an example. A constant force accelerates a bus (mass m) from speed v_1 to speed v_2 over a distance d. What work is done by the engine?



Recall that for constant acceleration, $\text{v}_2^2 - \text{v}_1^2 = 2a\,(x_2 - x_1)$ where: v_2 = final velocity, x_2 = final position, v_1 = initial velocity, x_1 = initial position. Hence, $a = \frac{\text{v}_2^2 - \text{v}_1^2}{2d}$. Now calculate the work done: $W = Fd = mad = m\frac{\text{v}_2^2 - \text{v}_1^2}{2d}\,d = \frac{1}{2}m\text{v}_2{}^2 - \frac{1}{2}m\text{v}_1{}^2$. So the work done has resulted in an increase in the quantity $\frac{1}{2}m\text{v}^2$, which is kinetic energy.

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Summary of Lecture 8 – CONSERVATION OF ENERGY

1. Potential energy is, as the word suggests, the energy "locked up" up somewhere and which can do work. An object can store energy as the result of its position. Stored energy of position is referred to as potential energy. **Potential energy** is the stored energy of position possessed by an object. It has the ability or capacity to do work like other forms of energies.Potential energy can be converted into kinetic energy, $\frac{1}{2}mv^2$. As I showed you earlier, this follows directly from Newton's Laws.

2. If you lift a stone of mass m from the ground up a distance x, you have to do work against gravity. The (constant) force is mg, and so $W = mgx$. By conservation of energy, the work done by you was transformed into gravitational potential energy whose values is exactly equal to mgx. Where is the energy stored? Answer: it is stored neither in the mass or in the earth - it is stored in the gravitational field of the combined system of stone+earth.

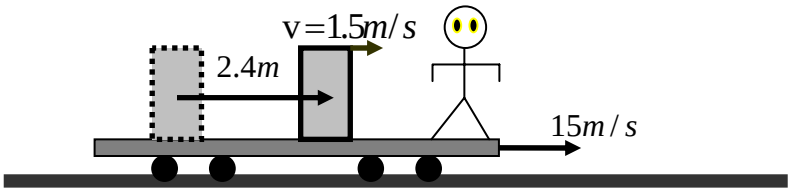
3. Suppose you pull on a spring and stretch it by an amount x away from its normal (equilibrium) position. How much energy is stored in the spring? Obviously, the spring gets harder and harder to pull as it becomes longer. When it is extended by length x and you pull it a further distance dx, the small amount of work done is $dW = Fdx = kxdx$. Adding up all the small bits of work gives the total work:

$$W = \int_0^x F dx = \int_0^x k x dx = \frac{1}{2} k x^2$$

This is the work you did. Maybe you got tired working so hard. What was the result of your working so hard? Answer: this work was transformed into energy stored in the spring. The spring contains energy exactly equal to $\frac{1}{2}kx^2$.

4. Kinetic energy obviously depends on the frame you choose to measure it in. If you are running with a ball, it has zero kinetic energy with respect to you. But someone who is standing will see that it has kinetic energy! Now consider the following situation: a box of mass 12kg is pushed with a constant force so that so that its speed goes from zero to 1.5m/sec (as measured by the person at rest on the cart) and it covers a distance of 2.4m. Assume there is no friction.

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mass of box = 12 *kg*

Let's first calculate the change in kinetic energy:

$$\Delta K = K_f - K_i = \frac{1}{2}(12kg)(1.5m/s)^2 - 0 = 13.5J$$

And then the (constant) acceleration:

$$a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{(1.5m/s)^2 - 0}{2(2 \cdot 4m)} = 0.469m/s^2$$

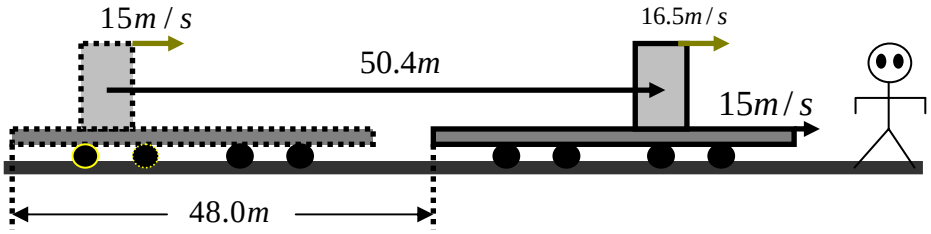
This acceleration results from a constant net force given by:

$$F = ma = (12kg)(0.469\,m/s^2) = 5.63N$$

From this, the work done on the crate is:

$$W = F\Delta x = (5.63N)(2.4m) = 13.5J \quad \text{(same as } \Delta K = 13.5J \text{ !)}$$

5. Now suppose there is somebody standing on the ground, and that the trolley moves at 15 m/sec relative to the ground:



Let us repeat the same calculation:

$$\Delta K' = K'_f - K'_i = \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_i^{\,2}$$

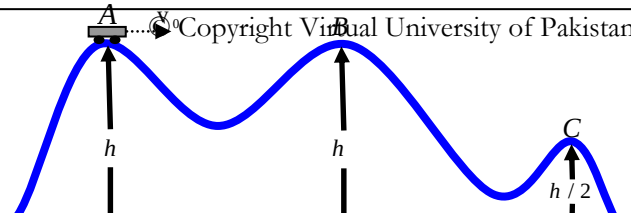
$$= \frac{1}{2}(12kg)(16.5m/s)^2 - \frac{1}{2}(12kg)(15.0m/s)^2 = 284J$$

This example clearly shows that work and energy have different values in different frames.

6. The total mechanical energy is: $E_{mech} = KE + PE$. If there is no friction then E_{mech} is conserved. This means that the sum does not change with time. For example: a ball is thrown upwards at speed v_0. How high will it go before it stops? The loss of potential energy is equal to the gain of potential energy. Hence, $\frac{1}{2}mv_0^2 = mgh \;\Rightarrow\; h = \frac{v_0^2}{2g}$.

Now look at the smooth, frictionless motion of a car over the hills below:

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Even though the motion is complicated, we can use the fact that the total energy is a constant to get the speeds at the points B,C,D:

At point A: $\frac{1}{2}mv_A^2 + mgh = \frac{1}{2}mv_B^2 + mgh \quad \Rightarrow \quad v_B = v_C$

At point C: $\frac{1}{2}mv_A^2 + mgh = \frac{1}{2}mv_C^2 + mg\frac{h}{2} \quad \Rightarrow \quad v_C = \sqrt{v_A^2 + gh}$

At point D: $\frac{1}{2}mv_A^2 + mgh = \frac{1}{2}mv_D^2 \quad \Rightarrow \quad v_D = \sqrt{v_A^2 + 2gh}$

7. Remember that potential energy has meaning only for a force that is conservative. A conservative force is that for which the work done in going from point A to point B is independent of the path chosen. Friction is an example of a non-conservative force and a potential energy cannot be defined. For a conservative force, $F = -\frac{dV}{dx}$. So, for a spring, $V = \frac{1}{2}kx^2$ and so $F = -kx$.

8. Derivation of $F = -\frac{dV}{dx}$: If the particle moves distance Δx in a potential V, then change in PE is ΔV where, $\Delta V = -F\,\Delta x$. From this, $F = -\frac{\Delta V}{\Delta x}$. Now let $\Delta x \to 0$. Hence, $F = -\lim_{\Delta x \to 0}\frac{\Delta V}{\Delta x} = -\frac{dV}{dx}$.

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Summary of Lecture 9 – MOMENTUM

1. Momentum is the "quantity of motion" possessed by a body. More precisely, it is defined as:

Mass of the body × Velocity of the body.

The dimensions of momentum are MLT^{-1} and the units of momentum are kg-m/s.

2. Momentum is a vector quantity and has both magnitude and direction, $p = m\vec{v}$. We can easily see that Newton's Second Law can be reexpressed in terms of momentum. When I wrote it down originally, it was in the form $m\vec{a} = \vec{\mathbf{F}}$. But since $\frac{d\vec{v}}{dt} = \vec{a}$, this can also be written as $\frac{d\vec{\mathbf{p}}}{dt} = \vec{\mathbf{F}}$ (new form). In words, the rate of change of momentum of a body equals the total force acting upon it. Of course, the old and new are exactly the same,

$$\frac{d\vec{\mathbf{p}}}{dt} = \frac{d(m\vec{\mathbf{v}})}{dt} = m\frac{d\vec{\mathbf{v}}}{dt} = m\vec{\mathbf{a}} = \vec{\mathbf{F}}.$$

3. When there are many particles, then the total momentum $\vec{\mathbf{P}}$ is,

$$\begin{aligned}\vec{\mathbf{P}} &= \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \cdots \vec{\mathbf{p}}_N \\ \frac{d}{dt}\vec{\mathbf{P}} &= \frac{d\vec{\mathbf{p}}_1}{dt} + \frac{d\vec{\mathbf{p}}_2}{dt} + \cdots \frac{d\vec{\mathbf{p}}_N}{dt} \\ &= \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \cdots \vec{\mathbf{F}}_N = \vec{\mathbf{F}}\end{aligned}$$

This shows that when there are several particles, the rate at which the total momentum changes is equal to the total force. It makes sense!

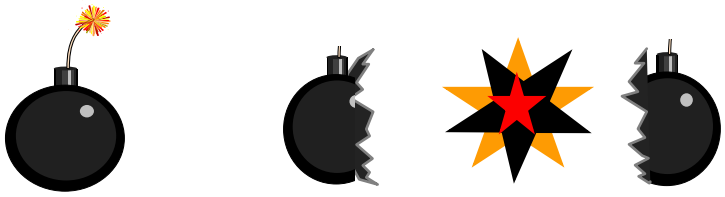
4. A very important conclusion of the above is that if the sum of the total external forces vanishes, then the total momentum is conserved, $\sum \vec{F}_{ext} = 0 \Rightarrow \frac{d}{dt}\vec{\mathbf{P}} = 0$. This is quite independent of what sort of forces act between the bodies - electric, gravitational, etc. - or how complicated these are. We shall see why this is so important from the following examples.

5. Two balls, which can only move along a straight line, collide with each other. The initial momentum is $P_i = m_1u_1 + m_2u_2$ and the final momentum is $P_f = m_1v_1 + m_2v_2$. Obviously one ball exerts a force on the other when they collide, so its momentum changes. But, from the fact that there is no external force acting on the balls, $P_i = P_f$, or $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$.

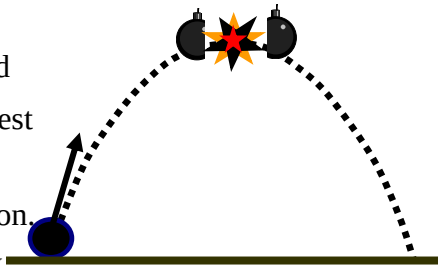
6. A bomb at rest explodes into two fragments. Before the explosion the total momentum is

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zero. So obviously it is zero after the explosion as well, $\mathbf{P}_f = \mathbf{0}$. During the time that the explosion happens, the forces acting upon the pieces are very complicated and changing rapidly with time. But when all is said and done, there are two pieces flying away with a total zero final momentum $\mathbf{P}_f = m_1\mathrm{v}_1 + m_2\mathrm{v}_2$. Hence $m_1\mathrm{v}_1 = -m_2\mathrm{v}_2$. In other words, the fragments fly apart with equal momentum but in opposite directions. The centre-of-mass stays at rest. So, knowing the velocity of one fragment permits knowing the velocity of the other fragment.



7. If air resistance can be ignored, then we can do some interesting calculations with what we have learned. So, suppose a shell is fired from a cannon with a speed 10 m/s at an angle 60^0 with the horizontal. At the highest point in its path it explodes into two pieces of equal masses. One of the pieces retraces its path to the cannon. Let us find the velocity of the other piece immediately after the explosion.



Solution: After the explosion: $P_{1x} = -5\dfrac{M}{2}$ (why?). But $P_{1x} + P_{2x} = P_x = M \times 10\cos 60$

$$\Rightarrow P_{2x} = 5M + 5\frac{M}{2}. \text{ Now use: } P_{2x} = \frac{M}{2}\mathrm{v}_{2x} \Rightarrow \mathrm{v}_{2x} = 15\, m/s.$$

8. When you hit your thumb with a hammer it hurts, doesn't it? Why? Because a large amount of momentum has been destroyed in a short amount of time. If you wrap your thumb with foam, it will hurt less. To understand this better, remember that force is the rate of change of momentum: $F = \dfrac{dp}{dt} \Rightarrow dp = Fdt$. Now define the **impulse** I as:

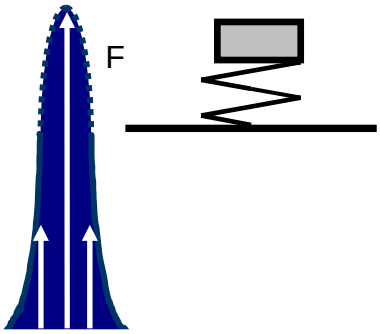
force × time over which the force acts.

If the force changes with time between the limits , then one should define I as,

$I = \int_{t_1}^{t_2} Fdt$. Since $\int_{t_1}^{t_2} Fdt = \int_{p_i}^{p_f} dp$, therefore $I = p_f - p_i$. In words, the change of momentum equals the impulse, which is equal to the area under the curve of force versus time. Even if you wrap your thumb in foam, the impulse is the same. But the force is definitely not!

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9. Sometimes we only know the force numerically (i.e. there is no expression like F=something). But we still know what the integral means: it is the area under the curve of force versus time. The curve here is that of a hammer striking a table. Before the hammer strikes, the force is zero, reaches a peak, and goes back to zero.



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Summary of Lecture 10 – COLLISIONS

1. Collisions are extremely important to understand because they happen all the time - electrons collide with atoms, a bat with a ball, cars with trucks, star galaxies with other galaxies,...In every case, the sum of the initial momenta equals the sum of the final momenta. This follows directly from Newton's Second Law, as we have already seen.

2. Take the simplest collision: two bodies of mass m_1 and m_2 moving with velocities u_1 and u_2. After the collision they are moving with velocities v_1 and v_2. From momentum conservation,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \Rightarrow m_1(u_1 - v_1) = m_2(v_2 - u_2)$$

This is as far as we can go. There are two unknowns but only one equation. However, if the collision is elastic then,

$$\tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 \Rightarrow \tfrac{1}{2}m_1(u_1^2 - v_1^2) = \tfrac{1}{2}m_2(v_2^2 - u_2^2).$$

Combine the two equations above,

$$u_1 + v_1 = v_2 + u_2 \Rightarrow u_1 - u_2 = v_2 - v_1.$$

In words, this says that in an elastic collision the relative speed of the incoming particles equals the relative speed of the outgoing particles.

3. One can solve for v_1 and v_2 (please do it!) easily and find that:

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

$$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$

Notice that if $m_1 = m_2$, then $v_1 = u_2$ and $v_2 = u_1$. So this says that after the collision, the bodies will just reverse their velocities and move on as before.

4. What if one of the bodies is much heavier than the other body, and the heavier body is at rest? In this case, $m_2 >> m_1$ and $u_2 = 0$. e can immediately see that $v_1 = -u_1$ and $v_2 = 0$. This makes a lot of sense: the heavy body continues to stay at rest and the light body just bounces back with the same speed. In the lecture, you saw a demonstration of this!

5. And what if the lighter body (rickshaw) is at rest and is hit by the heavier body (truck)? In this case, $m_2 << m_1$ and $u_2 = 0$. From the above equation we see that $v_1 = u_1$ and $v_2 = 2u_1$. So the truck's speed is unaffected, but the poor rickshaw is thrust in the direction of the truck at twice the truck's speed!

6. Sometimes we wish to slow down particles by making them collide with other particles.

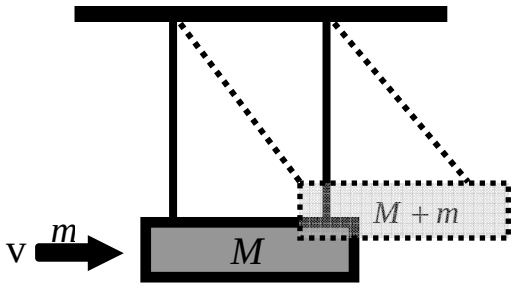
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In a nuclear reactor, neutrons can be slowed down in this way. let's calculate the fraction by which the kinetic energy of a neutron of mass m_1 decreases in a head-on collision with an atomic nucleus of mass m_2 that is initially at rest:

Solution: $\frac{K_i - K_f}{K_i} = 1 - \frac{K_f}{K_i} = 1 - \frac{v_f^2}{v_i^2}$

For a target at rest: $v_f = (\frac{m_1 - m_2}{m_1 + m_2})v_i$ $\quad\quad$ $\therefore \frac{K_i - K_f}{K_i} = \frac{4m_1m_2}{(m_1 + m_2)^2}.$

7. A bullet with mass m, is fired into a block of wood with mass M, suspended like a pendulum and makes a completely inelastic collision with it. After the impact, the block swings up to a maximum height y. What is the initial speed of the bullet?



Solution:

By conservation of momentum in the direction of the bullet,

$mv = (m + M)V, \Rightarrow v = \frac{(m + M)}{m}V$

The block goes up by distance y, and so gains potential energy. Now we can use the conservation of energy to give, $\frac{1}{2}(m + M)V^2 = (m + M)gy,$ where V is the velocity acquired by the block+bullet in the upward direction just after the bullet strikes. Now use $V = \sqrt{2gy}$. So finally, the speed of the bullet is: $v = \frac{(m + M)}{m}\sqrt{2gy}.$

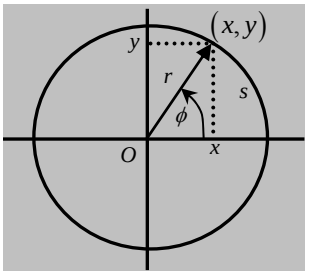
8. In 2 or 3 dimensions, you must apply conservation of momentum in each direction separately. The equation $\vec{P}_i = \vec{P}_f$ looks as if it is one equation, but it is actually 3 separate equations: $p_{ix} = p_{fx}, p_{iy} = p_{fy}, p_{iz} = p_{fz}$. On the other hand, suppose you had an elastic collision. In that case you would have only one extra equation coming from energy conservation, not three.

9. What happens to energy in an inelastic collision? Let's say that one body smashes into another body and breaks it into 20 pieces. To create 20 pieces requires doing work against the intermolecular forces, and the initial kinetic energy is used up for this.

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Summary of Lecture 11 – ROTATIONAL KINEMATICS

1. Any rotation is specified by giving two pieces of information:
   a) The point about which the rotation occurs, i.e. the origin.
   b) The angle of rotation is denoted by ϕ in the diagram.



2. The arc length = radius × angular displacement, or $s = r\phi$. Here ϕ is measured in radians. The maximum value of ϕ is 2π radians, which corresponds to 360 degrees or one full revolution. From this it follows that 1 radian = 57.3^0 or 1 radian = 0.159 revolution. Obviously, if $\phi = 2\pi$, then $s = 2\pi r$, which is the total circumference.

3. Suppose that there is a particle located at the tip of the radius vector. Now we wish to describe the rotational *kinematics* of this particle, i.e. describe its motion as goes around the circle. So, suppose that the particle moves from angle ϕ_1 to ϕ_2 in time $t_2 - t_1$. Then, the *average angular speed* $\bar{\omega}$ is defined as, $\bar{\omega} = \dfrac{\phi_2 - \phi_1}{t_2 - t_1} = \dfrac{\Delta\phi}{\Delta t}$. Suppose that we look at $\bar{\omega}$ over a very short time. Then, $\omega = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\phi}{\Delta t} = \dfrac{d\phi}{dt}$ is called the instantaneous angular speed.

4. To familarize ourselves with the notion of angular speed, let us compute ω for a clock second, minute and hour hands:

$$\omega_{\text{second}} = \frac{2\pi}{60} = 0.105\ rad/s,$$

$$\omega_{\text{minute}} = \frac{2\pi}{60 \times 60} = 1.75 \times 10^{-3}\ rad/s,$$

$$\omega_{\text{hour}} = \frac{2\pi}{60 \times 60 \times 12} = 1.45 \times 10^{-4}\ rad/s.$$

5. Just as we defined accelaration for linear motion, we also define acceleration for circular motion:

$$\bar{\alpha} \equiv \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t} \qquad \text{(average angular speed)}$$

Hence, $\alpha = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\omega}{\Delta t}$ becomes $\alpha = \dfrac{d\omega}{dt} = \dfrac{d}{dt}\dfrac{d\phi}{dt} = \dfrac{d^2\phi}{dt^2}$ (angular acceleration). Let us see what this means for the speed with which a particle goes around. Now use $s = r\phi$. Differentiate with respect to time t: $\dfrac{ds}{dt} = r\dfrac{d\phi}{dt}$. The rate of change of arc length s is clearly

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what we should call the circular speed, v. So $v = r\omega$. Since r is held fixed, it follows that $\frac{dv}{dt} = r\frac{d\omega}{dt}$. Now define $a_T = \frac{dv}{dt}$. Obviously, $a_T = r\alpha$. Here T stands for tangential, i.e. in the direction of increasing s.

6. Compare the formulae for constant linear and angular accelerations:

LINEAR	ANGULAR
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x = x_0 + v_0 t + \frac{1}{2}at^2$	$\phi = \phi_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\phi - \phi_0)$

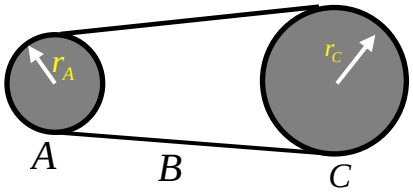
Why are they almost identical even though they describe two totally different physical situations. Answer: because the mathematics is identical!

7. The angular speed of a car engine is increased from 1170 rev/min to 2880 rev/min in 12.6 s. a)Find the average angular acceleration in rev/min^2. (b) How many revolutions does the engine make during this time?

SOLUTION: this is a straightforward application of the formulae in point 5 above.

$$\alpha = \frac{\omega_f - \omega_i}{t} = 8140\,\text{rev/min}^2, \quad \phi = \omega_i t + \frac{1}{2}\alpha t^2 = 425\,\text{rev}.$$

8. Wheel A of radius r_A = 10.0 cm is coupled by a chain B to wheel C of radius r_C = 25.0 cm. Wheel A increases its angular speed from rest at a uniform rate of 1.60 rad/s2. Determine the time for wheel C to reach a rotational speed of 100 rev/min.

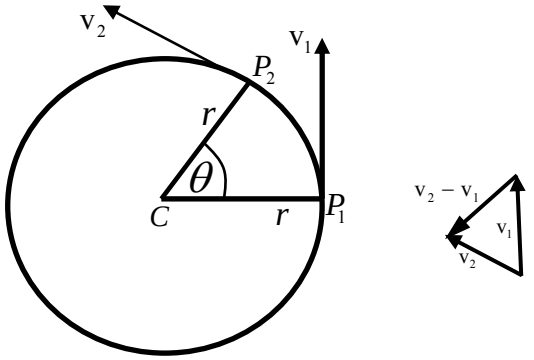


SOLUTION: Obviously every part of the chain moves with the same speed and so $v_A = v_C$. Hence $r_A\omega_A = r_C\omega_C \Rightarrow \omega_A = \frac{r_C\omega_C}{r_A}$. From the definition of acceleration, $\alpha = \frac{\omega_A - 0}{t}$. From this, $t = \frac{\omega_A}{\alpha} = \frac{r_C\omega_C}{r_A\alpha} = 16.4\ s$.

9. Imagine a disc going around. All particles on the disc will have same 'ω' and 'α' but different 'v' and 'a' . Clearly 'ω' and 'α' are simpler choices !!

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10. Now consider a particle going around a circle at constant speed. You might think that constant speed means no acceleration. Bu this is wrong! It is changing its direction and accelerating. This is called "centripetal acceleration", meaning acceleration directed towards the centre of the circle. Look at the figure below:



Note that the distance between points P_1 and P_2 is $\Delta r = \mathrm{v}\Delta t \approx r\theta$. Similarly, $\Delta \mathrm{v} \approx \mathrm{v}\theta \Rightarrow \bar{a} = \frac{\Delta \mathrm{v}}{\Delta t} \approx \frac{\mathrm{v}\theta}{r\theta/\mathrm{v}} = \frac{\mathrm{v}^2}{r}$. More generally, $a = \lim_{\Delta t \to 0} \frac{\Delta \mathrm{v}}{\Delta t} = \frac{\mathrm{v}^2}{r}$. In vector form, $\vec{\mathrm{a}}_R = -\frac{\mathrm{v}^2}{r}\hat{r}$. The negative sign indicates that the acceleration is towards the centre.

12. Vector Cross Products: The vector crossproduct of two vectors is defined as: $\vec{A} \times \vec{B} = AB\sin\theta\, \hat{n}$ where $\hat{n}$ is a unit vector that is perpendicular to both $\vec{A}$ and $\vec{B}$. Apply this definition to unit vectors in 3-dimensions: $\hat{i} \times \hat{j} = \hat{k},\ \hat{k} \times \hat{i} = \hat{j},\ \hat{j} \times \hat{k} = \hat{i}$.

13. Some key properties of the crossproduct:

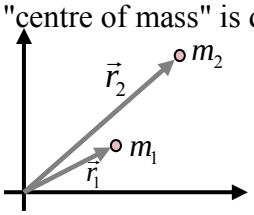
- $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
- $\vec{A} \times \vec{A} = 0$
- $\left(\vec{A} + \vec{B}\right) \times \vec{C} = \left(\vec{A} \times \vec{B}\right) + \left(\vec{A} \times \vec{C}\right)$
- $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$

14. The cross product is only definable in 3 dimensions and has no meaning in 2-d. This is unlike the dot product which as a meaning in any number of dimensions.

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Summary of Lecture 12 – PHYSICS OF MANY PARTICLES

1. A body is made of a collection of particles. We would like to think of this body having a "centre". For two masses the "centre of mass" is defined as: $\vec{r}_{cm} \equiv \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$.



In 2 dimensions (i.e. a plane) this is actually two equations:

$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$ and $y_{cm} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$. These give the coordinates of the centre of mass of the two-particle system.

2. Example: one mass is placed at $x = 2cm$ and a second mass, equal to the first, is placed at $x = 6cm$. The cm position lies halfway between the two as you can see from:

$$x_{cm} = \frac{mx_1 + mx_2}{m + m} = \frac{2m + 6m}{2m} = 4cm.$$

Note that there is no physical body that is actually located at $x_{cm} = 4cm$! So the centre of mass can actually be a point where there is no matter. Now suppose that the first mass is three times bigger than the first:

$$x_{cm} = \frac{(3m)x_1 + mx_2}{3m + m} = \frac{2(3m) + 6m}{4m} = 3cm$$

This shows that the cm lies closer to the heavier body. This is always true.

3. For N masses the obvious generalization of the centre of mass position is the following:

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_N\vec{r}_N}{m_1 + m_2 + \cdots + m_N} = \frac{1}{M}\left(\sum_{n=1}^{n=N} m_n\vec{r}_n\right).$$

In words, this says that the following: choose any origin and draw vectors $\vec{r}_1, \vec{r}_2, \cdots \vec{r}_N$ that connect to the masses $m_1, m_2, \cdots m_N$. Heavier masses get more importance in the sum. So suppose that m_2 is much larger than any of the others. If so, $\vec{r}_{cm} \approx \frac{m_2\vec{r}_2}{m_2} = \vec{r}_2$. Hence, the cm is very close to the position vector of m_2.

4. For symmetrical objects, it is easy to see where the cm position lies: for a sphere or circle it lies at the centre; for a cylinder it is on the axis halfway between the two faces, etc.

5. Our definition of the cm allows Newton's Second Law to be written for entire collection

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of particles:

$$\vec{v}_{cm} = \frac{d\vec{r}_{cm}}{dt} = \frac{1}{M}\left(\sum m_n \vec{v}_n\right)$$

$$\vec{a}_{cm} = \frac{d\vec{v}_{cm}}{dt} = \frac{1}{M}\left(\sum m_n \vec{a}_n\right)$$

$$\therefore \quad M\,\vec{a}_{cm} = \sum \vec{F}_n = \sum\left(\vec{F}_{ext} + \vec{F}_{int}\right) \qquad \text{use } \sum \vec{F}_{int} = 0$$

$$\Rightarrow \quad \sum \vec{F}_{ext} = M\,\vec{a}_{cm} \text{ (the sum of external forces is what causes acceleration)}$$

In the above we have used Newton's Third Law as well: $\vec{F}_{12} + \vec{F}_{21} = 0$ etc.

6. Consider rotational motion now for a rigid system of N particles. Rigid means that all particles have a fixed distance from the origin. The kinetic energy is,

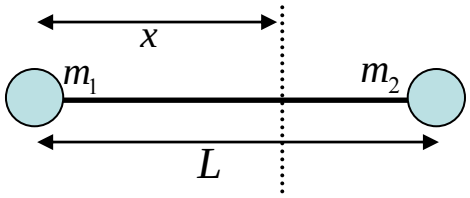
$$K = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}m_3 v_3^2 + \cdots$$

$$= \frac{1}{2}m_1 r_1^2\omega^2 + \frac{1}{2}m_2 r_2^2\omega^2 + \frac{1}{2}m_3 r_3^2\omega^2 + \cdots$$

$$= \frac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

Now suppose that we define the "moment of inertia" $I \equiv \sum m_i r_i^2$. Then clearly the kinetic energy is $K = \frac{1}{2} I\omega^2$. How similar this is to $K = \frac{1}{2}Mv^2$!

7. To familiarize ourselves with I, let us consider the following: Two particles m_1 and m_2 are connected by a light rigid rod of length L. Neglecting the mass of the rod, find the rotational inertia I of this system about an axis perpendicular to the rod and at a distance x from m_1.



Answer: $I = m_1 x^2 + m_2\left(L - x\right)^2$. Of course, this was quite trivial. Now we can ask a more interesting question: For what x is I the largest? Now, near a maximum, the slope of a function is zero. So calculate $\frac{dI}{dx}$ and then put it equal to zero:

$$\therefore \quad \frac{dI}{dx} = 2m_1 x - 2m_2\left(L - x\right) = 0 \quad \Rightarrow \quad x_{\max} = \frac{m_2 L}{m_1 + m_2}.$$

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8. Although matter is made up of discrete atoms, even if one takes small pieces of any body, there are billions of atoms within it. So it is useful to think of matter as being continuously distributed. Since a sum $\sum$ becomes an integral $\int$, it is obvious that the new definitions of I and $\vec{\mathrm{R}}_{\mathrm{cm}}$ becom:

$$I \equiv \int r^2 dm \quad \text{and} \quad \vec{\mathrm{R}}_{\mathrm{cm}} \equiv \frac{1}{M}\int \vec{r} dm.$$

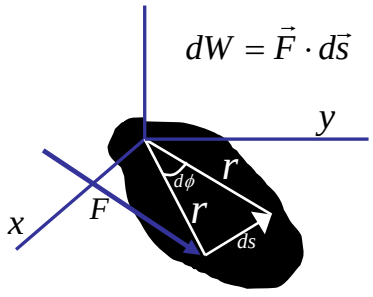
9. A simple application: suppose there is a hoop with mass distributed uniformly over it. The moment of inertia is: $I = \int r^2 dm = R^2 \int dm = MR^2$.

10. A less trivial application: instead of a hoop as above, now consider a solid plate:

$$I = \int r^2 dm \quad (dm = 2\pi r dr \rho_0)$$
$$= \int_0^R 2\pi r^3 dr \rho_0 = \frac{1}{2}(\pi R^2 \rho_0) R^2 = \frac{1}{2} M R^2$$

11. You have seen that it is easier to turn things (e.g. a nut, when changing a car's tyre after a puncture) when the applied force acts at a greater distance. This is because the *torque* τ is greater. We define $\vec{\tau} = \vec{r} \times \vec{F}$ from the magnitude is $\tau = rF\sin\theta$. Here θ is the angle between the radius vector and the force.

12. Remember that when a force $\vec{F}$ acts through a distance $d\vec{r}$ it does an amount of work equal to $\vec{F} \cdot d\vec{r}$. Now let us ask how much work is done when a torque acts through a certain angle as in the diagram below:



The small amount of work done is:

$$dW = \vec{F} \cdot d\vec{s} = F\cos\theta\, ds = \left(F\cos\theta\right)\left(rd\phi\right) = \tau d\phi$$

Add the contributions coming from from all particles,

$$dW_{net} = \left(F_1\cos\theta_1\right) r_1 d\phi + \left(F_2\cos\theta_2\right) r_2 d\phi + \cdots + \left(F_n\cos\theta_n\right) r_n d\phi$$

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$$= (\tau_1 + \tau_2 + \cdots + \tau_n)\, d\phi$$

$$\therefore dW_{net} = \left(\sum \tau_{ext}\right) d\varphi = \left(\sum \tau_{ext}\right) \omega dt \qquad .....(1)$$

Now consider the change in the kinetic energy K,

$$dK = d\left(\frac{1}{2} I\omega^2\right) = I\omega d\omega = (I\alpha)\omega dt \qquad .....(2)$$

By conservation of energy, the change in K must equal the work done, and so:

$$dW_{net} = dK \quad \Rightarrow \quad \sum \tau_{ext} = I\alpha$$

In words this says that the total torque equals the moment of inertia times the angular acceleration. This is just like Newton's second law, but for rotational motion !

13.A comparison between linear and rotational motion quantities and formulae:

LINEAR	ROTATIONAL
x, M	ϕ, I
$v = \frac{dx}{dt}$	$\omega = \frac{d\phi}{dt}$
$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$
$F = Ma$	$\tau = I\alpha$
$K = \frac{1}{2} Mv^2$	$K = \frac{1}{2} I\omega^2$
$W = \int F dx$	$W = \int \tau d\phi$

14. Rotational and translational motion can occur simultaneously. For example a car's wheel rotates and translates. In this case the total kinetic energy is clearly the sum of the energies of the two motions: $K = \frac{1}{2} M\, v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$.

15. It will take a little work to prove the following fact that I simply stated above: for a system of N particles, the total kinetic energy divides up neatly into the kinetic energy of rotation and translation. Start with the expression for kinetic energy and write $\vec{v}_{cm} + \vec{v}'_i$ where $\vec{v}'_i$ is the velocity of a particle with respect to the cm frame,

$$K = \sum \frac{1}{2} m_i v_i^{\,2} = \sum \frac{1}{2} m_i \vec{v}_i \cdot \vec{v}_i$$

$$= \sum \frac{1}{2} m_i \left(\vec{v}_{cm} + \vec{v}'_i\right) \cdot \left(\vec{v}_{cm} + \vec{v}'_i\right)$$

$$= \sum \frac{1}{2} m_i \left(v_{cm}^2 + 2\vec{v}_{cm} . \vec{v}'_i + v'^2_i\right)$$

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Now, $\sum m_i \vec{v}_{cm} \cdot \vec{v}'_i = \vec{v}_{cm} \cdot \sum m_i \vec{v}'_i = \vec{v}_{cm} \cdot \sum \vec{p}'_i = 0$. Why? because the total momentum is xero in the cm frame! So this brings us to our result that,

$$K = \sum \frac{1}{2} m_i v_{cm}^2 + \sum \frac{1}{2} m_i v_i'^2$$

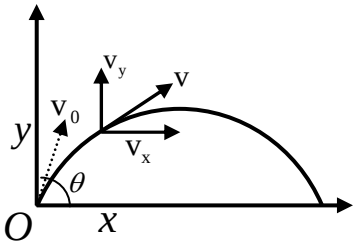
$$= \frac{1}{2} M v_{cm}^2 + \sum \frac{1}{2} m_i r_i'^2 \omega^2.$$

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Summary of Lecture 13 – ANGULAR MOMENTUM

1. Recall the definition of angular momentum: $\vec{L} = \vec{r} \times \vec{p}$. The magnitude can be written in several different but equivalent ways,
   (a) $L = rp\sin\theta$
   (b) $L = (r\sin\theta)\, p = r_{\perp} p$
   (c) $L = r(p\sin\theta) = r\, p_{\perp}$

2. Let us use this definition to calculate the angular momentum of a projectile thrown from the ground at an angle θ. Obviously, initial angular momentum is zero (why?).



We know what the projectile's coordinates will be at time t after launch,

$$x = (\mathrm{v}_0 \cos\theta)t, \quad y = (\mathrm{v}_0 \sin\theta)t - \frac{1}{2}gt^2$$

as well as the velocity components,

$$\mathrm{v}_x = \mathrm{v}_0 \cos\theta,\ \mathrm{v}_y = \mathrm{v}_0 \sin\theta - gt.$$

Hence, $\vec{L} = \vec{r} \times \vec{p} = (x\hat{i} + y\,\hat{j}) \times (\mathrm{v}_x \hat{i} + \mathrm{v}_y\,\hat{j})m = m(x\,\mathrm{v}_y - y\,\mathrm{v}_x)\hat{k}$

$$= m\left(\frac{1}{2}gt^2\mathrm{v}_0\cos\theta - gt^2\mathrm{v}_0\cos\theta\right)\hat{k} = -\frac{m}{2}gt^2\mathrm{v}_0\cos\theta\,\hat{k}.$$

In the above, $\hat{k} = \hat{i} \times \hat{j}$ is a unit vector perpendicular to the paper. You can see here that the angular momentum increases as t^2.

2. Momentum changes because a force makes it change. What makes angular momentum change? Answer: torque. Here is the definition again: $\vec{\tau} = \vec{r} \times \vec{F}$. Now let us establish a very important relation between torque and rate of change of L.
   Begin:
   $\vec{L} = \vec{r} \times \vec{p}$. At a slightly later time, $\vec{L} + \Delta\vec{L} = (\vec{r} + \Delta\vec{r}) \times (\vec{p} + \Delta\vec{p})$

$$= \vec{r} \times \vec{p} + \vec{r} \times \Delta\vec{p} + \Delta\vec{r} \times \vec{p} + \Delta\vec{r} \times \Delta\vec{p}$$

By subtracting, $\Delta\vec{L} = \vec{r} \times \Delta\vec{p} + \Delta\vec{r} \times \vec{p}$ $\dfrac{\Delta\vec{L}}{\Delta t} = \dfrac{\vec{r} \times \Delta\vec{p} + \Delta\vec{r} \times \vec{p}}{\Delta t} = \vec{r} \times \dfrac{\Delta\vec{p}}{\Delta t} + \dfrac{\Delta\vec{r}}{\Delta t} \times \vec{p}.$

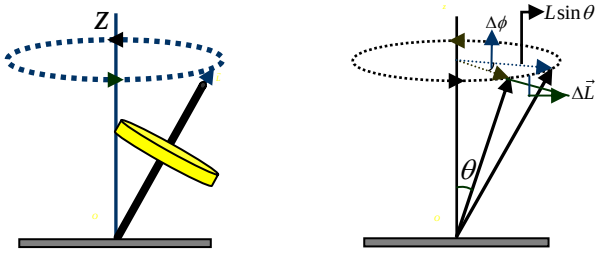
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Now divide by the time difference and then take limit as $\Delta t \to 0$:

$$\because \quad \lim_{\Delta t \to 0} \frac{\Delta \vec{L}}{\Delta t} = \frac{d\vec{L}}{dt} \quad \therefore \quad \frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt} + \frac{d\vec{r}}{dt} \times \vec{p}$$

But $\frac{d\vec{r}}{dt}$ is $\vec{\mathrm{v}}$ and $\vec{p} = m\vec{\mathrm{v}}$! Also, $\frac{d\vec{r}}{dt} \times \vec{p} = \vec{\mathrm{v}} \times m\vec{\mathrm{v}} = m\left(\vec{\mathrm{v}} \times \vec{\mathrm{v}}\right) = 0$. So we arrive at $\frac{d\vec{L}}{dt} = \vec{r} \times \frac{d\vec{p}}{dt}$. Now use Newton's Law, $\vec{F} = \frac{d\vec{p}}{dt}$. Hence we get the fundamental equation $\frac{d\vec{L}}{dt} = \vec{r} \times \vec{F}$, or $\frac{d\vec{L}}{dt} = \vec{\tau}$. So, just as a particle's momentum changes with time because of a force, a particle's *angular* momentum changes with time because of a torque.

3. As you saw in the lecture, the spinning top is an excelllent application of $\frac{d\vec{L}}{dt} = \vec{\tau}$.



Start from $\vec{\tau} = \vec{r} \times \vec{F}$ where $\vec{F} = m\vec{g}$ $\therefore$ $\tau = Mgr\sin\theta$. But $\vec{\tau}$ is perpendicular to $\vec{L}$ and so it cannot change the magnitude of $\vec{L}$. Only the direction changes. Since $\Delta\vec{L} = \vec{\tau}\ \Delta t$, you can see from the diagram that $\Delta\phi = \frac{\Delta L}{L\sin\theta} = \frac{\tau\ \Delta t}{L\sin\theta}$. So the precession speed ω_P is: $\omega_P = \frac{\Delta\phi}{\Delta t} = \frac{\tau}{L\sin\theta} = \frac{Mgr\sin\theta}{L\sin\theta} = \frac{Mgr}{L}$. As the top slows down due to friction and L decreases, the top precesses faster and faster.

4. Now consider the case of many particles. Choose any origin with particles moving with respect to it. We want to write down the total angular momentum,

$$\vec{L} = \vec{L}_1 + \vec{L}_2 + \cdots + \vec{L}_N = \sum_{n=1}^{N} \vec{L}_n$$

$$\frac{d\vec{L}}{dt} = \frac{d\vec{L}_1}{dt} + \frac{d\vec{L}_2}{dt} + \cdots + \frac{d\vec{L}_N}{dt} = \sum_{n=1}^{N} \frac{d\vec{L}_n}{dt}$$

Since $\frac{d\vec{L}_n}{dt} = \vec{\tau}_n$, it follows that $\frac{d\vec{L}}{dt} = \sum_{n=1}^{N} \vec{\tau}_n$. Thus the time rate of change of the total angular momentum of a system of particles equals the net torque acting on the system. I showed earlier that internal forces cancel. So also do internal torques, as we shall see.

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5. The torque on a system of particles can come both from external and internal forces. For example, there could be charged particles which attract/repel each other while they are all in an external gravitational field. Mathematically, $\sum \vec{\tau} = \sum \vec{\tau}_{\text{int}} + \sum \vec{\tau}_{ext}$. Now, if the forces between two particles not only are equal and opposite but are also directed along the line joining the two particles, then can easily show that the total internal torque, $\sum \vec{\tau}_{\text{int}} = 0$. Take the case of two particles,

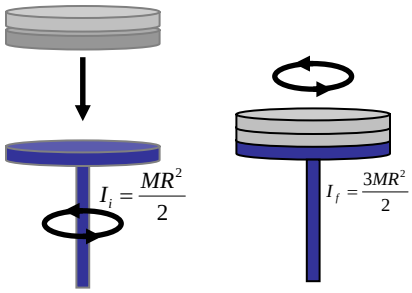
$$\sum \vec{\tau}_{\text{int}} = \vec{\tau}_1 + \vec{\tau}_2 = \vec{r}_1 \times \vec{F}_{12} + \vec{r}_2 \times \vec{F}_{21}$$

But $\vec{F}_{12} = -\vec{F}_{21} = F\hat{r}_{12}, \quad \therefore \sum \vec{\tau}_{\text{int}} = (\vec{r}_1 - \vec{r}_2) \times \vec{F}_{12} = \vec{r}_{12} \times (F\hat{r}_{12}) = F(\vec{r}_{12} \times \hat{r}_{12}) = 0$

Thus net external torque acting on a system of particles is equal to the time rate of change of the of the total angular momentum of the system.

6. It follows from $\frac{d\vec{L}}{dt} = \vec{\tau}$ that if no net external torque acts on the system, then the angular momentum of the system does not change with the time: $\frac{d\vec{L}}{dt} = 0 \quad \Rightarrow \quad \vec{L} =$ a constant.

This is simple but extremely important. Let us apply this to the system shown here. Two stationary discs, each with $I = \frac{1}{2}MR^2$, fall on top of a rotating disc. The total angular momentum is unchanged so, $I_i\omega_i = I_f\omega_f \;\Rightarrow\; \omega_f = \omega_i\left(\frac{I_i}{I_f}\right)$.



Hence, $\omega_f = \omega_i\left(\frac{MR^2}{2} \times \frac{2}{3MR^2}\right) = \frac{1}{3}\omega_i$.

7. You shoud be aware of the similarities and differences between the equations for linear and rotational motion: $\vec{F} = \frac{d\vec{p}}{dt} \Leftrightarrow \vec{\tau} = \frac{d\vec{L}}{dt}$. One big difference is that for momentum, $\vec{p} = m\vec{\text{v}}$, it does not matter where you pick the origin. But $\vec{L}$ definitely depends on the choice of the origin. So changing $\vec{r}$ to $\vec{c} + \vec{r}$ changes $\vec{L}$ to $\vec{L}'$:

$$\vec{L}' = \vec{r}' \times \vec{p} = (\vec{c} + \vec{r}) \times \vec{p} = \vec{c} \times \vec{p} + \vec{L}.$$

8. Linear and angular acceleration:

$$\vec{\text{a}} = \frac{d\vec{\text{v}}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r}) = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$
$$= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{\text{v}}$$

So the acceleration has a tangential and radial part, $\vec{\text{a}} = \vec{\text{a}}_{\text{T}} + \vec{\text{a}}_{\text{R}}$.

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Summary of Lecture 14 – EQUILIBRIUM OF RIGID BODIES

1. A rigid body is one where all parts of the body are fixed relative to each other (for example, a pencil). Fluids and gases are non-rigid.

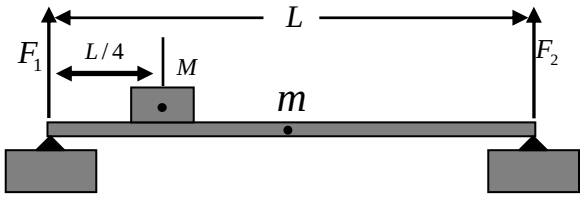
2. The translational motion of the centre of mass of a rigid body is governed by:

$$\frac{d\vec{P}}{dt} = \vec{F} \text{ where } \vec{F} = \sum \vec{F}_{ext} \text{ is the } \textit{net} \text{ external force.}$$

Similarly, for rotational motion, $\frac{d\vec{L}}{dt} = \vec{\tau}$ where $\vec{\tau} = \sum \vec{\tau}_{ext}$ is the net external torque.

3. A rigid body is in mechanical equilibrium if **both** the linear momentum $\vec{P}$ and angular momentum $\vec{L}$ have a constant value. i.e., $\frac{d\vec{P}}{dt} = 0$ **and** $\frac{d\vec{L}}{dt} = 0$. **Static equilibrium** refers to $\vec{P} = 0$ and $\vec{L} = 0$.

4. As an example of static equilibrium, consider a beam resting on supports:



We want to find the forces F_1 and F_2 with which the supports push on the rod in the upwards direction. First, balance forces in the vertical y direction:

$$\sum F_y = F_1 + F_2 - Mg - mg = 0$$

Now demand that the total torque vanishes:

$$\sum \tau_y = (F_1)(0) + (F_2)(L) - (Mg)(L/4) - (mg)(L/2) = 0$$

From these two conditions you can solve for F_1 and F_2,

$$F_1 = \frac{(3M + 2m)g}{4}, \quad \text{and} \quad F_2 = \frac{(M + 2m)g}{4}.$$

5. Angular momentum and torque depend on where you choose the origin of your coordinates. However, I shall now prove that for a body in equilibrium, the choice of origin does not matter. Let's start with the origin O and calculate the torque about O,

$$\vec{\tau}_O = \vec{\tau}_1 + \vec{\tau}_2 + \cdots + \vec{\tau}_N = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \cdots + \vec{r}_N \times \vec{F}_N$$

Now, if we take a second point P, then all distances will be measured from P and each

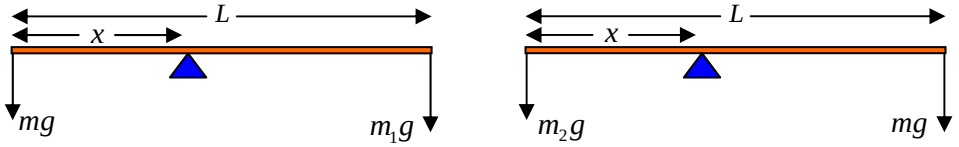
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vector will be shifted by an amount $\vec{r}_P$. Hence the torque about P is,

$$\begin{aligned}\vec{\tau}_P &= (\vec{r}_1 - \vec{r}_P)\times\vec{F}_1 + (\vec{r}_2 - \vec{r}_P)\times\vec{F}_2 + \cdots + (\vec{r}_N - \vec{r}_P)\times\vec{F}_N \\ &= \left[\vec{r}_1\times\vec{F}_1 + \vec{r}_2\times\vec{F}_2 + \cdots + \vec{r}_N\times\vec{F}_N\right] - \left[\vec{r}_P\times\vec{F}_1 + \vec{r}_P\times\vec{F}_2 + \cdots + \vec{r}_P\times\vec{F}_N\right] \\ &= \vec{\tau}_O - \left[\vec{r}_P\times\left(\vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N\right)\right] \\ &= \vec{\tau}_O - \left[\vec{r}_P\times\left(\sum\vec{F}_{ext}\right)\right]\end{aligned}$$

but $\sum\vec{F}_{ext} = 0,$ for a body in translational equilibrium $\therefore \quad \vec{\tau}_P = \vec{\tau}_O.$

6. Let us use the equilibrium conditions to do something of definite practical importance. Consider the balance below which is in equilibrium when two known weights are hung as shown. We want to know m in terms of m_1 and m_2.



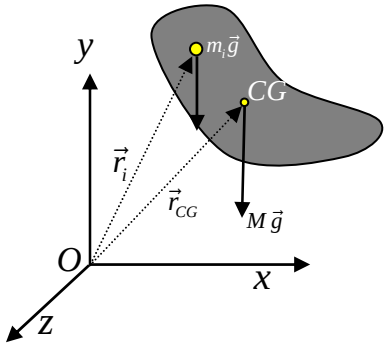
Taking the torques about the knife edge in the two cases, we have:

$$mgx = m_1 g(L - x) \text{ and } m_2 gx = mg(L - x)$$

$$\Rightarrow \frac{m}{m_2} = \frac{m_1}{m} \text{ or } m = \sqrt{m_1 m_2}.$$

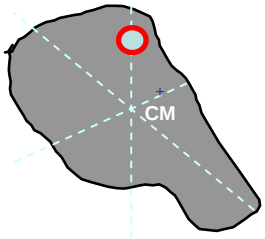
Remarkably, we do not need the values of x or L.

7. **Centre of Gravity.** The centre of gravity is the average location of the weight of an object. This is not quite the same as the centre of mass of a body (see lecture 12) but suppose the gravitational acceleration $\vec{g}$ has the same value at all points of a body. Then: 1) The weight is equal to $M\vec{g},$ and 2)the centre of gravity coincides with the centre of mass. Remember that weight is force, so the CG is really the centre of gravitational force acting on the body. The net force on the whole body = sum of forces over all individual particles, $\sum\vec{F} = \sum m_i\vec{g}$. If $\vec{g}$ has the same value at all points of the body, then $\sum\vec{F} = \vec{g}\sum m_i = M\vec{g}$. So the net torque about the origin O is $\sum\vec{\tau} = \sum(\vec{r}_i\times m_i\vec{g}) = \sum(m_i\vec{r}_i\times\vec{g})$ Hence, $\sum\vec{\tau} = M\vec{r}_{cm}\times\vec{g} = \vec{r}_{cm}\times M\vec{g}$. So the torque due to gravity about the centre of mass of a body (i.e. at $\vec{r}_{cm} = 0$) is zero !!



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8. In the demonstration I showed, you saw how to find the CG of an irregular object by simply suspending it on a pivot,

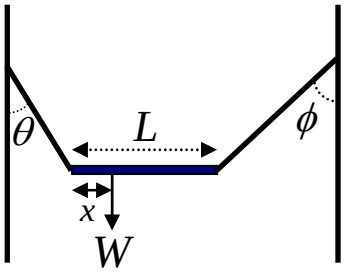


9. Let's solve a problem of static equilibrium: A non-uniform bar of weight W is suspended at rest in a horizontal position by two light cords. Find the distance *x* from the left-hand end to the center of gravity.

Solution : Call the tensions T_1 and T_2 . Put the forces in both directions equal to zero,

a) $T_2 \sin\phi - T_1 \sin\theta = 0$ (horizontal)

b) $T_2 \cos\phi + T_1 \cos\theta - W = 0$ (vertical) $\Rightarrow T_2 = \dfrac{W}{\sin(\theta+\phi)}$



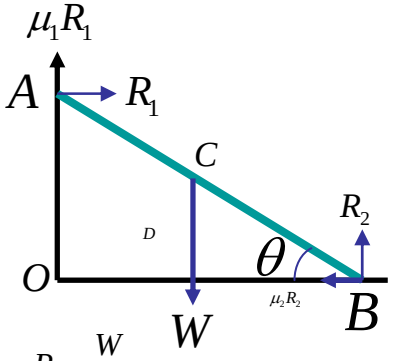
The torque about any point must vanish. Let us choose that point to be one end of the bar,

$$-Wx + (T_2 \cos\phi)L = 0 \Rightarrow x = \frac{(T_2 \cos\phi)L}{W} = \frac{L\cos\phi}{\sin(\theta+\phi)}$$

10.Here is another problem of the same kind: find the least angle θ at which the rod can lean to the horizontal without slipping.

Solution : Considering the translational equilibrium of the rod, $R_1 = \mu_2 R_2$ and $R_2 + \mu_1 R_1 = W$. This gives, $R_2 = \dfrac{W}{(1+\mu_1\mu_2)}$. Now consider rotational equilibrium about the point A: $R_2 \times OB = W \times OD + \mu_2 R_2 \times OA$

or, $R_2 \times AB\cos\theta = W \times \dfrac{AB\cos\theta}{2} + \mu_2 R_2 \times AB\sin\theta.$



This gives $\cos\theta\left(R_2 - \dfrac{W}{2}\right) = \mu_2 R_2 \sin\theta$ from which $\tan\theta = \dfrac{R_2 - \dfrac{W}{2}}{\mu_2 R_2}$ with $R_2 = \dfrac{W}{(1+\mu_1\mu_2)}$.

Using this value of R_2, we get $\tan\theta = \dfrac{1-\mu_1\mu_2}{2\mu_2}$.

10. **Types of Equilibrium.** In the lecture you heard about:

a)Stable equilibrium: object returns to its original position if displaced slightly.

b)Unstable equilibrium: object moves farther away from its original position if displaced slightly.

c)Neutral equilibrium: object stays in its new position if displaced slightly.

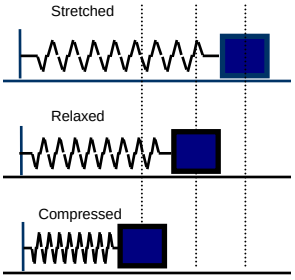
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Summary of Lecture 15 – OSCILLATIONS: I

1. An oscillation is any self-repeating motion. This motion is characterized by:
   a) The period T, which is the time for completing one full cycle.
   b) The frequency $f = 1/T$, which is the number of cycles per second. (Another frequently used symbol is ν).
   c) The amplitude A, which is the maximum displacement from equilibrium (or the size of the oscillation).

2. Why does a system oscillate? It does so because a force is always directed towards a central equilibrium position. In other words, the force always acts to return the object to its equilibrium position. So the object will oscillate around the equilibrium position. The restoring force depends on the displacement $F_{\text{restore}} = -k\,\Delta x,$ where Δx is the distance away from the equilibrium point, the negative sign shows that the force acts towards the equilibrium point, and k is a constant that gives the strength of the restoring force.

3. Let us understand these matters in the context of a spring tied to a mass that can move freely over a frictionless surface. The force $F(x) = -kx$ (or -$k\Delta x$ because the extension Δx will be called x for short). In the first diagram x is positive, x is negative in the second, and zero in the middle one. The energy stored in the spring, $U(x) = \frac{1}{2}kx^2,$ is positive in the first and third diagrams and zero in the middle one. Now we will use Newton's second law to derive a differential euation that describes the motion of the mass: From $F(x) = -kx$ and $ma = F$ it follows that $m\frac{d^2x}{dt^2} = -kx$, or $\frac{d^2x}{dt^2} + \omega^2 x = 0$ where $\omega^2 \equiv \frac{k}{m}$. This is the equation of motion of a simple harmonic oscillator (SHO) and is seen widely in many different branches of physics. Although we have derived it for the case of a mass and spring, it occurs again and again. The only difference is that $\omega,$ which is called the oscillator frequency, is defined differently depending on the situation.



4. In order to solve the SHO equation, we shall first learn how to differentiate some elementary trignometric functions. So let us first learn how to calculate $\frac{\mathrm{d}}{\mathrm{dt}}\cos\omega t$ starting from the basic definition of a derivative:

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Start : $x(t) = \cos \omega t$ and $x(t + \Delta t) = \cos \omega (t + \Delta t)$. Take the difference:

$$x(t + \Delta t) - x(t) = \cos \omega (t + \Delta t) - \cos \omega t$$
$$= -\sin \omega \Delta t \sin(\omega t + \omega \Delta t / 2)$$
$$\approx -\omega \Delta t \sin \omega t \quad \text{as } \Delta t \text{ becomes very small.}$$
$$\therefore \frac{d}{dt} \cos \omega t = -\omega \sin \omega t.$$

(Here you should know that $\sin \theta \approx \theta$ for small θ, easily proved by drawing triangles.)

You should also derive and remember a second important result:

$$\frac{d}{dt} \sin \omega t = \omega \cos \omega t \ .$$

(Here you should know that $\cos \theta \approx 1$ for small θ.)

5. What happens if you differentiate twice?

$$\frac{d^2}{dt^2}(\sin \omega t) = \omega \frac{d}{dt} \cos \omega t = -\ \omega^2 \sin \omega t$$
$$\frac{d^2}{dt^2}(\cos \omega t) = -\omega \frac{d}{dt} \sin \omega t = -\ \omega^2 \cos \omega t.$$

So twice differentiating either $\sin \omega t$ or $\cos \omega t$ gives the same function back!

6. Having done all the work above, now you can easily see that any function of the form $x(t) = a \cos \omega t + b \sin \omega t$ satisfies $\frac{d^2 x}{dt^2} = -\omega^2 x$. But what do ω, a, b represent ?

a) The significance of ω becomes clear if your replace t by $t + \frac{2\pi}{\omega}$ in either $\sin \omega t$ or $\cos \omega t$. You can see that $\cos \omega \left(t + \frac{2\pi}{\omega} \right) = \cos(\omega t + 2\pi) = \cos \omega t$. That is, the function merely repeats itself after a time $2\pi / \omega$. So $2\pi / \omega$ is really the period of the motion T, $T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}$. The frequency ν of the oscillator is the number of complete vibrations per unit time: $\nu = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ so $\omega = 2\pi\nu = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$.

Sometimes ω is also called the angular frequency. Note that $\dim[\omega] = T^{-1}$, from it is clear that the unit of ω is radian/second.

b) To understand what a and b mean let us note that from $x(t) = a \cos \omega t + b \sin \omega t$ it follows that $x(0) = a$ and that $\frac{\mathrm{d}}{\mathrm{dt}} x(t) = -\omega a \sin \omega t + \omega b \cos \omega t = \omega b$ (at $t = 0$). Thus, a is the initial position, and b is the initial velocity divided by ω.

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c) To understand what a and b mean let us note that from $x(t) = a\cos\omega t + b\sin\omega t$ it follows that $x(0) = a$ and that $\frac{\mathrm{d}}{\mathrm{dt}}x(t) = -\omega a\sin\omega t + \omega b\cos\omega t = \omega b$ (at $t = 0$). Thus, a is the initial position, and b is the initial velocity divided by ω.

d) The solution can also be written as: $x(t) = x_m\cos(\omega t + \phi)$. Since cos and sin never beome bigger than 1, or less than -1, it follows that $-x_m \le x \le +x_m$. For obvious reason x_m is called the amplitude of the motion. The frequency of the simple harmonic motion is independent of the amplitude of the motion.

e) The quantity $\theta = \omega t + \phi$ is called the phase of the motion. The constant ϕ is called the *phase constant.* A different value of ϕ just means that the origin of time has been chosen differently.

7. **Energy of simple harmonic motion**. Put $\phi = 0$ for convenience, and so imagine a mass whose position oscillates like $x = x_m\cos\omega t$. Let us first calculate the potential energy:

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kx_m^2\cos^2\omega t.$$

Now calculate the kinetic energy:

$$K = \frac{1}{2}m\mathrm{v}^2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = \frac{1}{2}m\omega^2x_m^2\sin^2\omega t = \frac{1}{2}kx_m^2\sin^2\omega t$$

The sum of potential + kinetic is:

$$E = K + U = \frac{1}{2}kx_m^2\cos^2\omega t + \frac{1}{2}kx_m^2\sin^2\omega t$$
$$= \frac{1}{2}kx_m^2\left(\cos^2\omega t + \sin^2\omega t\right) = \frac{1}{2}kx_m^2.$$

Note that this is independent of time and energy goes from kinetic to potential, then back to kinetic etc.

8. From the above, you can see that $\mathrm{v} = \frac{dx}{dt} = \pm\sqrt{\frac{k}{m}\left(x_m^2 - x^2\right)}$. From this it is clear that the speed is maximum at $x = 0$ and that the speed is zero at $x = \pm x_m$.

9. Putting two springs in parallel makes it harder to stretch them, and $k_{eff} = k_1 + k_2$. In series they are easier to stretch, and $k_{eff} = \left(\frac{k_1k_2}{k_1 + k_2}\right)$. So a mass will oscillate faster in the first case as compared to the second.

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