MIDTERM EXAMINATION
Spring 2010
MTH301- Calculus II (Session - 3)

Time: 60 min
Marks: 40

Student Info	
StudentID:	
Center:	OPKST
ExamDate:	6/3/2010 12:00:00 AM

For Teacher's Use Only									
Q No.	1	2	3	4	5	6	7	8	Total
Marks									
Q No.	9	10	11	12	13	14	15	16	
Marks									
Q No.	17	18	19	20	21	22	23	24	
Marks									
Q No.	25	26							
Marks									

Question No: 1 (Marks: 1) - Please choose one

Which of the following number is associated to each point on a co-ordinate line?

► An integer

► **A real number**

► A rational number

► A natural number

Question No: 2 (Marks: 1) - Please choose one

If $a>0$, then the parabola $y=ax^2+bx+c$ opens in which of the following direction?

► **Positive x - direction**

► Negative x - direction

► Positive y - direction

► Negative y - direction

Question No: 3 (Marks: 1) - Please choose one

Rectangular co-ordinate of a point is $(1, \sqrt{3}, -2)$. What is its spherical co-ordinate?

$\left(2\sqrt{2}, \frac{\pi}{3}, \frac{3\pi}{2}\right)$

. ►

$\left(2\sqrt{2}, \frac{\pi}{2}, \frac{3\pi}{4}\right)$

. ►

$\left(2\sqrt{2}, \frac{\pi}{3}, \frac{3\pi}{4}\right)$

. ►

$\left(\sqrt{2}, \frac{\pi}{3}, \frac{3\pi}{4}\right)$

. ►

Question No: 4 (Marks: 1) - Please choose one

If a function is not defined at some point, then its limit ---------- exist at that point.

. ► Always

. ► Never

. ▶ May

Question No: 5 (Marks: 1) - Please choose one

Suppose $f(x,\ y) = x^3 e^{xy}$. Which one of the statements is correct?

. ▶ $\frac{\partial f}{\partial y} = 3x^3 e^{xy}$

. ▶ $\frac{\partial f}{\partial y} = x^3 e^{xy}$

. ▶ $\frac{\partial f}{\partial y} = x^4 e^{xy}$

. ▶ $\frac{\partial f}{\partial y} = x^3 y e^{xy}$

Question No: 6 (Marks: 1) - Please choose one

If $f(x,\ y) = x^2 y - y^3 + \ln x$

$\frac{\partial^2 f}{\partial x^2}$

then =

. ▶ $2xy + \frac{1}{x^2}$

. ▶ $2y + \frac{1}{x^2}$

. ▶ $2xy - \frac{1}{x^2}$

. ▶ $2y - \frac{1}{x^2}$

Question No: 7 (Marks: 1) - Please choose one

Suppose $f(x,\ y) = xy - 2y^2 \ \ where\ \ x = 3t + 1 \ \ and \ \ y = 2t$. Which one of the following is true?

$\frac{df}{dt} = -4t + 2$

$\frac{df}{dt} = -16t - t$

$\frac{df}{dt} = 18t + 2$

$\frac{df}{dt} = -10t^2 + 8t + 1$

Question No: 8 (Marks: 1) - Please choose one

Is the function $f(x,\ y)$ continuous at origin? If not, why?

$$f(x,\ y) = \begin{cases} 0 & If\ \ x \geq 0\ and\ \ y \geq 0 \\ 1 & Otherwise \end{cases}$$

- $f(x,\ y)$ is continuous at origin
- $f(0,\ 0)$ is not defined
- $f(0,\ 0)$ is defined but $\lim_{(x,y)\to(0,\ 0)} f(x,\ y)$ does not exist
- $f(0,\ 0)$ is defined and $\lim_{(x,y)\to(0,\ 0)} f(x,\ y)$ exists but these two numbers are not equal.

Question No: 9 (Marks: 1) - Please choose one

What is the relation between the direction of gradient at any point on the surface to the tangent plane at that point ?

- parallel
- **perpendicular**
- opposite direction
- No relation between them.

Question No: 10 (Marks: 1) - Please choose one

Two surfaces are said to intersect orthogonally if their normals at every point common to them are ----------

- perpendicular
- parallel
- **in opposite direction**

Question No: 11 (Marks: 1) - Please choose one

By Extreme Value Theorem, if a function $f(x, y)$ is continuous on a closed and bounded set R, then $f(x, y)$ has both ---------- on R.

- ► **Absolute maximum and absolute minimum value**
- ► Relative maximum and relative minimum value

Question No: 12 (Marks: 1) - Please choose one

Let the function $f(x, y)$ has continuous second-order partial derivatives $(f_{xx}, f_{yy} \text{ and } f_{xy})$ in some circle centered at a critical point (x_0, y_0) and let $D = f_{xx}(x_0, y_0)\, f_{yy}(x_0, y_0) - f_{xy}{}^2(x_0, y_0)$

If $D > 0$ and $f_{xx}(x_0, y_0) < 0$ then f has ----------

- ► **Relative maximum at** (x_0, y_0)
- ► Relative minimum at (x_0, y_0)
- ► Saddle point at (x_0, y_0)
- ► No conclusion can be drawn.

Question No: 13 (Marks: 1) - Please choose one

Let the function $f(x, y)$ has continuous second-order partial derivatives $(f_{xx}, f_{yy} \text{ and } f_{xy})$ in some circle centered at a critical point (x_0, y_0) and let $D = f_{xx}(x_0, y_0)\, f_{yy}(x_0, y_0) - f_{xy}{}^2(x_0, y_0)$

If $D = 0$ then ----------

- ► f has relative maximum at (x_0, y_0)
- ► f has relative minimum at (x_0, y_0)
- ► f has saddle point at (x_0, y_0)
- ► **No conclusion can be drawn.**

Question No: 14 (Marks: 1) - Please choose one

The function $f(x, y) = \sqrt{y - x}$ is continuous in the region --------- and discontinuous elsewhere.

- $x \neq y$
- $x \leq y$
- $x > y$

Question No: 15 (Marks: 1) - Please choose one

Plane is an example of ----------

- Curve
- **Surface**
- Sphere
- Cone

Question No: 16 (Marks: 1) - Please choose one

If $R = R_1 \cup R_2$, where R_1 and R_2 are no overlapping regions then $\iint_{R_1} f(x,y)dA + \iint_{R_2} f(x,y)dA =$

- $\iint_R f(x,y)dA$
- $\iint_{R_1} f(x,y)dA \cup \iint_{R_2} f(x,y)dA$
- $\iint_R f(x,y)dV$
- $\iint_{R_1} f(x,y)dA \cap \iint_{R_2} f(x,y)dA$

Question No: 17 (Marks: 1) - Please choose one

If $R = \{(x, y) / 0 \le x \le 2 \, and \, 1 \le y \le 4\}, then$

$\iint_R (6x^2 + 4xy^3)dA =$

$\int_1^4 \int_0^2 (6x^2 + 4xy^3)dydx$

▶

$\int_0^2 \int_1^4 (6x^2 + 4xy^3)dxdy$

▶

$\int_1^4 \int_0^2 (6x^2 + 4xy^3)dxdy$

▶

$\int_2^4 \int_0^1 (6x^2 + 4xy^3)dxdy$

▶

Question No: 18 (Marks: 1) - Please choose one

If $R = \{(x, y) / 0 \le x \le 2 \, and \, -1 \le y \le 1\}, then$

$\iint_R (x + 2y^2)dA =$

$\int_{-1}^1 \int_0^2 (x + 2y^2)dydx$

▶

$\int_0^2 \int_1^{-1} (x + 2y^2)dxdy$

▶

$\int_{-1}^1 \int_0^2 (x + 2y^2)dxdy$

▶

$\int_1^2 \int_{-1}^0 (x + 2y^2)dxdy$

▶

Question No: 19 (Marks: 1) - Please choose one

If $R=\{(x,y)/0\le x\le 2 \text{ and } 0\le y\le 3\}$, *then*

$\iint_R (1-ye^{xy})dA=$

$\int_0^2 \int_0^3 (1-ye^{xy})dydx$

▶

$\int_0^2 \int_0^3 (1-ye^{xy})dxdy$

▶

$\int_2^3 \int_0^0 (1-ye^{xy})dxdy$

▶

$\int_0^2 \int_2^3 (4xe^{2y})dydx$

▶

Question No: 20 (Marks: 1) - Please choose one

If $R=\{(x,y)/0\le x\le 4 \text{ and } 0\le y\le 9\}$, *then*

$\iint_R (3x-4x\sqrt{xy})dA=$

$\int_0^9 \int_0^4 (3x-4x\sqrt{xy})dydx$

▶

$\int_0^4 \int_4^9 (3x-4x\sqrt{xy})dxdy$

▶

$\int_4^9 \int_0^0 (3x-4x\sqrt{xy})dxdy$

▶

$$\int_0^4 \int_0^9 (3x - 4x\sqrt{xy})dydx$$

▶

Question No: 21 (Marks: 2)

Evaluate the following double integral.

$\iint \left(2xy + y^3\right) dx\ dy$

Question No: 22 (Marks: 2)

$Let\ f(x,\ y) = 2 + x^2 + \frac{y^2}{4}$

$Find\ the\ gradient\ of\ f$

Question No: 23 (Marks: 3)

Evaluate the following double integral.

$\iint \left(3 + 2x - 3y^2\right) dx\ dy$

Question No: 24 (Marks: 3)

$Let\ f(x,\ y,\ z) = yz^3 - 2x^2$

$Find\ the\ gradient\ of\ f.$

Question No: 25 (Marks: 5)

Find Equation of a Tangent plane to the surface $f(x,\ y,\ z) = x^2 + 3y + z^3 - 9$ at the point $(2,\ -1,\ 2)$

Question No: 26 (Marks: 5)

Evaluate the iterated integral

$\int_2^4 \int_{\frac{x}{2}}^{\sqrt{x}} (xy)\, dy\, dx$