MTH: 603 Lec# 09 Date BY: M. Khalil

VU

Gauss Elimination Method:-

(Direct method)

"Consider the system of n-linear Algebraic Equation's with n-un. Knowns says $x_1, x_2, \dots, x_n$"

$$\left.\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \dots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \dots + a_{2n}x_n &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + \dots + a_{3n}x_n &= b_3 \\
\text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\
\text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\
\text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\
a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \dots + a_{nn}x_n &= b_n
\end{aligned}\right\}$$

Write augumented matrix Ab

$$Ab = \left[\begin{array}{ccccc}
a_{11} & a_{12} & a_{13} & \dots & a_{1n} \; b_1 \\
a_{21} & a_{22} & a_{23} & \dots & a_{2n} \; b_2 \\
\text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\
\text{---} & \text{---} & \text{---} & \text{---} & \text{---} \\
a_{n1} & a_{n2} & a_{n3} & \dots & a_{nn} \; b_n
\end{array}\right]$$

Reduce Ab into echelon form by using Row & Column operation.

$$Ab = \begin{bmatrix} 1 & t_{12} & t_{13} & \cdots & t_{1n} & K_1 \\ 0 & 1 & t_{23} & \cdots & t_{2n} & K_2 \\ 0 & 0 & 1 & \cdots & t_{3n} & K_3 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & 1 & K_n \end{bmatrix}$$

" In this way we obtain the system "

$$x_1 + t_{12}x_2 + t_{13}x_3 + \cdots + t_{1n}x_n = K_1$$

$$0 + x_2 + t_{23}x_3 + \cdots + t_{2n}x_n = K_2$$

$$0 + 0 + x_3 + \cdots + t_{3n}x_n = K_3$$

$$\cdots \quad \cdots \quad \cdots \quad \cdots \quad \cdots$$

$$x_n = K_n$$

The remaining un-known are evaluated by Backward substitutions.

Example:- Solve the following system by Gauss elimination method.

$$2x_1 + 2x_2 + 4x_3 = 18$$

$$x_1 + 3x_2 + 2x_3 = 13$$

$$3x_1 + x_2 + 3x_3 = 14$$

Sol:

Date:

$$Ab = \begin{bmatrix} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix}$$

Reduce Ab into echelon form

$$= R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix} \text{ by } \frac{1}{2} R_1$$

$$R_1 = \frac{1}{2}\cdot 2 \quad \frac{1}{2}\cdot 2 \quad \frac{1}{2}\cdot 4 \quad 18\cdot\frac{1}{2}$$
$$1 \quad 1 \quad 2 \quad 9$$

$$= R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -2 & -3 & -13 \end{bmatrix} \begin{matrix} \text{By} \\ R_2 - R_1 \\ R_3 + (-3) R_1 \end{matrix}$$

$R_2 = 1 \quad 3 \quad 2 \quad 13$

$-R_1 = -1 \quad -1 \quad -2 \quad -9$

$R_2 - R_1 = 0 \quad 2 \quad 0 \quad 4$

$R_3 = 3 \quad 1 \quad 3 \quad 14$

$-3R_1 = -3 \quad -3 \quad -6 \quad -27$

$R_3 - 3R_1 = 0 \quad -2 \quad -3 \quad -13$

Date:

$$\underset{\sim}{=} R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -3 & -13 \end{bmatrix} \quad \text{By } \frac{1}{2} R_2$$

$\frac{1}{2}R_2 = 0 \cdot \frac{1}{2} \quad 2 \cdot \frac{1}{2} \quad 0 \cdot \frac{1}{2} \quad 4 \cdot \frac{1}{2}$

$\frac{1}{2}R_2 = 0 \quad 1 \quad 0 \quad 2$

$$\underset{\sim}{=} R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -3 & -9 \end{bmatrix} \quad \text{By } R_3 + 2R_2$$

$R_3 = 0 \quad -2 \quad -3 \quad -13$

$2R_2 = 0 \quad 2 \quad 0 \quad 4$

$R_3 + 2R_2 = 0 \quad 0 \quad -3 \quad -9$

$$\underset{\sim}{=} R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad \text{By } -\frac{1}{3} R_3$$

$\Rightarrow x_1 + x_2 + 2x_3 = 9 \quad \text{— (i)}$

$x_2 = 2 \quad \text{— (ii)}$

Date:

$\Rightarrow \quad x_3 = 3 \quad$ — (i)

Put the value of x_2 & x_3 in Eq (i)

$$x_1 + 2 + 8 = 9$$

$$x_1 = 9 - 8 \Rightarrow x_1 = 1$$

Thus,

$$x_1 = 1, \quad x_2 = 2, \quad x_3 = 3$$

:- MTH : 603 :-

"Lec # 10"

BY

M. KHALIL

GAUSS JORDAN METHOD

Consider the system of Linear Equations

$$Ax = B$$

Write the augumented matrix

$$Ab$$

Convert Ab into reduced Echelon form

$$\left[\begin{array}{cccc|c} 1 & 0 & \cdots & 0 & K_1 \\ 0 & 1 & \cdots & 0 & K_2 \\ 0 & 0 & 1 & \cdots & K_3 \\ 0 & 0 & 0 & \cdots 1 & K_n \end{array}\right]$$

Which gives Direct solution:-

$x_1 = K_1$, $x_2 = K_2$, $x_3 = K_3$, $x_n = K_n$

Example:- Solve the following system by Gauss Jordan method

$$2x_1 + 2x_2 + 4x_3 = 18$$
$$x_1 + 3x_2 + 2x_3 = 13$$
$$3x_1 + x_2 + 3x_3 = 14$$

Sol: Here

$$Ab = \begin{bmatrix} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix}$$

$$\sim R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix} \begin{matrix} \text{By} \\ \frac{1}{2}R_1 \end{matrix}$$

$$\sim R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -2 & -3 & -13 \end{bmatrix} \begin{matrix} R_2 - R_1 \\ R_3 - 3R_1 \end{matrix}$$

$$\sim R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -3 & -13 \end{bmatrix} \frac{1}{2}R_2$$

$$= \underset{\sim}{R} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -3 & -9 \end{bmatrix} \quad \text{By } R_3 + 2R_2$$

$$= \underset{\sim}{R} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad \text{By } -\frac{1}{3} R_3$$

$$= \underset{\sim}{R} \begin{bmatrix} 1 & 1 & 2 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad \text{By } R_1 - 2R_3$$

$$= \underset{\sim}{R} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad \text{By } R_1 - R_2, \; R_1 - R_3$$

$$\Rightarrow x_1 = -2, \; x_2 = 2, \; x_3 = 3$$