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| | FINALTERM EXAMINATION<br>FALL 2006<br>MTH401 - DIFFERENTIAL EQUATIONS (Session - 1 ) | Marks: 53<br>Time: 120min |
|---|---|---|

StudentID/LoginID: _____

Student Name: _____

Center Name/Code: _____

Exam Date: Monday, February 12, 2007

**Please read the following instructions carefully before attempting any of the questions:**

**1**. Attempt all questions. Marks are written adjacent to each question.

**2**. Do not ask any question about the contents of this examination from anyone.

***a***. If you think that there is something wrong with any of the question, attempt it to the best of your understanding.

***b***. If you believe that some essential piece of information is missing, make an appropriate assumption and use it to solve the problem.

***c***. Write all steps, missing steps may lead to deduction of marks.

**3.** Calculator is allowed.

****WARNING: Please note that Virtual University takes serious note of unfair means. Anyone found involved in cheating will get an `F` grade in this course.**

| For Teacher's use only | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Question | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | Total |
| Marks | | | | | | | | | | | |
| Question | 11 | | | | | | | | | | |
| Marks | | | | | | | | | | | |

Question No: 1 ( Marks: 2 ) - Please choose one

If the variation of the path of the curves can be describrd by the concept of differential equations then which of the following differential equation describe the path for $y-axis$ .

- $\frac{dy}{dx}=1$
- $\frac{dy}{dx}=0$
- $\frac{dy}{dx}=-1$
- $\frac{dy}{dx}=\infty$

Question No: 2 ( Marks: 2 ) - Please choose one

Suggestive form of the constant input function for the non homogeneous differential equation under the method entitled as "**Method of the undetermined coefficient**" is

- $f(x)=e^{x}$
- $f(x)=a$
- $f(x)=e^{ax}\left(ACosx+BSinx\right)$
- Suggestive form is impossible.

Question No: 3 ( Marks: 2 ) - Please choose one

Which of the following function is linearly dependant to the exponential function $e^x$ ?

- $-e^x$
- $e^{-x}$
- $xe^x$
- $-xe^{-x}$

Question No: 4 ( Marks: 2 ) - Please choose one

Eigen values for the system of the differential equations $X' = AX$ are evaluated for the

- Solution vector $X$
- Coefficient matrix $A$
- Differentiated solution vector $X'$
- Transpose of the Coefficient matrix $A$

Question No: 5 ( Marks: 2 ) - Please choose one

Fundamental set of the solution vectors $X_1, X_2, \ldots\ldots, X_n$ for any system of the differential equations are obtained by

- $\{X\} = \{c_1 X_1, c_2 X_2, \ldots\ldots, c_n X_n\}$

  Developing the singleton set of the linear combinitions of the solution vectors.

- Taking derivative of the each solution vector and forming the set $\{X_1', X_2', \ldots\ldots, X_n'\}$

▶ Taking Integral of the each solution vector and forming the set

$$\left\{\int X_1 dx, \int X_2 dx, \cdots\cdots, \int X_n dx\right\}$$

▶ Just verifying their linear independance and establishing the set

$$\left\{X_1, X_2, \cdots\cdots, X_n\right\}$$

.

Question No: 6 ( Marks: 5 )

$$y = \frac{a}{x}$$

For the family with parameter '**a**', of rectangular hyperbola , find its corresponding orthogonal trajectorie and induce the parameter of the new family.

Question No: 7 ( Marks: 5 )

**(a)** Justify whether $D^2 + 9$ annihilates the function $f(x) = \sin 3x$ **(3)**

**(b)** Evaluate Wronskian of functions $Sin|x|$ and $|x|$ for $x < 0$ provided that $|x| = -x$ for $x < 0$ **(2)**

Question No: 8 ( Marks: 8 )

$$\sum_{k=0}^{\infty} \frac{2^k}{k} x^k$$

**(a)** Determine the interval of convergence of power series **(5)**

$$\sum_{n=0}^{\infty} nc_n x^n \quad and \quad \sum_{n=1}^{\infty} nc_{n-1} x^{n+1}$$

**(b)** If are power series solutions for a differential equation then find their sum by introducing a single summation symbol. **(3)**

Question No: 9 ( Marks: 10 )

**(a)** Develop a recurrence relation for the differential equation $y'' - xy = 0$ by applying power series method. **(8)**

**(b)** Discuss shortly the linear independence of the power series solutions

$$y_1 = A\sqrt{x}\left[2+\sum_{k=1}^{\infty}\frac{x^k}{(k+1)!5.8.11.....(3k+2)}\right]$$

*and*

$$y_2 = Bx\left[3+\sum_{k=1}^{\infty}\frac{x^k}{k!1.4.7.....(3k-2)}\right]$$

$$y''+\frac{1}{3x}(y'-y)=0$$

of the differential equation
**(2)**

Question No: 10 ( Marks: 10 )

$$y''-\frac{1}{x}y'+\frac{1}{(x-1)^3}y=0$$

**(a)** Determine the singular points of differential equation .Also classify each singular point as regular or irregular.
**(5)**

$$P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}\left(x^2-1\right)^n$$

**(b)**Using Rodrigues Formula; to generate fourth Legendre's polynomial $P_3(x)$ . **(5)**

Question No: 11 ( Marks: 5 )

$$\frac{dx}{dt}=3x-4y$$
$$\frac{dy}{dt}=4x-7y$$

$$X=\begin{bmatrix}1\\2\end{bmatrix}e^{-5t}$$

Verify that the vector is a solution of the system **;** .