z no 2 2022 solution by taleemi markaz part 1

MTH622: Quiz 2

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Question # 3 of 10

SOLVED BY: WAQAR HUSSAIN

If a particle of mass 4 moving with velocity $4\hat{i}$ collides another particle of mass m with velocity $3\hat{i}$. If after collision the velocity of first particle is $5\hat{i}$ and that of second particle is $2\hat{i}$. Find m

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Select the correct option

- 3
- 4
-
-

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Q.No.1: Write necessary and sufficient conditions for a Conservative Force Field.

(Module 97, Pg.34)

ANSWER: Conservative force fields conserve the following properties:

1- A force field F is conservative if and only if there exists a continuously differentiable scalar field V such that $\vec{F} = -\nabla V$ or, equivalently, if and only if

$$curl\ \vec{F} = \nabla \times \vec{F} = 0$$

identically

2- A continuously differentiable force field F is conservative if and only if for any closed non-intersecting curve C (simple closed curve)

$$W = \oint_C \vec{F}.d\vec{r} = 0$$

i.e. the total work done in moving a particle around any closed path is zero.

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Q.No.2: Write different examples of Conservative Forces. (Module 97, Pg.35)

ANSWER: Examples of Conservative Forces are given below:

1- Gravitational force is an example of a conservative force.

2- Elastic spring force is example of conservative force.

3- The work done of a particle moving along a closed path is zero and the force which causes such motion is conservative.

Q.No.3: State the Chasles' Theorem. (Module 109, Pg.23)

ANSWER: Theorem Statement:

Chasle's theorem states that the most general rigid body displacement can be produced by a translation along a line (called its screw axis) followed (or preceded) by a rotation about that line.

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Q.No.4: Write down the mathematical expression for angular momentum of a single particle. (Module 135, Pg.64)

ANSWER: The angular momentum of a single particle is defined as the cross product of linear momentum and position vector of concerned particle.
Mathematically $L = r \times mv$

Q.No.5: State Euler's Theorem. (Module 108, Pg.21)

ANSWER: Theorem Statement:
"A rotation of a rigid body about a fixed point of the body is equivalent to a rotation about a line which passes through the point."

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Q.No.6: Write down examples of Curvilinear Motion. (Module 112, Pg.27)

ANSWER: Examples of curvilinear motion are:
1- A stone thrown into the air at an angle.
2- A car driving along a curved road.
3- Throwing paper airplanes or paper darts is an example of curvilinear motion.

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Q.No.7: Show that $\vec{F} = xy\hat{i} + 2z\hat{k}$ is non-conservative force. **(Module 101, Pg.10)**

SOLUTION:

$$\vec{F} = xy\hat{i} + 2z\hat{k}$$

To verify whether the force field F is conservative or not, we will check whether $\nabla \times \vec{F} = 0$ or not.

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 0 & 2z \end{vmatrix}$$

$$= \hat{i}\left|\frac{\partial}{\partial y}(2z) - \frac{\partial}{\partial z}(0)\right| - \hat{j}\left|\frac{\partial}{\partial x}(2z) - \frac{\partial}{\partial z}(xy)\right| + \hat{k}\left|\frac{\partial}{\partial x}(0) - \frac{\partial}{\partial y}(xy)\right|$$

$$= (0 - 0)\hat{i}$$

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$$= (0-0)\hat{i} - (0-0)\hat{j} + (0-x)\hat{k}$$

$$= -x\hat{k}$$

As we obtain that $\nabla \times \vec{F} \neq 0$ so we Conclude that $\vec{F}$ is non-conservative.

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Q.No.8: If $r = a\cos \omega t\, i + b\sin \omega t\, j$ and $p = -am\omega \sin \omega t\, i + bm\omega \cos \omega t\, j$ Then find the angular momentum. (Module 120, Pg.39)

SOLUTION:

$$r = a\cos \omega t\, i + b\sin \omega t\, j$$

$$p = -am\omega \sin \omega t\, i + bm\omega \cos \omega t\, j$$

As we know that

$$r \times p = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a\cos \omega t & b\sin \omega t & 0 \\ -am\omega \sin \omega t & bm\omega \cos \omega t & 0 \end{vmatrix}$$

$$= \left(abm\omega \cos^2 \omega t + abm\omega \sin^2 \omega t\right)\hat{k}$$

$$= abm\omega \left(\cos^2 \omega t + \sin^2 \omega t\right)\hat{k}$$

$$= abm\omega (1)\hat{k}$$

$$= abm\omega \hat{k}$$

$$\tau = \frac{d\Omega}{dt} = \frac{d}{dt}(r \times p) = 0$$

We know that $\Omega = r \times p$

$$\Omega = abm\omega\, \hat{k}$$

It is the required angular momentum.

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Q.No.9: Particles of masses $2m, 3m$ and $4m$ are held in a rigid light framework at points $(1,0,1), (1,1,0)$ and $(0,1,1)$. Calculate moment of inertia of x-axis.

(Module 160, Pg.120)

SOLUTION:

Given that: $m_1 = 2, \quad m_2 = 3, \quad m_3 = 4$

Points are $P_1(1,0,1), \quad P_2(1,1,0), \quad P_3(0,1,1)$

Moment of Inertia about x-axis is

$$I_{xx} = \sum_i m_i(y_i^2 + z_i^2)$$

$$I_{xx} = m_1(y_1^2 + z_1^2) + m_2(y_2^2 + z_2^2) + m_3(y_3^2 + z_3^2)$$

$$I_{xx} = 3(0+1) + 5(1+0) + 7(1+0)$$

$$I_{xx} = 3 + 5 + 7 \qquad I_{xx} = 15$$

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Q.No.1: What is the impulse developed by a mass of $100\ gm$ which changes its velocity from $i + j + k\ m/sec$ to $2i + 2j + 3k\ m/sec$? (Module 118, Pg.37)

SOLUTION:

Mass $m = 100\ gm$

Initial velocity
$v_1 = i + j + k$

Final velocity
$v_2 = 2i + 2j + 3k$

Impulse $= m(v_2 - v_1)$

$= 100\big((2i + 2j + 3k) - (i + j + k)\big)$

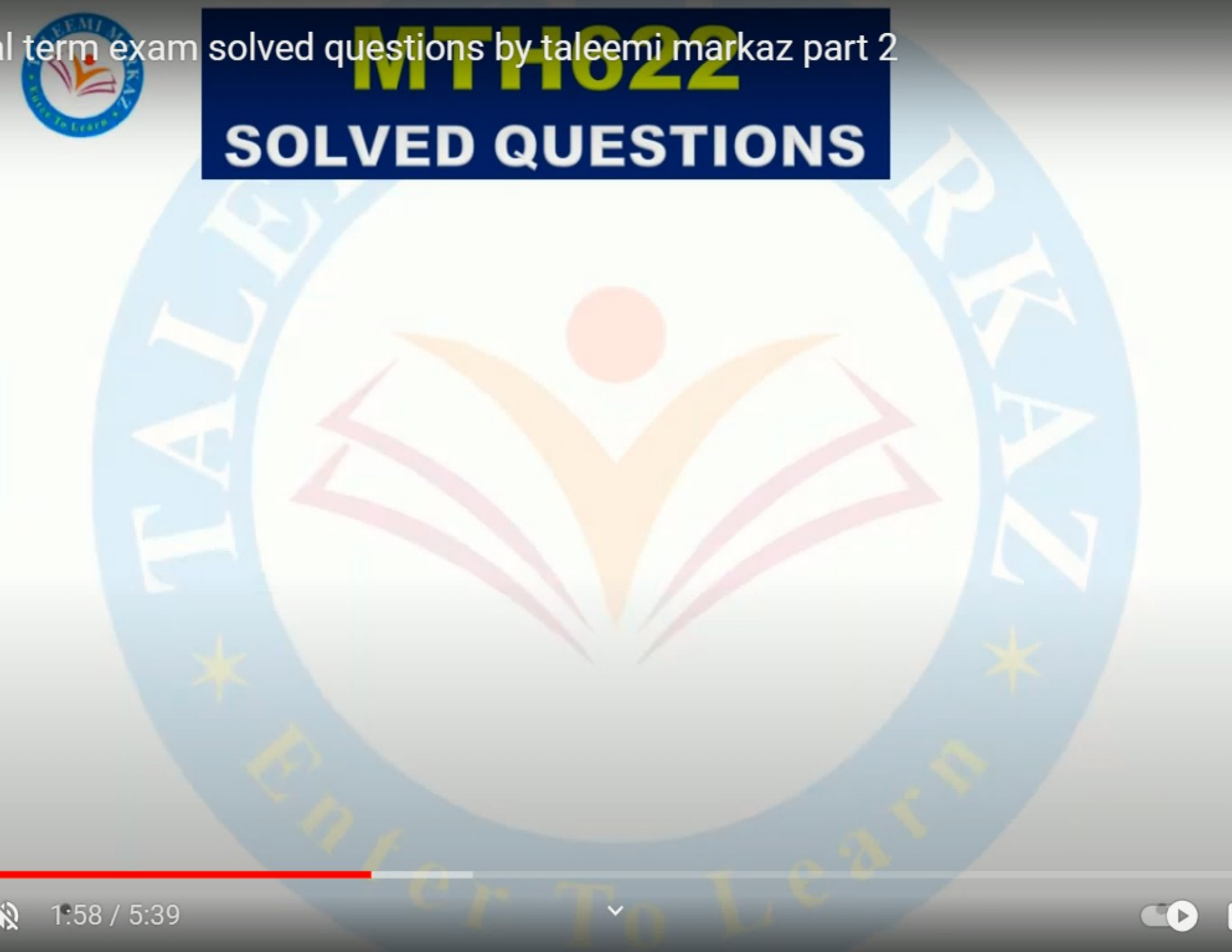
$= 100(2i + 2j + 3k - i - j - k)$

$= 100(i + j + 2k)$

Impulse Magnitude
$= 100\sqrt{(1)^2 + (1)^2 + (2)^2}$

Impulse Magnitude
$= 100\sqrt{1 + 1 + 4}$

Impulse Magnitude
$= 100\sqrt{6}$


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Q.No.2: If moment of Inertia of a uniform circular cylinder of length b and radius a about x-axis through the center and perpendicular to the central axis is:

$$I_x = m\left(\frac{1}{4}a^2 + \frac{1}{12}b^2\right)$$

Find the moment of Inertia about y-axis? (Module 168, Pg.138)

SOLUTION:

Moment of Inertia about x–axis $I_x = m\left(\frac{1}{4}a^2 + \frac{1}{12}b^2\right)$

Due to symmetry we have $I_x = I_y$

Moment of Inertia about y-axis will be $I_y = m\left(\frac{1}{4}a^2 + \frac{1}{12}b^2\right)$

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Q.No.3: Write the expression for total energy of a particle executing Simple Harmonic Motion. (Module 103, Pg.13)

ANSWER: If T is the kinetic energy and V is the potential energy then total energy will be $E = T + V$ where

$$K.E = T = \frac{1}{2}mv^2$$

and

$$V = \frac{1}{2}kx^2$$

Then the total energy of S.H.M will be

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

Q.No.4: Consider a small element of a uniform elliptic plate has moment of inertia $I = \delta m y^2$. Write down the expression for moment of inertia of whole plate. (Note: Do not solve the expression). (Module 172, Pg.146)

ANSWER: The moment of inertia of this element along x-axis will be equal to $I = \delta m y^2$. Then the moment of inertia of whole plate will be

$$I_{xx} = \int (\delta m) y^2 = \int_{plate} \rho ds y^2$$

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Q.No.5: To find the inertia matrix of uniform solid cuboid (parallelepiped) at one of its corner. If $I_{xx} = \frac{M}{3}(b^2 + c^2)$, then find I_{yy} and I_{zz}. **(Module 157, Pg.110)**

SOLUTION:

$$I_{xx} = \frac{M}{3}(b^2 + c^2)$$

Similarly, due to symmetry, we can write

$$I_{yy} = \frac{M}{3}(a^2 + c^2)$$

$$I_{zz} = \frac{M}{3}(a^2 + b^2)$$

Q.No.6: What is the degrees of freedom of the system of rigid bodies. **(Module 109, 122, Pg.21,42)**

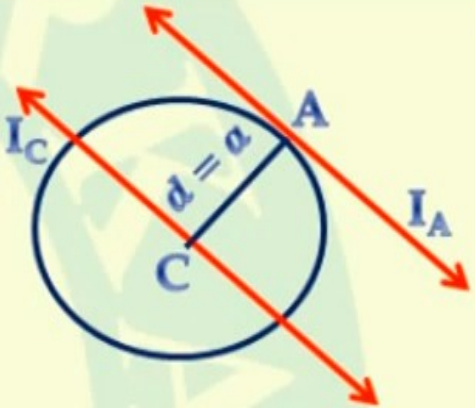
ANSWER: "The number of coordinates required to specify the position of a system of one or more particles is called the number of degrees of freedom of the system."

A rigid body has six degrees of freedom.

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Q.No.7: Use the parallel axis theorem to find the moment of inertia of a solid circular cylinder about a line on the surface of the cylinder and parallel to axis of cylinder.
(Module 120, Pg.39)



SOLUTION: By parallel axis theorem

$$I_A = I_C + Ma^2 \quad \ldots\ldots (1)$$

Since I_C which is the moment of inertia of a solid circular cylinder about an axis passing from the center of mass is defined by

$$I_C = \frac{1}{2}Ma^2 \quad \ldots\ldots (2)$$

By substituting equation (2) in equation (1), we have

$$I_A = \frac{1}{2}Ma^2 + Ma^2$$

Taking Ma^2 common, we get

$$I_A = Ma^2\left(\frac{1}{2} + 1\right)$$

$$I_A = \frac{3}{2}Ma^2$$

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Question # 1 of 10

SOLVED BY: WAQAR HUSSAIN

If the axis of rotation passes through the center of mass of the rigid body then it is called

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Select the correct option

- revolution
- spin
-
-

If the axis of rotation passes through the center of mass of the rigid body then body is said to be spin or rotate upon itself.

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Question # 1 of 10

SOLVED BY: WAQAR HUSSAIN

If the axis of rotation passes through the center of mass of the rigid body then it is called

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Select the correct option

- revolution
- spin

(Module 122, Pg.42)

-
-

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SOLVED BY: WAQAR HUSSAIN

If an object explodes, the center of mass will

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Select the correct option

- ○ cor
- ○ cor
- ○
- ○

If an object explodes, the different pieces of the object will follow seemingly independent paths after the explosion. The center of mass, however, will keep doing what it was doing before the explosion. This is because an explosion involves only internal forces.

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If an object explodes, the center of mass will

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Select the correct option

- continue its previous motion (Module 124, Pg.45)
- come into rest
-
-

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