Practice Question with Answer for Lecture # 2

Question # 1

Find the transpose of matrix $A=\begin{pmatrix}2 & 1\\ 3 & 1\end{pmatrix}$?

Solution:

$$A^t=\begin{pmatrix}2 & 3\\ 1 & 1\end{pmatrix}$$

Interchange the rows with columns.

Question # 2

What is the order of given matrix $\begin{bmatrix}5 & 2 & 1 & 0\\ 1 & 1 & 1 & 1\end{bmatrix}$?

Solution: Order is 2×4

Question # 3

Write the following single column matrix as the sum of three column vectors**:**

$$\begin{pmatrix}x^2+x\\ 3x+1\\ 9x^2+e^t\end{pmatrix}$$

Solution:

$$\begin{pmatrix}x^2+x\\ 3x+1\\ 9x^2+e^t\end{pmatrix}\begin{pmatrix}x^2\\ 0\\ 9x^2\end{pmatrix}+\begin{pmatrix}x\\ 3x\\ 0\end{pmatrix}+\begin{pmatrix}0\\ 1\\ e^t\end{pmatrix}=\begin{pmatrix}1\\ 0\\ 9\end{pmatrix}x^2+\begin{pmatrix}1\\ 3\\ 0\end{pmatrix}x+\begin{pmatrix}0\\ 1\\ e^t\end{pmatrix}$$

Question # 4

Find the derivative of the matrix $X(t)=\begin{pmatrix} t^2 \\ \sin t \end{pmatrix}$.

Solution:

$$X'(t)=\begin{pmatrix} 2t \\ \cos t \end{pmatrix}$$

Question # 5

Find, if possible, the multiplicative inverse of the matrix $A=\begin{pmatrix} 3 & 4 \\ 1 & 7 \end{pmatrix}$

Solution:

$$|A|=\begin{vmatrix} 3 & 4 \\ 1 & 7 \end{vmatrix}=21-4=17\neq 0$$

So inverse of A exists.

$$A^{-1}=\frac{1}{|A|}Adj(A)$$

$$Adj(A)=\begin{pmatrix} 7 & -4 \\ -1 & 3 \end{pmatrix}$$

$$A^{-1}=\frac{1}{|A|}Adj(A)=\frac{\begin{pmatrix} 7 & -4 \\ -1 & 3 \end{pmatrix}}{17}=\begin{pmatrix} \frac{7}{17} & \frac{-4}{17} \\ \frac{-1}{17} & \frac{3}{17} \end{pmatrix}$$

Question # 6

Find, if possible, the multiplicative inverse of the given matrices

1. $A=\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{pmatrix}$

Solution:

Solution Since $\det(A)=\begin{vmatrix}1 & 2 & 3\\ 0 & 4 & 5\\ 1 & 0 & 6\end{vmatrix}=1(24-0)-2(0-5)+3(0-4)=24+10-12=22\neq 0$

Therefore, the given matrix is non-singular. So, the multiplicative inverse A^{-1} of the matrix A exists. The cofactors corresponding to the entries in each row are

$$C_{11}=\begin{vmatrix}4 & 5\\ 0 & 6\end{vmatrix}=24, \qquad C_{12}=-\begin{vmatrix}2 & 3\\ 0 & 6\end{vmatrix}=-12, \qquad C_{13}=\begin{vmatrix}2 & 3\\ 4 & 5\end{vmatrix}=-2$$

$$C_{21}=-\begin{vmatrix}0 & 5\\ 1 & 6\end{vmatrix}=5, \qquad C_{22}=\begin{vmatrix}1 & 3\\ 1 & 6\end{vmatrix}=3, \qquad C_{23}=-\begin{vmatrix}1 & 3\\ 0 & 5\end{vmatrix}=-5$$

$$C_{31}=\begin{vmatrix}0 & 4\\ 1 & 0\end{vmatrix}=-4, \qquad C_{32}=-\begin{vmatrix}1 & 2\\ 1 & 0\end{vmatrix}=2, \qquad C_{33}=\begin{vmatrix}1 & 2\\ 0 & 4\end{vmatrix}=4$$

Hence $$A^{-1}=\frac{1}{22}\begin{pmatrix}24 & -12 & -2\\ 5 & 3 & -5\\ -4 & 2 & 4\end{pmatrix}=\begin{pmatrix}\frac{24}{22} & \frac{-12}{22} & \frac{-2}{22}\\ \frac{5}{22} & \frac{3}{22} & \frac{-5}{22}\\ \frac{-4}{22} & \frac{2}{22} & \frac{4}{22}\end{pmatrix}=\begin{pmatrix}\frac{12}{11} & \frac{-6}{11} & \frac{-1}{11}\\ \frac{5}{22} & \frac{3}{22} & \frac{-5}{22}\\ \frac{-2}{11} & \frac{1}{11} & \frac{2}{11}\end{pmatrix}$$

2. $A=\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 5 & 7 & 9\end{pmatrix}$

Solution

Since $\det(A)=\begin{vmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 5 & 7 & 9\end{vmatrix}=1(45-42)-2(36-30)+3(28-25)=3-12+9=0$

Hence inverse does not exist.

Question # 7

For the matrices $A=\begin{pmatrix}3 & 5\\ 1 & 1\end{pmatrix}$ and $B=\begin{pmatrix}0 & 1\\ 4 & 1\end{pmatrix}$, evaluate *BA*?

Solution:

$$BA=\begin{pmatrix}0 & 1\\ 4 & 1\end{pmatrix}\begin{pmatrix}3 & 5\\ 1 & 1\end{pmatrix}=\begin{pmatrix}0\times3+1\times1 & 0\times5+1\times1\\ 4\times3+1\times1 & 4\times5+1\times1\end{pmatrix}=\begin{pmatrix}1 & 1\\ 13 & 21\end{pmatrix}$$

Question # 8

For the matrices $A=\begin{pmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{pmatrix}$ and $B=\begin{pmatrix}1 & 1\\ 0 & 1\\ 1 & 1\end{pmatrix}$, evaluate *AB*?

Solution:

$$AB=\begin{pmatrix}4 & 6\\ 10 & 15\\ 16 & 24\end{pmatrix}$$

Question # 9

For the matrices $A=\begin{pmatrix}1 & 6\\ 3 & 7\\ 5 & 9\end{pmatrix}$ and $B=\begin{pmatrix}5 & 0\\ 3 & 0\\ 1 & 0\end{pmatrix}$, evaluate *AB*?

Solution:

Multiplication is not possible as number of columns of first matrix is not equal to the number of rows of second matrix.

Practice Questions Lecture # 3

Question # 1

Which of the following is a non-linear equation?

a. $x_1 + x_2 + x_3 = 0$
b. $x_1 + \sqrt{3}\, x_2 - x_3 = 1$
c. $4x_1x_2 = 1$
d. $x_1 = x_2$

Solution:

$4x_1x_2 = 1$ is non-linear equation because there is product of variables involved. So part c is the correct option.

Question # 2

Which of the following is the correct Coefficient matrix for the system $\begin{matrix} x_1 - x_2 - x_3 = 6 \\ x_1 + 2x_2 = 8 \\ 9x_3 = 1 \end{matrix}$?

a. $\begin{bmatrix} 1 & -1 & 6 \\ 1 & 2 & 8 \\ 0 & 0 & 1 \end{bmatrix}$
b. $\begin{bmatrix} 1 & -1 & -1 \\ 1 & 2 & 0 \\ 0 & 0 & 9 \end{bmatrix}$
c. $\begin{bmatrix} 1 & -1 & -1 \\ 1 & 2 & 8 \\ 9 & 0 & 1 \end{bmatrix}$
d. $\begin{bmatrix} 1 & -1 & -1 & 6 \\ 1 & 2 & 0 & 8 \\ 0 & 0 & 9 & 1 \end{bmatrix}$

Solution:

Part b is the correct option.

Question # 3. Write a system of linear equations corresponding to the following augmented matrices

a. $\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 4 & 9 & 2 & 6 \\ 6 & 0 & 5 & 3 \end{bmatrix}$

Solution:

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{bmatrix} \qquad \begin{aligned} x_1 &= 2 \\ x_2 &= 1 \\ x_3 &= 4 \end{aligned}$$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 4 & 9 & 2 & 6 \\ 6 & 0 & 5 & 3 \end{bmatrix} \qquad \begin{aligned} x_1 + 2x_2 + 3x_3 &= 0 \\ 4x_1 + 9x_2 + 2x_3 &= 6 \\ 6x_1 + 5x_3 &= 3 \end{aligned}$$

Question # 4

Evaluate whether the following system is consistent or inconsistent.

a. $\begin{aligned} 2x + y &= 1 \\ -x - 7y &= 3 \end{aligned}$

b. $\begin{aligned} 2x + 2y &= 5 \\ -2x - 2y &= 3 \end{aligned}$

Solution:

a.

Since $\begin{aligned} 2x + y &= 1 \\ -x - 7y &= 3 \end{aligned}$

The augmented matrix of the system is

$$\begin{pmatrix} 2 & 1 & 1 \\ -1 & -7 & 3 \end{pmatrix}$$

Using row operation, it becomes

$\begin{pmatrix} 1 & -6 & 4 \\ -1 & -7 & 3 \end{pmatrix}$ using $R_1 + R_2$

$\begin{pmatrix} 1 & -6 & 4 \\ 0 & -13 & 7 \end{pmatrix}$ using $R_2 + R_1$

$\begin{pmatrix} 1 & -6 & 4 \\ 0 & 1 & -\frac{7}{13} \end{pmatrix}$ using $-\frac{1}{13}R_2$

$\begin{pmatrix} 1 & 0 & \frac{10}{13} \\ 0 & 1 & -\frac{7}{13} \end{pmatrix}$ using $R_1 + 6R_2$

This corresponds to the system:

$$x = 10/13$$
$$y = -7/13$$

Hence system is consistent.

b.

$$\begin{aligned} 2x + 2y &= 5 \\ -2x - 2y &= 3 \end{aligned}$$

The augmented matrix of the system is

$$\begin{pmatrix} 2 & 2 & 5 \\ -2 & -2 & 3 \end{pmatrix}$$

Using row operation, it becomes

$$\begin{pmatrix} 1 & 1 & 5/2 \\ -2 & -2 & 3 \end{pmatrix} \text{ using } 1/2R_1$$

$$\begin{pmatrix} 1 & 1 & 5/2 \\ 0 & 0 & 8 \end{pmatrix} \text{ using } R_2 + 2R_1$$

This corresponds to the system:

$$x + y = 5/2$$
$$0 = 8$$

Since $0 = 8$ is an mathematically incorrect, the system has no solution. Therefore it is an inconsistent system.

Question # 5

Is $x = 3,\ y = 1$ a solution of the following linear system? If not then give the reason.

$$x - 3y = 1$$
$$5x - 2y = 3$$

Solution:

Put the given values of x and y in the above equation:

$$3 - 3(1) = 0 \neq 1$$
$$5(3) - 2(1) = 13 \neq 3$$

Which shows that these values do not satisfies the above equations so it cannot be solution of the given linear system.

Question # 6

Determine whether the following system is consistent or inconsistent?

$$2x + 3y + 4z = 4$$
$$4x + 5y = 6$$
$$2y + 7z = 1$$

Solution:

$$\begin{aligned} & 2x+3y+4z=4 \\ \text{Since} \quad & 4x+5y=6 \\ & 2y+7z=1 \end{aligned}$$

The augmented matrix of the system is

$$\begin{pmatrix} 2 & 3 & 4 & 4 \\ 4 & 5 & 0 & 6 \\ 0 & 2 & 7 & 1 \end{pmatrix}$$

After applying row operations we get

$$\begin{pmatrix} 1 & 0 & 0 & 7/3 \\ 0 & 1 & 0 & -2/3 \\ 0 & 0 & 1 & 1/3 \end{pmatrix}$$

This corresponds to the system

$$x = 7/3$$
$$y = -2/3$$
$$z = 1/3$$

This shows that system is consistent.

Question # 7

Determine whether the following system is consistent or inconsistent?

$$\begin{aligned} & 3x-2y+3z=8 \\ & x+3y+6y=-3 \\ & 2x+6y+12z=-6 \end{aligned}$$

Solution:

Solution:

$$3x-2y+3z=8$$

Since $x+3y+6y=-3$

$$2x+6y+12z=-6$$

The augmented matrix of the system is

$$\begin{pmatrix} 3 & -2 & 3 & 8 \\ 1 & 3 & 6 & -3 \\ 2 & 6 & 12 & -6 \end{pmatrix}$$

After applying row operations we get

$$\begin{pmatrix} 1 & 0 & 21/11 & 18/11 \\ 0 & 1 & 15/11 & -17/11 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

As the last row is all zero which means that two of the planes formed by the equations in the system of equations are parallel, and thus the system of equations is said to have an infinite set of solutions.

Practice Questions for Lecture # 4

Question # 1

Identify if the following matrices are in echelon form or reduced echelon form or not in both forms. Also give the reason to justify your answer.

a. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Echelon Form

b. $\begin{bmatrix} 6 & 2 & 1 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 \end{bmatrix}$

Neither Echelon nor Reduced echelon forms because row with all zero elements is not the last row. All non-zero rows are above any rows of all zeros.

c. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Reduced echelon form because:

The leading entry in each nonzero row is 1.

Each leading 1 is the only nonzero entry in its column

d. $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{bmatrix}$

Echelon form:

Each leading entry of a row is in a column to the right of the leading entry of the row above it.

All entries in a column below a leading entry are zero

e. $\begin{bmatrix} -1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Echelon form:

f. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

Echelon form:

Question # 2

For the given linear system $\begin{bmatrix} 2 & 3 & 5 & 7 \\ 0 & 2 & 3 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, find the solution when free variable $z = 0$.

Solution:

Corresponding system of equation is

$2x + 3y + 5z = 7 \quad ---1$

$2y + 3z = 9 \qquad ---2$

as given $z = 0$

put this value of z *in equation* 2 *we get*

$2y + 0 = 9$

$y = 4.5$

put the value of y *and* z *in equation* 1

$2x + 3(4.5) + 0 = 7$

$2x + 13.5 = 7$

$2x = 7 - 13.5 = -6.5$

$x = 3.25$

Question # 3

Apply elementary row operations to transform the following matrix into the reduced echelon form:

$$A=\begin{bmatrix}3 & 6 & 9\\ 1 & 3 & 3\\ 4 & 8 & 5\end{bmatrix}$$

Solution:

$$A=\begin{bmatrix}3 & 6 & 9\\ 1 & 3 & 3\\ 4 & 8 & 5\end{bmatrix}$$

$1/3R_1$

$$\begin{bmatrix}1 & 2 & 3\\ 1 & 3 & 3\\ 4 & 8 & 5\end{bmatrix}$$

$R_2 - R_1$

$$\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 4 & 8 & 5\end{bmatrix}$$

$R_4 - 4R_1$

$$\begin{bmatrix}1 & 2 & 3\\ 0 & 1 & 0\\ 0 & 0 & -7\end{bmatrix}$$

$R_1 - 2R_2$

$$\begin{bmatrix}1 & 0 & 3\\ 0 & 1 & 0\\ 0 & 0 & -7\end{bmatrix}$$

$-1/7R_3$

$$\begin{bmatrix}1 & 0 & 3\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$R_1 - 3R_3$

$$\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

Question # 4

Apply elementary row operations to transform the following matrix into the echelon form**:**

$$A=\begin{bmatrix}1 & 3 & 2\\ 1 & 2 & 2\\ 4 & -1 & -4\end{bmatrix}$$

Solution:

$$A=\begin{bmatrix}1 & 3 & 2\\ 1 & 2 & 2\\ 4 & -1 & -4\end{bmatrix}$$

R_2-R_1, R_3-4R_1

$$\begin{bmatrix}1 & 3 & 2\\ 0 & -1 & -1\\ 0 & -13 & -12\end{bmatrix}$$

$-R_2$

$$\begin{bmatrix}1 & 3 & 2\\ 0 & 1 & 1\\ 0 & -13 & -12\end{bmatrix}$$

R_3+13R_2

$$\begin{bmatrix}1 & 3 & 2\\ 0 & 1 & 1\\ 0 & 0 & 1\end{bmatrix}$$

Question # 5

Apply elementary row operations to transform the following matrix into

a. Echelon form**:**
b. Reduced Echelon form

$$A=\begin{bmatrix}1 & -3 & -2 & 1\\ -6 & 2 & 5 & 3\\ 2 & 1 & 1 & -3\end{bmatrix}$$

Solution:

$$A=\begin{bmatrix}1 & -3 & -2 & 1\\ -6 & 2 & 5 & 3\\ 2 & 1 & 1 & -3\end{bmatrix}$$

add **6** *times the 1st row to the 2nd row*

add **–2** *times the 1st row to the 3rd row*

multiply the 2nd row by **–1 / 16**

add **–7** *times the 2nd row to the 3rd row*

multiply the 3rd row by **16 / 31**

$$\begin{bmatrix}1 & -3 & -2 & 1\\ 0 & 1 & 7/16 & -9/16\\ 0 & 0 & 1 & -17/31\end{bmatrix}$$

The matrix is in echelon form

To convert this in reduced echelon form

add -7/16 times the 3rd row to the 2nd row

add 2 times the 3rd row to the 1st row

add 3 times the 2nd row to the 1st row

$$\begin{bmatrix}1 & 0 & 0 & -33/31\\ 0 & 1 & 0 & -10/31\\ 0 & 0 & 1 & -17/31\end{bmatrix}$$

Question # 6

Find the general solution of a linear system whose augmented matrix is $\begin{pmatrix}1 & 1 & 3\\ 3 & 2 & 1\end{pmatrix}$

Solution:

Corresponding system of linear equation

$$x + y = 3$$
$$3x + 2y = 1$$

Multiply equation 1 *with* 3 *and subtract from* sec *ond*

$$3x + 3y = 9$$
$$3x + 2y = 1$$
$$\underline{- \quad - \qquad -}$$
$$0 \quad + y \; = 8$$

put the value of y in first equation

$$x + 8 \; = 3$$
$$x = 3 - 8 = -5$$

Question # 7

Find the general solution of a linear system whose augmented matrix is $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

Corresponding system of linear equation

$$x + 0y = 0$$
$$0x + y = 0$$

$$x = 0$$
$$y = 0$$

Practice Question Lecture # 5

Question:

Find a vector equation of the plane whose parametric equations are given below**:**

$x = 1 - 2t_1 + 3t_2$, $y = 4 - 5t_1 + 6t_2$, $z = 7 - 8t_1 - 9t_2$

Solution:

Since $x = 1 - 2t_1 + 3t_2$, $y = 4 - 5t_1 + 6t_2$, $z = 7 - 8t_1 - 9t_2$

To find the vector equation of the plane, we have to rewrite the three equations as the single vector equation as following:

$$(x, y, z) = (1 - 2t_1 + 3t_2, 4 - 5t_1 + 6t_2, 7 - 8t_1 - 9t_2)$$

$$\Rightarrow \quad = (1,4,7) + (-2t_1, -5t_1, -8t_1) + (3t_2, 6t_2, -9t_2)$$

$$\Rightarrow \quad = (1,4,7) + t_1(-2,-5,-8) + t_2(3,6,-9)$$

This is the required equation of the plane that passes through the point $(1,4,7)$.

Question:

Find a vector equation of the line in R^2 that passes through the point (1, 3) and is parallel to the vector $\vec{v} = (3,4)$

Solution:

Let $\vec{x} = (x, y)$ and $x_0 = (1,3)$, the vector equation of the line is determined as**:**

$$\vec{x} = x_0 + \vec{v}t$$

$$(x, y) = (1, 3) + (3, 4)t$$

$$(x, y) = \begin{bmatrix} 1 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} t$$

Question:

Write the vector $\vec{a} = (2,3)$ as a linear combination of the vectors $(1,0)$ and $(0,1)$.

Solution:

To write the vector $\vec{a} = (2,3)$ as a linear combination of the vectors $(1,0)$ and $(0,1)$, we need two scalars

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Question:

If $a_1 = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$, $a_2 = \begin{pmatrix} 1 \\ -5 \\ -2 \end{pmatrix}$ and $b = \begin{pmatrix} 3 \\ -4 \\ 6 \end{pmatrix}$. Determine whether b can be generated as a linear combination of a_1 and a_2?

Solution:

First we see the equation $x_1a_1 + x_2a_2 = b$ has a solution.

To answer this, reduce the augmented matrix $\begin{bmatrix} a_1 & a_2 & b \end{bmatrix}$ in echelon form:

$$\begin{bmatrix} 1 & 1 & 3 \\ 2 & -5 & -4 \\ -3 & -2 & 6 \end{bmatrix}$$

$R_2 - 2R_1, R_3 + 3R_1$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & -7 & -10 \\ 0 & 1 & 15 \end{bmatrix}$$

$-1/7R_2$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & 10/7 \\ 0 & 1 & 15 \end{bmatrix}$$

$R_1 - R_2, R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 11/7 \\ 0 & 1 & 10/7 \\ 0 & 0 & 95/7 \end{bmatrix}$$

We can write this system as**:**

$x_1 = 11/7$

$x_2 = 10/7$

$0.x_1 + 0.x_2 = 95/7$

Which cannot be true for any value of $x_1, x_2 \in \mathbb{R}$.

$\Rightarrow$ Given system has no solution.

$\therefore b \notin Span\{a_1, a_2\}$ i.e. vector b does not lie in the plane spanned by vectors a_1 and a_2.

$\Rightarrow$ b Cannot be generated as a linear combination of a_1 and a_2.

Question:

If $\vec{s} = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$ and $\vec{t} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$. Determine whether $\vec{b} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}$ is in $Span\{\vec{s}, \vec{t}\}$ or not?

Note: Show the complete steps.

Solution:

To show that $\vec{b}$ is in $Span\{\vec{s}, t\}$, we have to show that $\vec{b}$ can be written as a linear combination of $\vec{s}$ and $\vec{t}$. For this the linear system with augmented matrix $\begin{bmatrix} \vec{s} & \vec{t} & \vec{b} \end{bmatrix}$ should be consistent. So,

$$\begin{bmatrix} 2 & 1 & 5 \\ 8 & 4 & 15 \end{bmatrix}$$

$1/2R_1$

$$\begin{bmatrix} 1 & 1/2 & 5/2 \\ 8 & 4 & 15 \end{bmatrix}$$

$R_2 - 8R_1$

$$\begin{bmatrix} 1 & 1/2 & 5/2 \\ 0 & 0 & -5 \end{bmatrix}$$

The linear system is not consistent because $0 = -5$ is never true. So, $\vec{b}$ can not be written as a linear combination of $\vec{s}$ and $\vec{t}$. Therefore $\vec{b} \notin Span\{\vec{s}, \vec{t}\}$.

Solution of Practice Questions Lecture # 6

Question # 1:

Express $\vec{b}=\begin{bmatrix}2\\4\end{bmatrix}$ as a linear combination of $\vec{s}=\begin{bmatrix}1\\2\end{bmatrix}$ and $\vec{t}=\begin{bmatrix}3\\4\end{bmatrix}$

Solution:

To express $\vec{b}$ as a linear combination of $\vec{s}$ and $\vec{t}$, the scalars c_1 and c_2 should be determined as following:

$$\vec{b} = c_1\vec{s} + c_2\vec{t}$$
$$(2,4) = c_1(1,2) + c_2(3,4)$$
$$(2,4) = (c_1 + 3c_2, 2c_1 + 4c_2)$$

The following system of equations is obtained by equating the corresponding entries:

$$c_1 + 3c_2 = 2$$
$$2c_1 + 4c_2 = 4$$

multiply first equation by 2 *and subtract from* second

$$2c_1 + 6c_2 = 4$$
$$2c_1 + 4c_2 = 4$$
$$- \quad - \quad -$$
$$2c_2 = 0$$
$$c_2 = 0$$

Put in first equation

$$\therefore \quad c_1 = 2$$

Hence $(2,4) = 2(1,2) + 0(3,4)$

Question # 2

Determine whether the set of vectors $\vec{v}_1 = (1,2,-1)$, $\vec{v}_2 = (3,-3,4)$ and $\vec{v}_3 = (2,-1,-2)$ will span R^3?

Solution:

To show that the given vectors span R^3, choose a general vector from R^3 , let $\vec{u}=(u_1,u_2,u_3)\in R^3$ and determine if we can find scalars c_1 , c_2 , c_3 so that $\vec{u}\in R^3$ can be written as a linear combination of the given vectors. That is ,

$$\vec{u}=c_1\vec{v}_1+c_2\vec{v}_2+c_3\vec{v}_3$$

$$(u_1\ ,\ u_2\ ,\ u_3)=c_1(1\ ,2\ ,-1)+c_2(3\ ,-3\ ,4)+c_3(2\ ,-1\ ,-2)$$

The following system of equations is obtained by doing some vector algebra:

$$u_1=c_1+3c_2+2c_3$$

$$u_2=2c_1-3c_2-c_3$$

$$u_3=-c_1+4c_2-2c_3$$

Writing in matrix form

$$\begin{bmatrix}1 & 3 & 2\\ 2 & -3 & -1\\ -1 & 4 & -2\end{bmatrix}\begin{bmatrix}c_1\\ c_2\\ c_3\end{bmatrix}=\begin{bmatrix}u_1\\ u_2\\ u_3\end{bmatrix}$$

Now we determine if this system is consistent (i.e have at least one solution) for every possible choice of $\vec{u}=(u_1,u_2,u_3)\in R^3$. We know that the system will be consistent for every possible choice of $\vec{u}=(u_1,u_2,u_3)$ provided the coefficient matrix is invertible and that will be checked by computing the determinate of the coefficient matrix. So that

$$\det(A)=45\neq 0$$

Therefore the coefficient matrix is invertible and so this system will have solution for every choice of $\vec{u}=(u_1,u_2,u_3)$ which in turns determine that the given set of vectors span R^3.

Question # 3

Determine whether the set of vectors $\vec{v}_1=(1,3,1,1)$, $\vec{v}_2=(1,2,1,0)$ and $\vec{v}_3=(1,1,0,0)$ will span R^3?

Solution:

$$\begin{pmatrix}1 & 1 & 1\\ 3 & 2 & 1\\ 1 & 1 & 0\\ 1 & 0 & 0\end{pmatrix}\rightarrow\begin{pmatrix}1 & 0 & 0\\ 3 & 2 & 1\\ 1 & 1 & 0\\ 1 & 1 & 1\end{pmatrix}\rightarrow\begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 1\\ 0 & 1 & 0\\ 0 & 1 & 1\end{pmatrix}\rightarrow\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 2 & 1\\ 0 & 1 & 1\end{pmatrix}\rightarrow\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 1\end{pmatrix}$$

At this point, it is clear the rank of the matrix is 3, so the vectors span a subspace of dimension 3, hence they span R^3.

Question # 4

Determine whether the vectors $v_1 = (1, -1, 4)$, $v_2 = (-2, 1, 3)$, and $v_3 = (4, -3, 5)$ span R^3.

Solution:

$$\begin{bmatrix} 1 & -2 & 4 \\ -1 & 1 & -3 \\ 4 & 3 & 5 \end{bmatrix} \xrightarrow{R_2+R_1,\ R_3-4R_1} \begin{bmatrix} 1 & -2 & 4 \\ 0 & -1 & 1 \\ 0 & 11 & -11 \end{bmatrix} \xrightarrow{-R_1,\ 1/11R_3} \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix} \xrightarrow{R_3-R_2} \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

It is in Echelon form, where it can be seen that 3rd row does not contain any Pivot . so it cannot span R^3.

Question # 5

Let $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}$ and $\vec{z} = \begin{bmatrix} h \\ 2 \\ -3 \end{bmatrix}$. If $\vec{z}$ can be generated by $\vec{v}_1$ and $\vec{v}_2$, then find value of 'h'.

Solution:

Consider $\vec{c}_1\vec{v}_1 + \vec{c}_2\vec{v}_2 = \vec{z}$.

Therefore, $\vec{c}_1 \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} + \vec{c}_2 \begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} h \\ 2 \\ -3 \end{bmatrix}$

$$\vec{c}_1 - \vec{c}_2 = h \quad .....e.q.(1)$$
$$2\vec{c}_1 + c_2 = 2 \quad .....e.q.(2)$$
$$-3\vec{c}_1 + 4\vec{c}_2 = -3 \quad .....e.q.(3)$$

Multiply equation 2 by 3 and multiply equation 3 by 2 and then add both equations

$$6\vec{c}_1 + 3c_2 = 6$$
$$\underline{-6\vec{c}_1 + 8\vec{c}_2 = -6}$$
$$11\vec{c}_2 = 0$$
$$\vec{c}_2 = 0$$

Put the value of $\vec{c}_2$ in equation 3 we get

$$\vec{c}_1 = 1$$

Now put the value of $\vec{c}_1$ *and* $\vec{c}_2$

$$1 - 0 = h$$
$$h = 1$$

Practice Questions Lecture # 7 and 8

Question # 1

Determine whether the following system has a trivial solution or non-trivial solution**:**

$$x_1 - 2x_2 + x_3 = 0$$
$$3x_2 - 3x_3 = 0$$
$$x_1 - 3x_2 = 0$$

Solution:

The coefficient matrix is

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 3 & -3 & 0 \\ 1 & -3 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & -1 & 0 \end{bmatrix} 1/3R_2, R_3 - R_1$$

$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -2 & 0 \end{bmatrix} R_3 + R_2$$

The corresponding system of equations will be

$$x_1 - 2x_2 + x_3 = 0$$
$$x_2 - x_3 = 0$$
$$-2x_3 = 0$$

By backward substitution, we get $x_1 = 0,\ x_2 = 0,\ x_3 = 0$. Thus, the given system has only the trivial solution.

Question # 2

Solve the following system using the reduced echelon form**:**

$$x_1 + 4x_2 + 6x_3 = 0$$
$$-2x_1 - 5x_3 = 0$$
$$3x_2 - 7x_3 = 0$$

Solution:

Given system of equation could be translated into matrix form as

$$\begin{matrix} 1 & 4 & 6 \\ -2 & 0 & -5 \\ 0 & 3 & -7 \end{matrix}$$

add 2 times the 1st row to the 2nd row

$$\begin{matrix} 1 & 4 & 6 \\ 0 & 8 & 7 \\ 0 & 3 & -7 \end{matrix}$$

Multiply the 2nd row by 1/8

$$\begin{matrix} 1 & 4 & 6 \\ 0 & 1 & \frac{7}{8} \\ 0 & 3 & -7 \end{matrix}$$

add -3 times the 2nd row to the 3rd row

$$\begin{matrix} 1 & 4 & 6 \\ 0 & 1 & \frac{7}{8} \\ 0 & 0 & \frac{-77}{8} \end{matrix}$$

Multiply the 3rd row by -8/77

$$\begin{matrix} 1 & 4 & 6 \end{matrix}$$

$$\begin{bmatrix} & & 7 \\ 0 & 1 & \frac{}{8} \\ 0 & 0 & 1 \end{bmatrix}$$

add -7/8 times the 3rd row to the 2nd row

$$\begin{bmatrix} 1 & 4 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

add -6 times the 3rd row to the 1st row

$$\begin{bmatrix} 1 & 4 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

add -4 times the 2nd row to the 1st row

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question # 3

Check whether $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent or not?

where $\vec{v}_1 = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ and $\vec{v}_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

Solution:

The set S = $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ of vectors in R^3 is **linearly independent** if the only solution of

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = 0$$

is

$$c_1, c_2, c_3 = 0$$

Otherwise (i.e., if a solution with at least some nonzero values exists), S is **linearly dependent**.

With our vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, becomes:

$$c_1 \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Rearranging the left hand side yields

$$\begin{bmatrix} -2\,c_1 + 1\,c_2 + 2\,c_3 \\ 1\,c_1 + 0\,c_2 - 2\,c_3 \\ 3\,c_1 + 2\,c_2 + 1\,c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The matrix equation above is equivalent to the following **homogeneous system of equations**

$$\begin{aligned} -2\,c_1 + 1\,c_2 + 2\,c_3 &= 0 \\ 1\,c_1 + 0\,c_2 - 2\,c_3 &= 0 \\ 3\,c_1 + 2\,c_2 + 1\,c_3 &= 0 \end{aligned}$$

We now transform the coefficient matrix of the homogeneous system above to the reduced row echelon form to determine whether the system has

- the trivial solution only (meaning that S is linearly independent), or
- the trivial solution as well as nontrivial ones (S is linearly dependent).

$$\begin{bmatrix} -2 & 1 & 2 \\ 1 & 0 & -2 \\ 3 & 2 & 1 \end{bmatrix}$$

can be transformed by a sequence of elementary row operations to the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The reduced row echelon form of the coefficient matrix of the homogeneous system is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which corresponds to the system

$$\begin{aligned} 1\,c_1 &= 0 \\ 1\,c_2 &= 0 \\ 1\,c_3 &= 0 \end{aligned}$$

Since each column contains a leading entry (highlighted in yellow), then the system has only the trivial solution, so that the only solution is $c_1, c_2, c_3 = 0$.

Therefore the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is **linearly independent**.

Question # 4

Determine, without solving, whether the following set of vectors is linearly independent or dependent. $S = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 3 \end{bmatrix} \right\}$

Solution:

Using the same rule in previous question, with our vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$

$$c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 4 \end{bmatrix} + c_3 \begin{bmatrix} 4 \\ 6 \end{bmatrix} + c_4 \begin{bmatrix} 7 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Rearranging the left hand side yields

$$\begin{bmatrix} 1\,c_1 + 3\,c_2 + 4\,c_3 + 7\,c_4 \\ 2\,c_1 + 4\,c_2 + 6\,c_3 + 3\,c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The matrix equation above is equivalent to the following **homogeneous system of equations**

$$1\,c_1 + 3\,c_2 + 4\,c_3 + 7\,c_4 = 0$$

$2\,c_1 + 4\,c_2 + 6\,c_3 + 3\,c_4 = 0$

- We now transform the coefficient matrix of the homogeneous system above to the reduced row echelon form

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 2 & 4 & 6 & 3 \end{bmatrix}$$

can be transformed by a sequence of elementary row operations to the matrix

$$\begin{bmatrix} 1 & 0 & 1 & \frac{-19}{2} \\ 0 & 1 & 1 & \frac{11}{2} \end{bmatrix}$$

The reduced row echelon form of the coefficient matrix of the homogeneous system is

$$\begin{bmatrix} \mathbf{1} & 0 & 1 & \frac{-19}{2} \\ 0 & \mathbf{1} & 1 & \frac{11}{2} \end{bmatrix}$$

which corresponds to the system

$$\begin{aligned} 1\,c_1 \qquad + 1\,c_3 + (-19/2)\,c_4 &= 0 \\ 1\,c_2 + 1\,c_3 + (11/2)\,c_4 &= 0 \end{aligned}$$

The leading entries have been highlighted in yellow.

Since some columns do not contain leading entries, then the system has nontrivial solutions, so that some of the values c_1, c_2, c_3, c_4 solving the system of equation may be nonzero.

Therefore the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is **linearly dependent**.

Question # 5

Show that the columns of $A = \begin{bmatrix} 2 & 4 \\ 4 & -3 \end{bmatrix}$ are linearly independent.

Solution:

Vectors in R^2 is **linearly independent** if the only solution of

$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 = 0$

is $c_1, c_2 = 0$

$$c_1 \begin{bmatrix} 2 \\ 4 \end{bmatrix} + c_2 \begin{bmatrix} 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Rearranging the left hand side yields

$$\begin{bmatrix} 2\,c_1 + 4\,c_2 \\ 4\,c_1 - 3\,c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The matrix equation above is equivalent to the following **homogeneous system of equations**

$$\begin{aligned} 2\,c_1 + 4\,c_2 &= 0 \\ 4\,c_1 - 3\,c_2 &= 0 \end{aligned}$$

- We now transform the coefficient matrix of the homogeneous system above to the reduced row echelon form

$$\begin{bmatrix} 2 & 4 \\ 4 & -3 \end{bmatrix}$$

can be transformed by a sequence of elementary row operations to the matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The reduced row echelon form of the coefficient matrix of the homogeneous is

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

which corresponds to the system

$$\begin{aligned} 1\,c_1 &= 0 \\ 1\,c_2 &= 0 \end{aligned}$$

Since each column contains a leading entry (highlighted in yellow), then the system has only the trivial solution, so that the only solution is $c_1, c_2 = 0$.

Therefore columns of A are **linearly independent**.

❖ **JOIN FOR TIPS, UPDATES & FREE RESOURCES:**

Click Here to Join WhatsApp Channel

(https://whatsapp.com/channel/0029VbAVi0F4CrfaQrxmPg3H)

And

Follow my Channel for BOOST your Final-Term Exam:

https://whatsapp.com/channel/0029Vb6WUs0KwqSLZB7ERj1n

Best of luck for everything ahead!

May Allah bless you with success, strength, and peace.

Please **remember me in your precious prayers**

and I'll do the same, In Sha Allah.

— Qalb-e-Momin

For Paid Service You can contact us directly.

Reliable & Fast help guaranteed.

03174545120

Practice Questions

Chapter No. 09

Question no 1

Consider a linear transformation $T : R^2 \to R^2$ defined by $T(\vec{x}) = A\ \vec{x}$ then verify that $T(\vec{u} + \vec{v}) = T(\vec{u}) + T(\vec{v})$, where

$$A=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix},\ \vec{u}=\begin{bmatrix}2\\ 0\end{bmatrix} \text{ and } \vec{v}=\begin{bmatrix}6\\ -1\end{bmatrix}.$$

Solution

$$\vec{u}+\vec{v}=\begin{bmatrix}2\\ 0\end{bmatrix}+\begin{bmatrix}6\\ -1\end{bmatrix}=\begin{bmatrix}8\\ -1\end{bmatrix}$$

$$T(u+v)=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}\begin{bmatrix}8\\ -1\end{bmatrix}=\begin{bmatrix}24\\ -3\end{bmatrix}$$

now

$$T(\vec{u})=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}\begin{bmatrix}2\\ 0\end{bmatrix}=\begin{bmatrix}6\\ 0\end{bmatrix}$$

and

$$T(\vec{v})=\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}\begin{bmatrix}6\\ -1\end{bmatrix}=\begin{bmatrix}18\\ -3\end{bmatrix}$$

$$\Rightarrow\ T(\vec{u})+T(\vec{v})=\begin{bmatrix}6\\ 0\end{bmatrix}+\begin{bmatrix}18\\ -3\end{bmatrix}=\begin{bmatrix}24\\ -3\end{bmatrix}$$

$$\therefore\ T(\vec{u}+\vec{v})=T(\vec{u})+T(\vec{v}) \text{ verified.}$$

Question no 2

Consider a linear transformation $T: R^2 \to R^2$ defined by $T(\vec{x})=A\ \vec{x}$ then find the value of $\vec{x}$ such that $A\ \vec{x}=\vec{b}$, where

$$A=\begin{bmatrix}3 & 3\\ 1 & 4\end{bmatrix} \text{ and } \vec{b}=\begin{bmatrix}4\\ 6\end{bmatrix}.$$

Solution

We have to find $\vec{x}$ such that $T(x)=\vec{b}$ or $A\vec{x}=\vec{b}$. That is,

$$\begin{bmatrix}3 & 3\\ 1 & 4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}4\\ 6\end{bmatrix}$$

$$\Rightarrow 3x_1+3x_2=4$$

$$x_1+4x_2=6$$

Solving them simultaneously, it gives

$x_1 = -\frac{2}{9}$, $x_2 = \frac{14}{9}$

Hence $\vec{x} = \begin{bmatrix} -\frac{2}{9} \\ \frac{14}{9} \end{bmatrix}$

Note: $\vec{x}$ can also be calculated by reducing the augmented matrix of the system $Ax = \vec{b}$.

Question no 3

Consider a linear transformation $T : R^2 \to R^2$ defined by $T(\vec{x}, \vec{y}) = (2x - y, 0)$ then find $T(\vec{u})$, where $\vec{u} = (-1 , 4)$.

Solution

$$T(-1 , 4) = (-1 - 4 , 0)$$
$$= (-5 , 0)$$

Question no 4

Consider a linear transformation $T : R^2 \to R^2$ defined by

$$T(\vec{u}) = A\vec{u} = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 + 3x_2 \\ 2x_1 - x_2 \end{bmatrix}$$

Find $T(\vec{u})$, where

$\vec{u} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$.

Solution

$$T(\vec{u}) = A\vec{u} = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 + 3 \\ 2(3) + 1(-1) \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

Question no 5

Show that the transformation $L : R^3 \to R^2$ defined by $L(x,\ y,\ z) = (3y - z,\ x)$ is linear by proving $L(\vec{u} + \vec{v}) = L(\vec{u}) + L(\vec{v})$ and $L(c\,\vec{u}) = c\,L(\vec{u})$, *where c is scalar*.

Solution

Let $\vec{u} = (x_1,\ y_1,\ z_1)$ and $\vec{v} = (x_2, y_2, z_2)$, then

$$L(\vec{u} + \vec{v}) = L((x_1,\ y_1, z_1) + (x_2, y_2, z_2))$$

$$= L(x_1 + x_2,\ y_1 + y_2,\ z_1 + z_2)$$

$$= (3(y_1 + y_2) - (z_1 + z_2)\ ,\ x_1 + x_2)$$

$$= (3y_1 - z_1 + 3y_2 - z_2\ ,\ x_1 + x_2)$$

$$= (3y_1 - z_1,\ x_1) + (3y_2 - z_2, x_2)$$

$$= L(\vec{u}) + L(\vec{v})$$

$$L(c\,\vec{u}) = L(c\,(x_1,\ y_1, z_1))\ \ ;\ \textit{where c is scalar}$$

$$= L(c\,x_1,\ c\,y_1, c\,z_1))$$

$$= (3c\,y_1 - c\,z_1,\ c\,x_1)$$

$$= c\,(3y_1 - z_1,\ x_1)$$

$$= c\,L\,(x_1, y_1, z_1)$$

$$= c\,L\,(\vec{u})$$

Question no 6

Consider a linear transformation $L : R^2 \to R^2$ for which $L(i) = L\,(1\ ,\ 0) = (1\ ,\ 2)$ and $L(j) = L\,(0\ ,\ 1) = (3\ ,\ -2)$

are given. Find $L\,(-3\ ,\ 4)$, where $(-3\ ,\ 4) = -3i + 4\,j$.

Since $(-3\ ,\ 4) = -3i + 4\,j$,

$$
\begin{aligned}
L(-3\ ,\ 4) &= L(-3i + 4j) \\
&= -3L(i) + 4L(j) \\
&= -3(1\ ,\ 2) + 4(3\ ,\ -2) \\
&= (-3\ ,\ -6) + (12\ ,\ -8) \\
&= (\ 9\ ,\ -14)
\end{aligned}
$$

❖ **JOIN FOR TIPS, UPDATES & FREE RESOURCES:**

Click Here to Join WhatsApp Channel

(https://whatsapp.com/channel/0029VbAVi0F4CrfaQrxmPg3H)

And

Follow my Channel for BOOST your Final-Term Exam:

https://whatsapp.com/channel/0029Vb6WUs0KwgSLZB7ERj1n

Best of luck for everything ahead!

May Allah bless you with success, strength, and peace.

Please **remember me in your precious prayers**

and I'll do the same, In Sha Allah.

— Qalb-e-Momin

For Paid Service You can contact us directly.

Reliable & Fast help guaranteed.

03174545120

Practice Questions

Chapter No. 10 to 12

Question

Let $L : R^2 \to R^2$ be a linear operator defined by $L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x + 2y \\ x - y \end{bmatrix}$. Find the standard matrix A representing L *i.e*; $L = [L(\mathrm{e}_1), L(\mathrm{e}_2)]$.

Solution

The standard matrix A representing L is the 2×2 matrix whose columns are $L(e_1)$ and $L(e_2)$ respectively.

So,

$$L(e_1) = L\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$$
$$= \begin{bmatrix} 2+2(0) \\ 1-0 \end{bmatrix}$$
$$= \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Similarly

$$L(e_2) = L\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$$
$$= \begin{bmatrix} 2(0)+2(1) \\ 0-1 \end{bmatrix}$$
$$= \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

Therefore

$$A = \begin{bmatrix} 2 & 2 \\ 1 & -1 \end{bmatrix}$$

Question

Show that $L(\vec{x}) = A\vec{x}$ for the linear operator $L\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - y \\ 2x + y \end{bmatrix}$, where A is a standard matrix and

$L(e_1)=\begin{bmatrix}1\\2\end{bmatrix}$, $L(e_2)=\begin{bmatrix}-1\\1\end{bmatrix}$

Solution

To show that $L(x)=Ax$, we have to find standard matrix first which is of order 2×2 with columns are $L(e_1)$ and $L(e_2)$ respectively.
So,

Therefore, $A=\begin{bmatrix}1 & -1\\2 & 1\end{bmatrix}$

Now,

$$Ax=\begin{bmatrix}1 & -1\\2 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

$$\Rightarrow \quad Ax=\begin{bmatrix}x-y\\2x+y\end{bmatrix}=L(x)$$

Question

If $A=B$, then find the values of x and y, where $A=\begin{bmatrix}1 & 2 & x-2\\3 & 4 & 5\\6 & 5+y & 0\end{bmatrix}$ and

$B=\begin{bmatrix}1 & 2 & 1-x\\3 & 4 & 5\\6 & -y & 0\end{bmatrix}$.

Solution

Since both the matrices are equal, their corresponding entries will also be equal.

That is

$$x-2=1-x$$
$$x+x=1+2$$
$$2x=3$$
$$x=\frac{3}{2}$$

Also,

$$5+y=-y$$
$$2y=-5$$
$$y=\frac{-5}{2}$$

Question

If $A=\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}1\\ 6\end{bmatrix}$, then show that $(AB)^t=B^tA^t$.

Solution

$$AB=\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1\\ 6\end{bmatrix}$$
$$=\begin{bmatrix}2+0\\ 0+6\end{bmatrix}$$
$$=\begin{bmatrix}2\\ 6\end{bmatrix}$$

Now

$$(AB)^t=\begin{bmatrix}2 & 6\end{bmatrix}$$

$$B^tA^t=\begin{bmatrix}1 & 6\end{bmatrix}\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}2+0 & 0+6\end{bmatrix}$$
$$=\begin{bmatrix}2 & 6\end{bmatrix}$$
$$=(AB)^t$$

Therefore, $(AB)^t = B^t A^t$

Evaluate the determinant of the matrix $A = \begin{bmatrix} 2 & 3 & 1 & -2 \\ 0 & 4 & 6 & 8 \\ 0 & 2 & 2 & 3 \\ 0 & 1 & 1 & 3 \end{bmatrix}$.

The determinant of the given matrix is calculated as

$$\det(A) = \begin{vmatrix} 2 & 3 & 1 & -2 \\ 0 & 4 & 6 & 8 \\ 0 & 2 & 2 & 3 \\ 0 & 1 & 1 & 3 \end{vmatrix}$$

Expanding it for first column, it gives

$$= 2 \begin{vmatrix} 4 & 6 & 8 \\ 2 & 2 & 3 \\ 1 & 1 & 3 \end{vmatrix}$$

$$= 2[4(6-3) - 6(6-3) + 8(2-2)]$$

$$= 2[12 - 18 + 0]$$

$$= -12$$

So, $\det(A) = -12$

Question

If $A = \begin{bmatrix} -1 & 2 \\ 3 & -7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix}$, then calculate the inverse of AB (if exists) ; where $A^{-1} = \begin{bmatrix} -7 & -2 \\ -3 & -1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}$.

Solution

We know that $(AB)^{-1} = B^{-1}A^{-1}$

Prove it by using simple matrix multiplication.

To find the inverse of AB, use the matrix identity:

$$(AB)^{-1} = B^{-1}A^{-1}$$

$(AB)^{-1} = B^{-1}A^{-1}$

Given

$$A^{-1} = \begin{bmatrix} -7 & -2 \\ -3 & -1 \end{bmatrix}, \qquad B^{-1} = \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix}$$

we compute

$$(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 4 & -3 \\ -5 & 4 \end{bmatrix} \begin{bmatrix} -7 & -2 \\ -3 & -1 \end{bmatrix}$$

Now multiply:

$$\begin{aligned} (1,1) &= 4(-7) + (-3)(-3) = -28 + 9 = -19 \\ (1,2) &= 4(-2) + (-3)(-1) = -8 + 3 = -5 \\ (2,1) &= (-5)(-7) + 4(-3) = 35 - 12 = 23 \\ (2,2) &= (-5)(-2) + 4(-1) = 10 - 4 = 6 \end{aligned}$$

Therefore,

$$(AB)^{-1} = \begin{bmatrix} -19 & -5 \\ 23 & 6 \end{bmatrix}$$

$(AB)^{-1} = \begin{bmatrix} -19 & -5 \\ 23 & 6 \end{bmatrix}$

❖ **JOIN FOR TIPS, UPDATES & FREE RESOURCES:**
Click Here to Join WhatsApp Channel
(https://whatsapp.com/channel/0029VbAVi0F4CrfaQrxmPg3H)
And
Follow my Channel for BOOST your Final-Term Exam:
https://whatsapp.com/channel/0029Vb6WUs0KwqSLZB7ERj1n

Best of luck for everything ahead!
May Allah bless you with success, strength, and peace.
Please **remember me in your precious prayers**
and I'll do the same, In Sha Allah.
— *Qalb-e-Momin*
For Paid Service You can contact us directly.
Reliable & Fast help guaranteed.
03174545120

Practice Questions
Chapter No. 13 to 15

MCQs

What is the maximum possible number of pivots in a 3×5 matrix ?

(a) 1
(b) 2
(c) 3
(d) 4

Which of the following is true for the partitioned matrices $P=[X \quad Y]$ and $Q=[L \quad M]$, where sub-matrices X and Y have the same sizes as L and M respectively ?

(a) $P+Q=[XL \quad YM]$

(b) $P+Q=\begin{bmatrix} X+L \\ Y+M \end{bmatrix}$

(c) $P+Q=[X+L \quad Y+M]$

(d) $P+Q=\begin{bmatrix} XL \\ YM \end{bmatrix}$

Short

Question No. 1. Show that the following system of linear equations has unique solution **:**

$$2x_1 + 3x_2 = 4$$
$$4x_1 + 7x_2 = 8$$

Question No. 2. If a matrix A is decomposed into $L = \begin{bmatrix} 3 & 0 \\ 2 & -4 \end{bmatrix}$ and $U = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ by LU factorization , then find A **.**

Answer $A = \begin{bmatrix} 3 & -6 \\ 2 & -8 \end{bmatrix}$

Long

Question No. 3. If $A = \left[\begin{array}{cc|cc|c} 4 & 7 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 0 \\ \hline 0 & 0 & 3 & 4 & 0 \\ 0 & 0 & 2 & 3 & 0 \\ \hline 0 & 0 & 0 & 0 & 2 \end{array}\right]$ is a block diagonal matrix , then find A^{-1} **.**

Answer $A^{-1} = \begin{bmatrix} 2 & -7 & 0 & 0 & 0 \\ -1 & 4 & 0 & 0 & 0 \\ 0 & 0 & 3 & -4 & 0 \\ 0 & 0 & -2 & 3 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$

❖ **JOIN FOR TIPS, UPDATES & FREE RESOURCES:**
Click Here to Join WhatsApp Channel
(https://whatsapp.com/channel/0029VbAVi0F4CrfaQrxmPg3H)
And
Follow my Channel for BOOST your Final-Term Exam:
https://whatsapp.com/channel/0029Vb6WUs0KwqSLZB7ERj1n

Best of luck for everything ahead!
May Allah bless you with success, strength, and peace.
Please **remember me in your precious prayers**
and I'll do the same, In Sha Allah.
— *Qalb-e-Momin*
For Paid Service You can contact us directly.
Reliable & Fast help guaranteed.
03174545120

Practice Questions
Chapter No. 16 to 18

MCQs

If $A=\begin{bmatrix} 3 & b \\ 5 & d \end{bmatrix}$, then which of the following is the determinant of A ?

(a) $3d+5b$
(b) $3d-5b$
(c) $5d+3b$
(d) $5d-3b$

If the matrix $A=\begin{bmatrix} 2 & 2 & 5 \\ 2 & 2 & 1 \\ 2 & 2 & 7 \end{bmatrix}$, then which of the following is true about it?

(a) Its determinant is -2 .
(b) Its determinant is 0 .
(c) Its determinant is 4 .
(d) The information is not sufficient to calculate the determinant .

Short

Question No. 1. If the following system of linear equations is solved using the Jacobi's method, then construct the matrix M from the coefficient matrix of the following linear system:

$$10x_1 + 2x_2 + 3x_3 = 5$$
$$3x_1 + 8x_2 + x_3 = 8$$
$$x_1 + 3x_2 + 7x_3 = 6$$

Answer: $M = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

Question No. 2. If the first iteration $y_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ **,** then use the Jacobi Iteration to find next two iterations of the following system of equations**:**

$$10x_1 + x_2 = 30$$
$$x_1 + 12x_2 = 48$$

Answer: $y_2 = \begin{bmatrix} 2.7 \\ 3.8 \end{bmatrix}$, $y_3 = \begin{bmatrix} 2.6 \\ 3.8 \end{bmatrix}$

Question No. 3. Evaluate the determinant $\begin{vmatrix} 2 & 0 & 0 & 0 \\ 14 & 5 & 0 & 0 \\ -6 & 6 & -3 & 0 \\ 4 & 7 & -4 & -7 \end{vmatrix}$.

Answer: 210

Long

Question No. 4. Verify that $(\det A)(\det B) = \det(AB)$ **,** where $A = \begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$.

Answer:

Given

$$A = \begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix}, \qquad B = \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$$

First calculate the determinants.

For matrix A:

$$\det(A) = (4)(3) - (2)(5) = 12 - 10 = 2$$

For matrix B:

$$\det(B) = (3)(4) - (7)(2) = 12 - 14 = -2$$

Thus,

$$(\det A)(\det B) = (2)(-2) = -4$$

Now compute AB:

$$AB = \begin{bmatrix} 4 & 2 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 16 & 36 \\ 21 & 47 \end{bmatrix}$$

Now find its determinant:

$$\det(AB) = (16)(47) - (36)(21)$$

$$\det(AB) = 16 \cdot 47 - 36 \cdot 21$$

$$= 752 - 756 = -4$$

Therefore,

$$\boxed{(\det A)(\det B) = \det(AB) = -4}$$

Hence verified.