MIDTERM EXAMINATION
Spring 2010
MTH501- Linear Algebra (Session - 4)
Solved by well wisher
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Question No: 1 (Marks: 1) - Please choose one

If for a linear transformation the equation T(x)=0 has only the trivial solution then T is

- **one-to-one** **Page no : 113**
- onto

Question No: 2 (Marks: 1) - Please choose one

Which one of the following is an elementary matrix?

- $\begin{bmatrix} 1 & 0 \\ 0 & -3 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 & 1 \\ 0 & -3 & -3 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix}$
- $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ can be converted into $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ by R2+(-R1)=>R2

Ref: An elementary matrix is a matrix that results from applying a single elementary row operation to an identity matrix.

Question No: 3 (Marks: 1) - Please choose one

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$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Let and let k be a scalar .A formula that relates det kA to k and det A is

- ▶ det kA= k det A Page no : 100
- ▶ det kA = det (k+A)
- ▶ det k A = k^2 det A
- ▶ det kA = det A

Question No: 4 (Marks: 1) - Please choose one

The equation x = p + t v describes a line

- ▶ Through v parallel to p
- ▶ Through p parallel to v Page no : 55
- ▶ Through origin parallel to p

Question No: 5 (Marks: 1) - Please choose one

Determine which of the following sets of vectors are linearly dependent.

$$v_1=\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad v_2=\begin{bmatrix} 6 \\ 2 \end{bmatrix}$$

- ▶

$$v_1=\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \quad v_2=\begin{bmatrix} 6 \\ 2 \\ 1 \end{bmatrix}$$

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- ▶

$$v_1=\begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix}, \quad v_2=\begin{bmatrix} 10 \\ 4 \\ 6 \end{bmatrix}$$

- ▶

Question No: 6 (Marks: 1) - Please choose one

Every linear transformation is a matrix transformation

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► True
► False Page no : 100

Ref: Every matrix transformation is a linear transformation.

Question No: 7 (Marks: 1) - Please choose one

A null space is a vector space.

► **True** **page no: 270**
► False

Question No: 8 (Marks: 1) - Please choose one

If two row interchanges are made in succession, then the new determinant

► equals to the old determinant

► **equals to -1 times the old determinant** **Page no : 386**

Ref: A row interchange changes the sign of the determinant

Question No: 9 (Marks: 1) - Please choose one

The determinant of A is the product of the pivots in any echelon form U of A, multiplied by $(-1)^r$, Where r is

► the number of rows of A

► The number of row interchanges made during row reduction from A to U
► The number of rows of U

► The number of row interchanges made during row reduction U to A

Ref: $If\ there\ are\ r\ interchanges,\ then \det(A) = (-1)^r \det(U)$

Question No: 10 (Marks: 1) - Please choose one

If A is invertible, then det(A)=det(A^{-1})=1.

► **True**
► False

Question No: 11 (Marks: 1) - Please choose one

A square matrix $A = [a_{ij}]$ is lower triangular if and only if $a_{ij} = 0$ for

► $i > j$
► $i < j$

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- ▶ $i \leq j$
- ▶ $i = j$

Question No: 12 (Marks: 1) - Please choose one

The product of upper triangular matrices is

- ▶ lower triangular matrix
- ▶ upper triangular matrix
- ▶ **Diagonal matrix**

Question No: 13 (Marks: 1) - Please choose one

The matrix multiplication is associative

- ▶ **True** **Page no : 17**
- ▶ False

Question No: 14 (Marks: 1) - Please choose one

We can add the matrices of _____.

- ▶ **same order** **Page no : 13**
- ▶ same number of columns.
- ▶ same number of rows
- ▶ different order

Question No: 15 (Marks: 1) - Please choose one

By solving system of equations with iterative method, we stop the process when the entries in two successive iterations are _____.

- ▶ Repeat
- ▶ Large difference
- ▶ Different
- ▶ **Same** **Page no : 196**

Ref: Stop the process when the entries in two successive iterations are the same.

Question No: 16 (Marks: 1) - Please choose one

Jacobi's Method is _____ converges to solution than Gauss Siedal Method.

- ▶ **slow** **Page no : 201**
- ▶ fast
- ▶ better

Question No: 17 (Marks: 1) - Please choose one

A system of linear equations is said to be homogeneous if it can be written in the form _____.

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► AX = B
► **AX = 0** **Page no : 75**
► AB = X
► $X = A^{-1}$

Question No: 18 (Marks: 1) - Please choose one

The row reduction algorithm applies only to augmented matrices for a linear system.

► **True** **Page no : 43**
► False

Question No: 19 (Marks: 1) - Please choose one

Whenever a system has no free variable, the solution set contains many solutions.

► **True** **Page no : 75**
► False

Question No: 20 (Marks: 1) - Please choose one

Which of the following is not a linear equation?

► $x_1 + 4x_2 + 1 = x_3$
► $x_1 = 1$
► $x_1 + 4x_2 - \sqrt{2}x_3 = \sqrt{4}$
► $x_1 + 4x_1x_2 - \sqrt{2}x_3 = \sqrt{4}$ **Page no : 24**

Question No: 1 (Marks: 1) - Please choose one

If A is a 2×2 matrix, the area of the parallelogram determined by the columns of A is

► A^{-1}
► **det A** **Page no : 240**
► adj A

Question No: 2 (Marks: 1) - Please choose one

Cramer's rule leads easily to a general formula for

► **The inverse of an $n \times n$ matrix A** **Page no : 237**

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► the adjugate of an $n \times n$ matrix A

► the determinant of an $n \times n$ matrix A

Question No: 3 (Marks: 1) - Please choose one

The transpose of an lower triangular matrix is

► lower triangular matrix
► **Upper triangular matrix**

► diagonal matrix

Question No: 4 (Marks: 1) - Please choose one

The transpose of an upper triangular matrix is

► **Lower triangular matrix** **Page no : 216**
► upper triangular matrix

► diagonal matrix

Question No: 5 (Marks: 1) - Please choose one

Let A be a square matrix of order 3×3 with $\det(A) = 21$, then $\det(2A) =$

► **168** **$\det(2A)=2^3\det(A)=8*21=168$**
► 186
► 21
► 126

Question No: 7 (Marks: 1) - Please choose one

Let A be an $m \times n$ matrix. If for each b in $\mathbb{R}^m$ the equation Ax=b has a solution then

► **A has pivot position in only one row (may be this option is true)**

► Columns of A span $\mathbb{R}^m$
► Rows of A span $\mathbb{R}^m$

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Question No: 8 (Marks: 1) - Please choose one

Reduced echelon form of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$ is

- $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$ **Page no : 37**
- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

Question No: 6 (Marks: 1) - Please choose one

Col A is all of $\mathbb{R}^m$ if and only if

- the equation $Ax = 0$ has a solution for each b in $\mathbb{R}^m$
- the equation $Ax = b$ has a solution for each b in $\mathbb{R}^m$
- the equation $Ax = b$ has a solution for a fixed b in $\mathbb{R}^m$.

Question No: 7 (Marks: 1) - Please choose one

If $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ and $B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$, then the partitions of A and B

- are not conformable for block multiplication
- **are conformable for AB block multiplication** **Page no : 163**
- are not conformable for BA block multiplication

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Question No: 8 (Marks: 1) - Please choose one

Two vectors are linearly dependent if and only if they lie

► on a line parallel to x-axis

► **On a line through origin** **Page no : 87**

► on a line parallel to y-axis

Question No: 9 (Marks: 1) - Please choose one

The equation x = p + t v describes a line

► through v parallel to p

► **through p parallel to v** **repeat**

► through origin parallel to p

Question No: 10 (Marks: 1) - Please choose one

Let A be an $m \times n$ matrix. If for each b in $\mathbb{R}^m$ the equation Ax=b has a solution then

► **A has pivot position in only one row** **repeat**

► Columns of A span $\mathbb{R}^m$

► Rows of A span $\mathbb{R}^m$

Question No: 11 (Marks: 1) - Please choose one

$$x_1 - 2x_2 + x_3 = 8$$
$$2x_2 - 7x_3 = 0$$
$$-4x_1 + 3x_2 + 9x_3 = -6$$

Given the system the augmented matrix for the system is

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -7 \\ -4 & 3 & 9 \end{bmatrix}$$

►

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$$\begin{bmatrix} 1 & -2 & 1 & 0 \\ 0 & 2 & -7 & 8 \\ -4 & 3 & 9 & -6 \end{bmatrix}$$

▶

$$\begin{bmatrix} 1 & -2 & 1 \\ 0 & 2 & -8 \\ -4 & 5 & 9 \end{bmatrix}$$

▶

$$\begin{bmatrix} 1 & -2 & 1 & 8 \\ 0 & 2 & -7 & 0 \\ -4 & 3 & 9 & -6 \end{bmatrix}$$

▶

Question No: 12 (Marks: 1) - Please choose one

Consider the linear transformation T such that $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ is the matrix of linear transformation then $T \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ is

$$\begin{bmatrix} 10 \\ 4 \\ 2 \end{bmatrix}$$

▶

$$\begin{bmatrix} 1 \\ 0 \\ 9 \end{bmatrix}$$

▶

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$$\begin{bmatrix} 10 \\ 4 \\ 1 \end{bmatrix}$$

▶

$$\begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix}$$

▶

Question No: 13 (Marks: 1) - Please choose one

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 5 \qquad \begin{vmatrix} a & b & c \\ 3d & 3e & 3f \\ g & h & i \end{vmatrix}$$

If then will be

▶ **15** **Page no : 220**
▶ 45
▶ 135
▶ 60

Ref:

c. If one row of $\boldsymbol{A}$ is multiplied by k to produce $\boldsymbol{B}$, then $det\ \boldsymbol{B} = k \cdot det\ \boldsymbol{A}$.

Question No: 14 (Marks: 1) - Please choose one

For an n×n matrix $(A^t)^t$ =

▶ A^t
▶ **A** **Page no : 18**
▶ A^{-1}
▶ $(A^{-1})^{-1}$

Ref:

- The transpose of the transpose of a matrix is the matrix itself:

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Question No: 15 (Marks: 1) - Please choose one

Each Linear Transformation T from R^n to R^m is equivalent to multiplication by a matrix A of order

- ► **m*n** **Page no : 99**
- ► n´m
- ► n´n
- ► m´m

Question No: 16 (Marks: 1) - Please choose one

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$$

Reduced echelon form of the matrix is

- $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 1 \end{bmatrix}$
- ► $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}$
- ► $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$
- ► $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$
- ►

Question No: 17 (Marks: 2)

Find vector and parametric equations of the plane that passes through the origin of $\boldsymbol{R^3}$ and is parallel to the vectors $\boldsymbol{v_1}$ = (1, 2, 5) and $\boldsymbol{v_2}$ = (5, 0, 4).

Solution:

$$\boldsymbol{x} = t_1 \boldsymbol{v_1} + t_2 \boldsymbol{v_2}$$

X= (1, 2, 5).t_1+(5, 0, 4). t_2 This is vector equation of the plane.

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$$x = x_0 + a_1 t_1 + a_2 t_2$$

$$y = y_0 + b_1 t_1 + b_2 t_2$$

$$z = z_0 + c_1 t_1 + c_2 t_2$$

These are parametric equations of the plane.

X= t_1+5 t_2

Y= $2t_1$

Z= $5t_1$+4 t_2

Question No: 22 (Marks: 3)

Find that is invertible or not $T(X_1,X_2)=T(6X_1+8X_2 , 5X_1-8X_2)$

Solution:

T(6,8)=T(5, -8)

$$\begin{bmatrix} 6 & 8 \\ 5 & -8 \end{bmatrix} = -48 + 40 = -8$$

This is invertible.

Question No: 23 (Marks: 3)

Find the volume of parallelogram of the vertices (1,2,4) (2,4,-7) and (-1,-3,20).

Solution:
Let
A(1,2,4)
B(2,4,-7)
C(-1,-3,20)
Let's fix A

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A(1,2,4)

B(2,4,-7)

C(-1,-3,20)

$$AB = \begin{bmatrix} 2-1 \\ 4-2 \\ -7-4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -11 \end{bmatrix}$$

$$AC = \begin{bmatrix} -1-1 \\ -3-23 \\ 20-4 \end{bmatrix} = \begin{bmatrix} -2 \\ -26 \\ 16 \end{bmatrix}$$

Question No: 24 (Marks: 2)

Which of the following is true? If V is a vector space over the field F.(justify your answer)

(a) $\{x+y / x \in V, y \in V\} = V$

(b) $\{x+y / x \in V, y \in V\} = V \times V$

(c) $\{\lambda v / v \in V, \lambda \in F\} = F \times V$

Question No: 26 (Marks: 5)

Justify that $A^2=I$ if $A=\begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$, if and only $M^2=I$. justify your answer by portioned matrix of M

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$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -3 & 1 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -3 & 1 \end{bmatrix}$$

$$\text{M} = M^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+0 & 0+0 \\ 3-3 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = I$$

Question No: 21 (Marks: 2)

If a square idempotent matrix A is non singular then show that A is equal to the identity matrix I.

Answer:

let $A = I$

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

lets see whether it is non sin *gular or not ..*

$$|A| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1 \neq 0$$

hence it is obvious that A is non sin *gular while* $A = I$

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Question No: 23 (Marks: 3)

$$\begin{vmatrix} 1 & 2 & 3 \\ -4 & 5 & 6 \\ 7 & -8 & 9 \end{vmatrix}$$

Find

Answer:

Example3

Find the determinant of the matrix $A=\begin{bmatrix} 1 & 4 & 3 \\ 5 & 2 & 4 \\ 3 & 6 & 3 \end{bmatrix}$

Solution Given $A=\begin{bmatrix} 1 & 4 & 3 \\ 5 & 2 & 4 \\ 3 & 6 & 3 \end{bmatrix}$

$$|A|=\begin{vmatrix} 1 & 4 & 3 \\ 5 & 2 & 4 \\ 3 & 6 & 3 \end{vmatrix}$$

$$=1\begin{vmatrix} 2 & 4 \\ 6 & 3 \end{vmatrix}-4\begin{vmatrix} 5 & 4 \\ 3 & 3 \end{vmatrix}+3\begin{vmatrix} 5 & 2 \\ 3 & 6 \end{vmatrix}$$

$$=1(2\times 3-4\times 6)-4(5\times 3-4\times 3)+3(5\times 6-2\times 3)$$

$$=1(6-24)-4(15-12)+3(30-6)$$

$$=1(-18)-4(3)+3(24)$$

$$=-18-12+72$$

$$=42$$

Question No: 25 (Marks: 5)

Show that is invertible and find its inverse.

Answer:

$$A = \begin{bmatrix} I & 0 \\ A & I \end{bmatrix}$$

$$\det A = I^2 - 0$$

$$\det A = I^2$$

$$I^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\det A \neq 0$$

so A is invertible

$$A^{-1} = \begin{bmatrix} I & 0 \\ A & I \end{bmatrix}$$

$$A^{-1} = \frac{adjA}{\det A} = \frac{\begin{bmatrix} I & 0 \\ -A & I \end{bmatrix}}{I^2} = 1/I^2 \begin{bmatrix} I & 0 \\ -A & I \end{bmatrix}$$

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